Discrete Complex Images in Modeling Antennas Over, Below or Penetrating the Ground

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Abstract. In this paper discrete complex images (DCI) are used to obtain approximate, efficient and fast solution of Sommerfeld integrals that appear in the analysis of vertical electric dipole (VED) in presence of air-ground half-space. The results are used to model vertical antenna above, below or penetrating the ground using the moment method technique with triangular expansion functions. Thus, the time consuming direct numerical evaluation of the Sommerfeld integrals is completely or partially avoided.

1. INTRODUCTION

The Sommerfeld integrals are frequently encountered in a number of electromagnetic problems involving current elements in the presence of half-space, including various combinations of the parameters and different integral formulations. Being computationally inefficient to integrate exactly, Sommerfeld integrals have been studied extensively for the last 50 years and are still an objective of interest. Several numerical difficulties due to oscillations, divergent behavior and singularities of the integrands are usually involved, and the satisfactory solution of Sommerfeld integrals is still lacking. Many numerical and analytical techniques have been developed and published giving numerous exact and approximate solutions.

In this paper discrete complex images (DCI) are used to obtain approximate, efficient and fast solution of Sommerfeld integrals that appear in the analysis of vertical electric dipole (VED) in presence of air-ground half-space. The results show less then 3% difference from the exact solution for a wide frequency range and various parameters. By this, the time consuming direct numerical evaluation of the SI are completely or partially avoided. It may be applied very efficiently in many other scattering and antenna radiation problems, where the Green's function is used to formulate the integral equation that solved numerically by the moment method. The rigorous Sommerfeld theory is used for modeling antennas over, below or penetrating the air-ground interface. The mathematical model is based on the procedures from the moment methods [1-3]. The basic idea was introduced in [4-6] and our previous studies presented in [7].

2. SOMMERFELD INTEGRALS

Consider a VED of unit strength positioned above or within the ground (ground = medium 1, air = medium 2) (characterized by conductivity σ and relative permittivity ε_r at source point (x',y',z') and field observed in point (x,y,z). The effect of the air-ground interface presented by the reflected field from the interface is formulated as

Sommerfield integral V_{11} (source below and observer below the ground) and V_{22} (source above and observer above the ground) [8]:

$$V_{22} = \int_0^{\infty} \frac{2}{k_1^2\gamma_0 + k_0^2\gamma_1} \exp[-\gamma_0(z+z')] J_0(k_\rho \rho) k_\rho dk_\rho \quad \text{for } z>0 \text{ and } z'>0 \tag{1}$$

$$V_{11} = \int_0^{\infty} \frac{2}{k_1^2\gamma_0 + k_0^2\gamma_1} \exp[-\gamma_1 |z+z'|] J_0(k_\rho \rho) k_\rho dk_\rho \quad \text{for } z<0 \text{ and } z'<0 \tag{2}$$

where $\gamma_0 = \sqrt{k_\rho^{\ 2} - k_0^2}$ and $\gamma_1 = \sqrt{k_\rho^{\ 2} - k_1^2}$

and $k_0^2 = \omega^2 \mu_0 \varepsilon_0$, $k_1^2 = k_0^2 \underline{\varepsilon}_r$ and $\underline{\varepsilon}_r = \varepsilon_r - j\sigma/\omega\varepsilon_0$.

After some mathematical manipulations, and normalization which eliminates the k_0 dependence the integrals are rewritten as:

$$I_1 = \frac{1}{k_0} \int_0^{\infty} \frac{2u_0}{\underline{\varepsilon}_r u_0 + u_1} \frac{1}{u_0} \exp[-k_0 u_0 (z+z')] J_0(\lambda k_0 \rho) \lambda d\lambda \quad \text{for } z>0 \text{ and } z'>0 \tag{3}$$

$$I_2 = \frac{1}{k_0} \int_0^{\infty} \frac{2u_1}{\underline{\varepsilon}_r u_0 + u_1} \frac{1}{u_1} \exp[k_0 u_1 (z+z')] J_0(\lambda k_0 \rho) \lambda d\lambda \quad \text{for } z<0 \text{ and } z'<0 \tag{4}$$

where $u_0 = \sqrt{\lambda^2 - 1}$, and $u_1 = \sqrt{\lambda^2 - \underline{\varepsilon}_r}$.

In the paper only a brief description of the procedure to get the closed-form approximate solution of the integral I_2 using DCI will be presented.

3. DISCRETE COMPLEX IMAGES APPROXIMATION

The idea of DCI application was initiated in the papers [3,4]. Our approach has the same foundation but uses different path approximation on the complex λ plane (Fig.1a). The integration contour is truncated by a point TP of flexible position elsewhere on the complex λ plane (TP_1, TP_2,…,TP_n) along the original path in the first quadrant [3-4]. Because of the multivalue character of function u_1, there are four different choices for the values. To obtain the result of square roots the following conditions are introduced: $\mathrm{Re}(u_1)>0$, $\mathrm{Im}(u_1)>0$ (Fig.1a).

To obtain the closed-form solution of the integral I_2 it is necessary to transform the integrand to a form that is analytically inverse-transformable. Here, the integrand is expressed in terms of a finite sum of complex exponential functions of u_1:

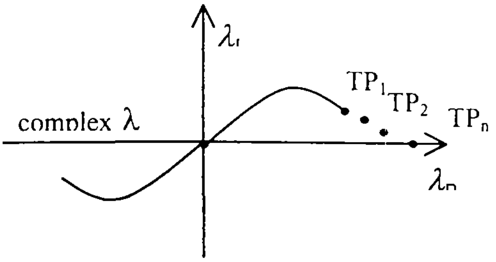

Fig.1 a) Integration path on the complex λ plane,

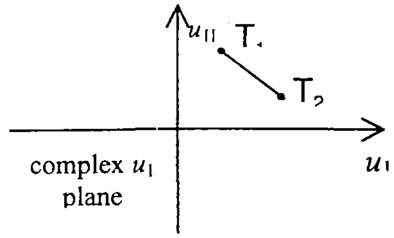

Fig.1 b) Integration path on the complex u_1 plane

$$\frac{2u_1}{\underline{\varepsilon}_r u_0 + u_1} \cong \sum_{i=1}^{N} a_i e^{b_i u_1} \tag{5}$$

where a_i and b_i are coefficients to be determined.
By this approximation the closed-form solution of the integral I_2 is obtained as

$$I_2 = \frac{1}{k_0}\sum_{i=1}^{N} a_i \int_0^{\infty} J_0(k_0\lambda\rho)\frac{1}{u_1}\exp^{b_i u_1}\exp^{k_0 u_1 (z+z')}\lambda d\lambda' = \frac{1}{k_0^2}\sum_{i=1}^{N} a_i \frac{e^{-jk_1 r_i}}{r_i} \tag{6}$$

$$r_i = \sqrt{\rho^2 + (z + z' + b_i / k_0)^2} \quad (i=1,2,..,N)$$

here ρ is the radial distance between the source and the field point. Each term in (4) represents a complex image.

In order to find the complex coefficient a_i and b_i the exponential approximation (Prony's method) is used.

$$\sum_{i=1}^{N} a_i e^{b_i u_1} \cong \sum_{i=1}^{N} A_i e^{B_i t} \tag{7}$$

Because of the complex type of the variable u_1 a linear function is used to transform it into a real variable t [3].

$$u_1 = c_1 t + c_2 \qquad \text{where } t \in [0, T_0] \tag{8}$$

where c_1 and c_2 are constants to be determined, and T_0 is the truncation point of the approximation process, which should be positive and greater than $\sqrt{(\varepsilon_r)}$.

Using the relation $u_1=\sqrt{(\lambda^2-\underline{\varepsilon}_r)}$, the starting point $\lambda=0$ corresponds to the point $T_1= u_1=j\sqrt{(\underline{\varepsilon}_r)}$ on the complex u_1 plane. Also, the ending point (TP) $\lambda=L_1+jL_2$ on the complex λ plane corresponds to the point $T_2= u_1=\sqrt{[(L_1+jL_2)^2-\underline{\varepsilon}_r]}$ on the complex u_1 plane as shown on Fig.1b. From (6), $t=0$ and $t=T_0$ give the starting and ending points the transformed path on the complex u_1 plane. Equation (8) can be written as

$$u_1(t) = \frac{\sqrt{(L_1 + jL_2)^2 - \underline{\varepsilon}_r}}{T_0}t + j\sqrt{\underline{\varepsilon}_r}(1 - \frac{t}{T_0}) \tag{9}$$

Approximation paths are shown on Fig.1b. The coefficients L_1 and L_2 are chosen arbitrarily. Using (5), (7) and (9) coefficients A_i and B_i are determined. After that, using (7) and (9) a_i and b_i are obtained.

$$a_i = A_i \exp^{-jb_i\sqrt{\varepsilon_r}} \quad \text{and} \quad b_i = \frac{B_i}{\sqrt{(L_1 + jL_2)^2 - \underline{\varepsilon}_r} - j\sqrt{\underline{\varepsilon}_r}} T_0 \tag{10}$$

Comparing to approximation path defined in [3], it can be shown that it is obtained if L_1^2=T_0^2+1 and L_2=0.

4. NUMERICAL RESULTS

Consider a vertical wire penetrating the air-ground interface with radius of 0.00025λ_0, extending from a height of 0.25λ_0 to a depth of -0.3λ_0 (test example presented in [1] which is usually used for comparison). The ground relative complex constant is $\underline{\varepsilon}_r$=16-j16. If the calculation is performed using n_g=30 segments for the ground stake, and n_a=25 segments for the upper part of the wire, the exact Sommerfeld integrals have to be calculated $\sum_{i=1}^{(n_g+n_a)} i$ times. Numerical integration can be reduced to $\sum_{i=1}^{10!} i$ times using the approximate solution,. It is shown that if vertical distance (z+z') is larger than 0.05λ_0 the approximate DCI solution can be used instead of the numerical exact integration. Errors grater that 3% are obtained for vertical distances less than 0.05λ_0.

As an example, this procedure is used for a set of frequencies from 8 MHz to $2\cdot10^3$ MHz. The results show errors less than 3% as presented in Table 1. The results are obtained using N=9 discrete complex images, with T_0=16, and constants L_1=1.273 and L_2=2.489.

Table 1. Comparison of the results: numerical integration of I_2 and DCI closed-form solution expressed in % error for arbitrary set of frequencies and vertical distances.

f (MHz)	z+z'=-0.05λ_0	z+z'=-0.01λ_0	z+z'=0.3λ_0
8	3%	0.55%	0.0073%
50	2.58%	0.42%	0.0044%
90	2.56%	0.41%	0.0040%
200	2.54%	0.40%	0.0042%
700	2.53%	0.398%	0.0039%
2000	2.53%	0.397%	0.0038%

5. CONCLUSION

In this paper discrete complex images (DCI) are used to obtain approximate fast solution of Sommerfeld integrals that appear in the analysis of VED in presence of air-ground half-space. The results are used to model vertical antenna above, below or penetrating the ground using the moment method technique with triangular expansion functions. Thus, the time consuming direct numerical evaluation of the Sommerfield integrals is completely or partially avoided. The results show difference from the exact solution less than 3%.

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