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Optimization of a Primary Circuit of the Nuclear Power Plant from the Vibration Point of View

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ABSTRACT

The primary circuit of the nuclear power plant (NPP) as a dynamical vibrating system can be disturbanced by various excitation including earthquake or pressure pulsation generated by main circulation pumps (MCP). Especially, unpleasant pulsation vibration growth can be caused by the small differences of revolutions between main circulation pumps of the individual coolant loops. This growth corresponds to the well known beats. The paper deals an approach to the improving and optimization of dynamical properties of the whole primary circuit system including the reactor and coolant loops under pressure pulsation.

KEYWORDS: vibration, nuclear reactor, piping system, finite element method, parameter optimization, modal synthesis

INTRODUCTION

The modal synthesis method is used for the assembling of whole mathematical model of the primary circuit. This approach means to perform a modal analysis of the uncoupled FEM+rigid body model of each individual subsystem (reactor, coolant loops) as the first step of operation sequence. The following step is assembling all subsystem models to the total primary circuit model respecting reduced number of eigenvalues (eigenfrequences and corresponding eigenvectors) of the individual subsystems. The number of respected eigenvalues should correspond to the highest frequency contained in excitating generalized forces. The compliance of couplings between individual subsystems was obtained by means of separated FEM analysis. Inverting compliance matrices we can come to stiffness matrices of the couplings. The use of the modal synthesis method is especially effective when the optimization parameters belongs to the couplings or only one subsystem. In the last case the modal analysis of only one uncoupled subsystem has to be performed in each optimization iteration and the modal parameters of the other subsystems and coupling matrices stay unchanged.

MATHEMATICAL MODEL

A scheme of the whole primary circuit (PC) of a nuclear power plant with Russian type of PWR-VVER 440/213 reactor is depicted in fig. 1.

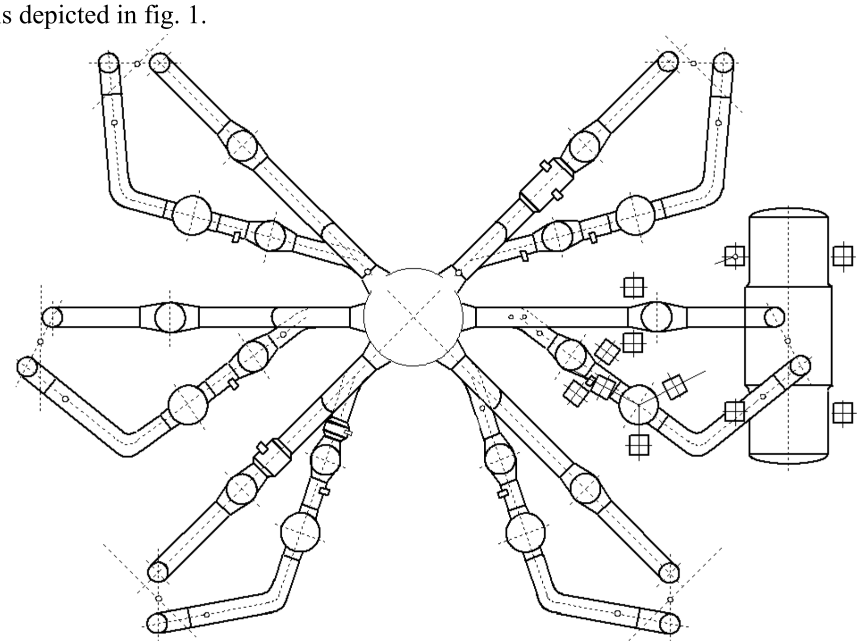

Fig. 1 The scheme of a primary circuit of the nuclear power plant with dampers

The PC consists of reactor and six coolant loops each of which contains a special viscous-elastic dampers (GERB) whose stiffness and damping parameter change will be gain of optimization. These GERBs are marked only at one coolant loop by squares in fig. 1. As a starting point of the whole PC model assembling is modal analysis of the individual isolated coolant loops without GERBs and the other connection. It means that GERBs are included into couplings as mentioned above in introduction. The mathematical model of the whole PC in modal coordinates of the uncoupled subsystems can be written in form [1]

$$\ddot{\boldsymbol{x}}(t)+\left[\boldsymbol{B}_0+\boldsymbol{B}(\omega)+\boldsymbol{V}^T\boldsymbol{B}_C\boldsymbol{V}\right]\dot{\boldsymbol{x}}(t)+\left[\Lambda+\boldsymbol{K}(\omega)+\boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V}\right]\boldsymbol{x}(t)=\boldsymbol{f}(t), \tag{1}$$

where $\boldsymbol{V}$ and Λ are modal and spectral matrices of the whole model consisting of uncoupled subsystems, respectively. These matrices have a form

$$\boldsymbol{V}=\begin{bmatrix} {}^m\boldsymbol{V}_0 & & & \\ & {}^m\boldsymbol{V}_1 & & \\ & & \ddots & \\ & & & {}^m\boldsymbol{V}_6 \end{bmatrix}\in \boldsymbol{R}^{n,m}, \Lambda=\begin{bmatrix} {}^m\Lambda_0 & & & \\ & {}^m\Lambda_1 & & \\ & & \ddots & \\ & & & {}^m\Lambda_6 \end{bmatrix}\in \boldsymbol{R}^{m,m}, m\in\sum_{j=0}^{6}m_j,\ n\in\sum_{j=0}^{6}n_j, \tag{2}$$

where ${}^m\boldsymbol{V}_j\in\boldsymbol{R}^{n_i,m_i}$ marks a modal matrix of the j-th subsystem consisting of the first m_j columns of the original uncoupled subsystem modal matrix, ${}^m\Lambda_j\in\boldsymbol{R}^{m_j,m_j}$ is a spectral matrix of the j-th subsystem consisting of the first m_j columns and rows of the original uncoupled subsystem spectral matrix. The total number of degree of freedom (DOF) is marked by n and total number of respected eigenvectors is m. The subscript j correspond to the subsystem (reactor ... 0, coolant loops ... 1,2, ... ,6). The symbols $\boldsymbol{B}(\omega)$ and $\boldsymbol{K}(\omega)$ correspond to the frequency dependent damping and stiffness matrices of GERBs, respectively, $\boldsymbol{B}_C$, $\boldsymbol{K}_C$ mark frequency independent damping and stiffness coupling matrices, respectively. The vector of transformed exciting forces generated by pressure pulsation of MCP has form

$$\boldsymbol{f}(t)=\sum_{k=0}^{6}\tilde{\boldsymbol{f}}_k e^{i\omega_k t},\ \tilde{\boldsymbol{f}}_0=\begin{bmatrix} {}^m\boldsymbol{V}_0^T\boldsymbol{f}_0^R \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix},\ \tilde{\boldsymbol{f}}_1=\begin{bmatrix} {}^m\boldsymbol{V}_0^T\boldsymbol{f}_1^R \\ {}^m\boldsymbol{V}_1^T\boldsymbol{f}_1 \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix},......,\tilde{\boldsymbol{f}}_6=\begin{bmatrix} {}^m\boldsymbol{V}_0^T\boldsymbol{f}_6^R \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \\ {}^m\boldsymbol{V}_6^T\boldsymbol{f}_6 \end{bmatrix}, \tag{3}$$

where ${}^m\boldsymbol{V}_j$ are modal matrices of isolated reactor $(j=0)$ and isolated coolant loops $(j=1\div 6)$, respectively, $\omega_j\,(j=1\div 6)$ is angular velocity of individual MCP and ω_0 is the average angular velocity which corresponds to the excitation of the reactor. Vectors $\boldsymbol{f}_j^R, j=0\div 6$ whose dimension is n_0 (number of reactor DOF) express effect of the pressure pulsation to the reactor, $\boldsymbol{f}_j$, $j=1\div 6$ (dimension n_j) express effect of the pressure pulsation of the coolant in the pipe elbows, inputs and outputs of the steam generators and MCP. A starting point for determination of force vector is pressure distribution in dependence on the distance from puslation source (output from MCP) which is represented by relation [2]

$$p(x,t)=\Delta p_j\left[\frac{\frac{2}{\pi}\sin\left(\frac{\pi}{L}x_i\right)}{\left(\frac{\omega_A}{\omega_j}\right)^2-1}+1-\frac{x_i}{L}\right]\cos\omega_j t,\quad j=1,2,...,6, \tag{4}$$

where Δp_j is the amplitude of the harmonically varying pressure generated by MCP of the j-th loop. Values of $\omega_j,\ j = 1 \div 6$ are generated as random variables with uniform probability density in the region $f_j \in \langle 24.90 \div 24.92 \rangle Hz$, where $f_j = \omega_j / 2\pi$. This narrow band of pump revolutions was obtained as a result of the long term measurements in EDU (Nuclear power station Dukovany, EDU is Czech abbreviation).

The following table contains the pressure pulsation frequencies of the individual MCP.

Tab. 1

coolant loop	**1**	**2**	**3**	**4**	**5**	**6**
$\delta_j\,[^\circ]$	45	0	315	225	180	135
$f_k\,[Hz]$	24.919	24.905	24.912	24.910	24.918	24.915

STEADY STATE VIBRATION OF THE SUBSYSTEMS AND THE COMPONENTS OF PC

A steady state of PC is determinated by particular solution of (1) respecting the excitation vector in form (3). As a result of small differences between exciting frequencies we can determine the frequency dependent matrices $\boldsymbol{B}(\omega)$ and $\boldsymbol{K}(\omega)$ for the average frequency ω_0. The particular solution

$$\boldsymbol{x}(t) = \sum_{k=0}^{6} \boldsymbol{x}_k e^{i\omega_k t} \tag{5}$$

has vectors of complex amplitudes

$$\boldsymbol{x}_k = \left\{ -\boldsymbol{I}\omega_k^2 + i\omega_k \left[\boldsymbol{B}_0 + \boldsymbol{B}(\omega_0) + \boldsymbol{V}^T \boldsymbol{B}_C \boldsymbol{V} \right] + \Lambda + \boldsymbol{K}(\omega_0) + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V} \right\}^{-1} \tilde{\boldsymbol{f}}_k, \tag{6}$$

where $\boldsymbol{I}$ is unity matrix. The modal displacement amplitude vectors $\boldsymbol{x}_k$ are decomposed into the subvectors $\boldsymbol{x}_k^{(j)},\ k = 0,1,...,6$ corresponding to the individual subsystems $j = 0,1,2,...,6$ and transformed back into configuration space of complex generalized displacements

$$\tilde{\boldsymbol{q}}_j(t) = \sum_{k=0}^{6} \boldsymbol{q}_k^{(j)} e^{i\omega_k t},\ \ \boldsymbol{q}_k^{(j)} = {}^m\boldsymbol{V}_j \boldsymbol{x}_k^{(j)}. \tag{7}$$

The time dependence of generalized displacements and velocities of subsystem points correspond to the real part of (7) as follows

$$\boldsymbol{q}(t) = \mathrm{Re}\{\tilde{\boldsymbol{q}}_j(t)\} = \sum_{k=0}^{6} \left[\mathrm{Re}\{\boldsymbol{q}_k^{(j)}\} \cos\omega_k t - \mathrm{Im}\{\boldsymbol{q}_k^{(j)}\} \sin\omega_k t \right]. \tag{8}$$

$$\begin{aligned} \dot{\boldsymbol{q}}(t) &= \sum_{k=0}^{6} \left[\mathrm{Re}\{i\omega_k \boldsymbol{q}_k^{(j)}\} \cos\omega_k t - \mathrm{Im}\{i\omega\ \boldsymbol{q}_k^{(j)}\} \sin\omega_k t \right] = \\ &= \sum_{k=0}^{6} -\omega_k \left[\mathrm{Im}\{\boldsymbol{q}_k^{(j)}\} \cos\omega_k t + \mathrm{Re}\{\boldsymbol{q}_k^{(j)}\} \sin\omega_k t \right] \end{aligned} \tag{9}$$

The transversal displacement

$$q_{i,j}^P(t) = \sqrt{\left(v_i^{(j)}(t)\right)^2 + \left(w_i^{(j)}(t)\right)^2} \tag{10}$$

and the transversal vibrating velocity

$$\dot{q}_{i,j}^{P}(t)=\sqrt{\left(\dot{v}_i^{(j)}(t)\right)^2+\left(\dot{w}_i^{(j)}(t)\right)^2} \tag{11}$$

in point i of the j-th loop are very important quantities. They can be calculated from the components of vectors of generalized displacements $\boldsymbol{q}_j(t)$ and velocities $\dot{\boldsymbol{q}}_j(t)$. The deformation energy caused by only one excitation vector $\widetilde{\boldsymbol{f}}_{\boldsymbol{k}} e^{i\omega_k t}$, $k=0,1,2,....,6.$ is the indicator expressing the vibrating level and influence of the excitation to one subsystem. It can be expressed by means of the quadratic form

$$E_k^{(j)}=\frac{1}{2}\left(\boldsymbol{q}_k^{(j)}\right)^H \boldsymbol{K}_j \boldsymbol{q}_k^{(j)}, \quad j,k\in\{0,1,....,6\}, \tag{12}$$

where j corresponds to the subsystem and k to the excitation frequency. Having expressed $\boldsymbol{q}_k^{(j)}$ in form (7) and respecting condition of orthogonality we can write

$$E_k^{(j)}=\frac{1}{2}\left(\boldsymbol{x}_{\boldsymbol{k}}^{(j)}\right)^H {}^m\Lambda_j \boldsymbol{x}_{\boldsymbol{k}}^{(j)}, \quad j,k\in\{0,1,2,...,6\}. \tag{13}$$

INFLUENCE OF THE VISCOUS-ELASTIC DAMPERS TO STEADY STATE RESPONSE OF PC

The approach mentioned above enabled to create a programme whose outputs include numerical and graphical results of dynamic deformation, displacement and velocity calculation of the inner components of the reactor and arbitrary node of the chosen coolant loop. The figures 2 and 3 show the chosen displacements and velocities of the PC without and with GERBs. Comparing these two figures it can be seen that reduction of the velocities and displacements caused by GERBs is approximatelly 75 percent.

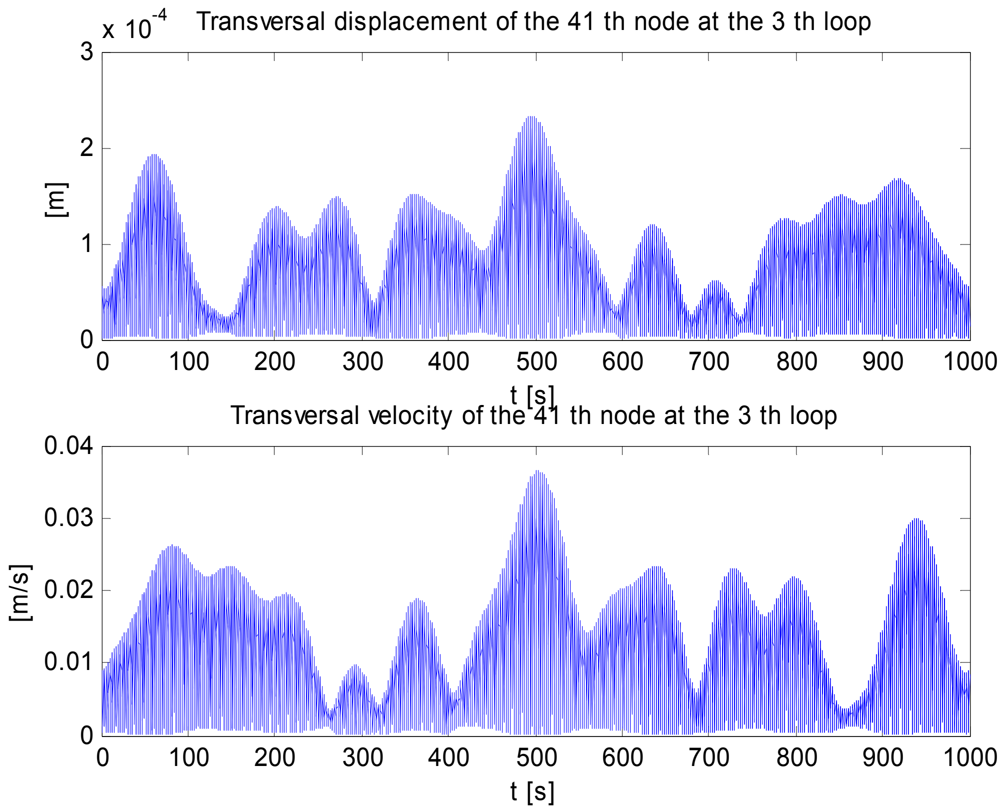

Fig. 2 Transversal displacement and velocity of 41 th node at the 3 th loop of the PC without GERBs

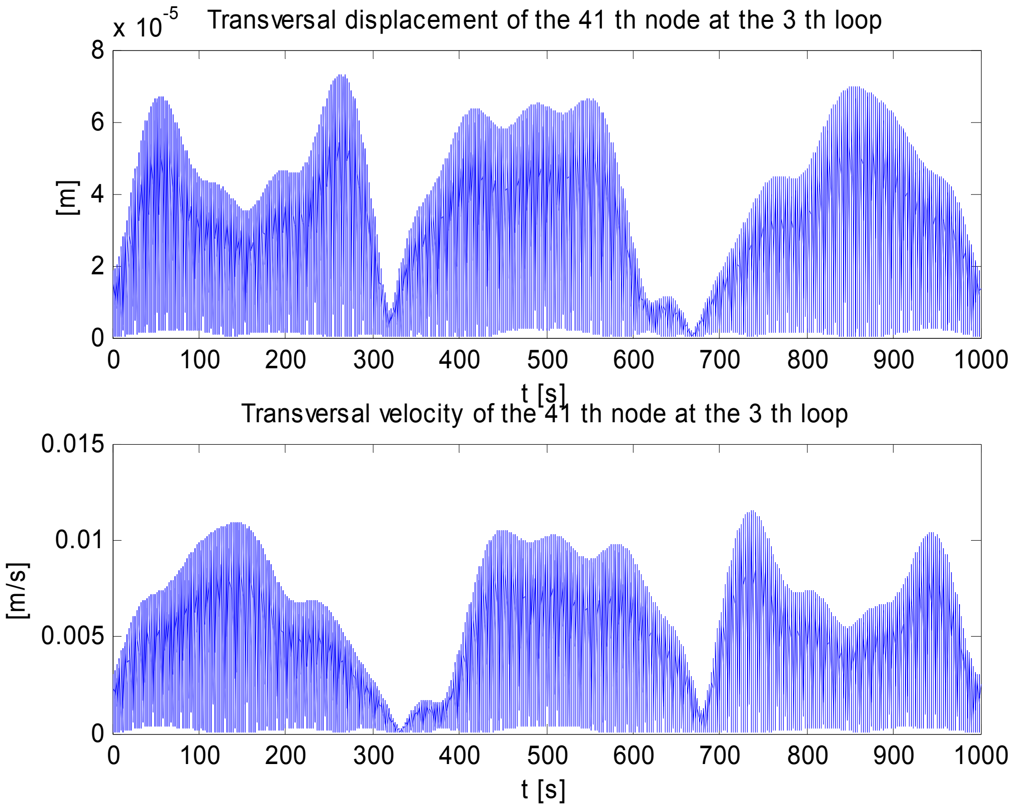

Fig. 3 Transversal displacement and velocity of 41 th node at the 3 th loop of the PC with GERBs

PARAMETER OPTIMIZATION OF VISCOUS-ELASTIC DAMPERS AND PIPE LOOP SUPPORT STFFNESSES FROM THE STEADY STATE RESPONSE POINT OF VIEW

The steady state of the displacements and velocities depicted in figs. 2 and 3 does not correspond to the periodical functions. It is caused by incommensurate exciting frequencies of MCPs. Let suppose an objective function in form

$$\psi(\overline{\boldsymbol{p}}) = \max_{j=1\div 6}\left\{\max_{t\in\langle 0,T\rangle}\left[\dot{q}_{i,j}^{P}(\overline{\boldsymbol{p}},t)\right]\right\}, \tag{14}$$

where $\dot{q}_{i,j}^{P}$ is transversal velocity of the i-th node at the j-th coolant loop which is governed by (11). The gain of optimization is to minimize the maximum of the transversal velocity of the i-th node in the time interval $t \in \langle 0,T\rangle$ over all PC coolant loops. As the optimization parameters were chosen relative stiffnesses of the GERBs of the same type to the values at the start of the optimization process

$$\overline{\boldsymbol{p}} = \left[\overline{k}_{100}^{H}, \overline{k}_{100}^{V}, \overline{k}_{30}^{H}, \overline{k}_{30}^{V}, \overline{k}_{20}^{H}, \overline{k}_{20}^{V}\right]^{T} \tag{15}$$

in the horizontal (superscript H) and vertical (superscript V) direction. The subscript marks type of the GERB which was used in EMO (Nuclear power plant Mochovce-Slovak Republic). The feasible domain of the optimization parameters is defined as follows

$$\bar{\boldsymbol{p}} = \langle 0.5;2 \rangle \bar{\boldsymbol{p}}_0, \quad \bar{\boldsymbol{p}}_0 = [1,1,1,1,1,1]^T. \tag{16}$$

The real values of optimized stiffnesses of the GERBs can be determined as follows

$$k_x^X = \bar{k}_x^X \left(k_x^X\right)_0, \; X = H,V, \quad x = 100, 30, 20, \tag{17}$$

where $\left(k_x^X\right)_0$ are stiffnesses at the start corresponding to the installed GERBs in EMO at the frequency 25 Hz. The horizontal and vertical stiffnesses $\left[10^6 \, N/m\right]$ and damping coefficients are depicted in tab. 2

Tab. 2

Type of GERB	$\left(k_x^H\right)_0$	$\left(k_x^V\right)_0$	$\left(b_x^H\right)_0$	$\left(b_x^V\right)_0$
VES-100	41.13	31.93	209.5	162.6
VES-30	17.12	14.14	87.2	72.0
VES-20	10.88	11.76	55.4	59.9

Because the ratio of the stiffnesses and damping coefficients are approximatelly the same

$$\frac{k_x^X}{b_x^X} \approx 196.3\left[s^{-1}\right], \; X = H,V, \; x = 100, 30, 20 \tag{18}$$

for all types of GERBs we have taken into account this ratio as constant in the optimization process. From this reason the dimension of the optimization parameter vector is 6 instead of 12.

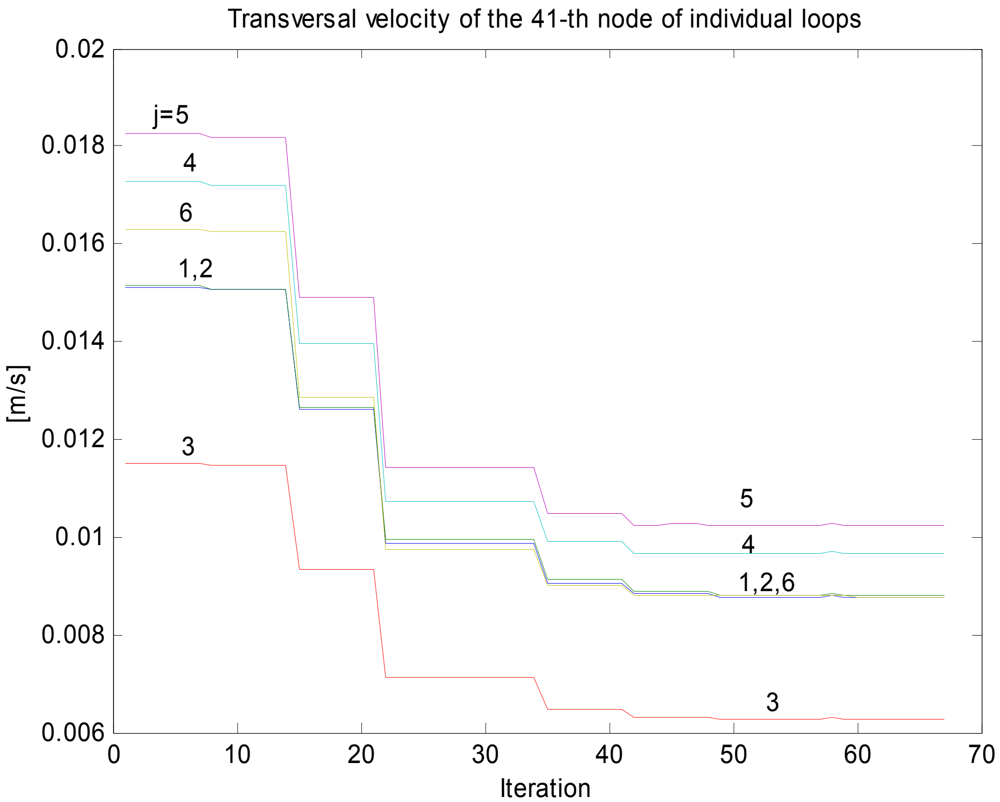

Fig. 4 The extreme transversal velocities of the 41-th nodes of the individual loops excited by sligthly different revolutions of MCPs

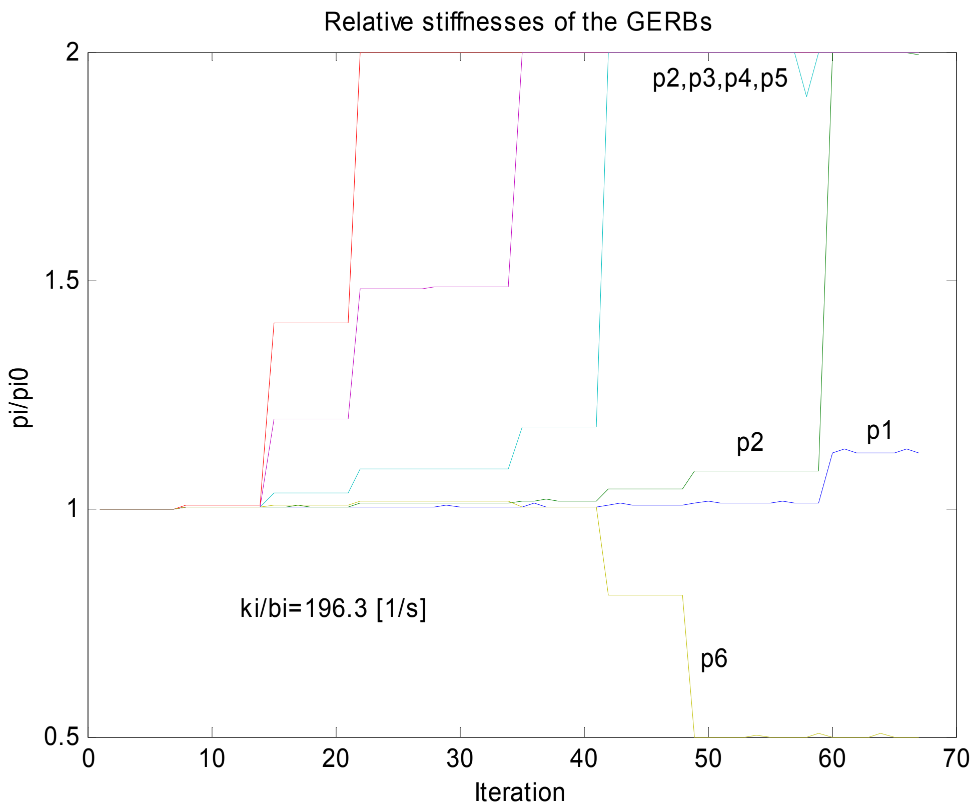

Fig. 5 The relative stiffnesses of the GERBs respecting sligthly different revolutions

The developed programme can be easily modificated for different types of the objective functions. The objective function

$$\psi(\bar{\boldsymbol{p}}) = \max_{j=1\div 6}\left\{\max_{t\in\langle 0,T\rangle}\sqrt{\dot{\xi}^2_{SGj}(\bar{\boldsymbol{p}},t)+\dot{\eta}^2_{SGj}(\bar{\boldsymbol{p}},t)+\dot{\zeta}^2_{SGj}(\bar{\boldsymbol{p}},t)}\right\} \tag{19}$$

expresses the maximal absolute velocity of the centre of gravity of the steam generator taken over all coolant loops.

The next parameters which can significantly effect the dynamic response are support stiffnesses of the pipe loops. To avoid doing modal analysis in each optimization iteration we can include these parameters into the coupling matrices. In this case the condensed model of the PC has a form

$$\ddot{\boldsymbol{x}}(t)+\left[\boldsymbol{B}_0+\boldsymbol{B}(\omega)+\boldsymbol{V}^T\boldsymbol{B}_C\boldsymbol{V}\right]\dot{\boldsymbol{x}}(t)+\left[\Lambda+\boldsymbol{K}_P+\boldsymbol{K}(\omega)+\boldsymbol{V}^T\boldsymbol{K}_C\boldsymbol{V}\right]\boldsymbol{x}(t)=\boldsymbol{f}(t), \tag{20}$$

where

$$\boldsymbol{K}_P = diag\left({}^m\boldsymbol{V}_j^T\boldsymbol{K}_j^P\,{}^m\boldsymbol{V}_j\right), \quad j=0,1,2,....,6 \tag{21}$$

is transformated stiffness matrix of all pipe supports which are to be optimized. When damping parameters of the supports should be optimized the matrices of the GERBs are parameter dependent too, it means

$$\boldsymbol{B}(\omega)=\boldsymbol{B}(\bar{\boldsymbol{p}},\omega),\ \boldsymbol{K}(\omega)=\boldsymbol{K}(\bar{\boldsymbol{p}},\omega)$$

$$\boldsymbol{K}_P = diag\left({}^m\boldsymbol{V}_j^T\boldsymbol{K}_j^P(\bar{\boldsymbol{p}})\,{}^m\boldsymbol{V}_j\right) \tag{22}$$

When we want to optimize simultaneously GERBs and pipe support stiffnesses under valves (HUAH) and in the 41-th nodes of loops the vector of optimization parameters has form

$$\bar{\boldsymbol{p}} = \left[\bar{k}_{100}^{H}, \bar{k}_{100}^{V}, \bar{k}_{30}^{H}, \bar{k}_{30}^{V}, \bar{k}_{20}^{H}, \bar{k}_{20}^{V}, \bar{k}_{HUAH}^{(1,2,3)}, \bar{k}_{41}^{(1,2,3)}, \bar{k}_{HUAH}^{(4,5,6)}, \bar{k}_{41}^{(4,5,6)}\right]^{T}. \tag{23}$$

We have taken into account the objective function in form (19). The changes of the maximal velocities of the SG (steam generator) centre of gravity with respect to optimization iteration are depicted in fig. 6.

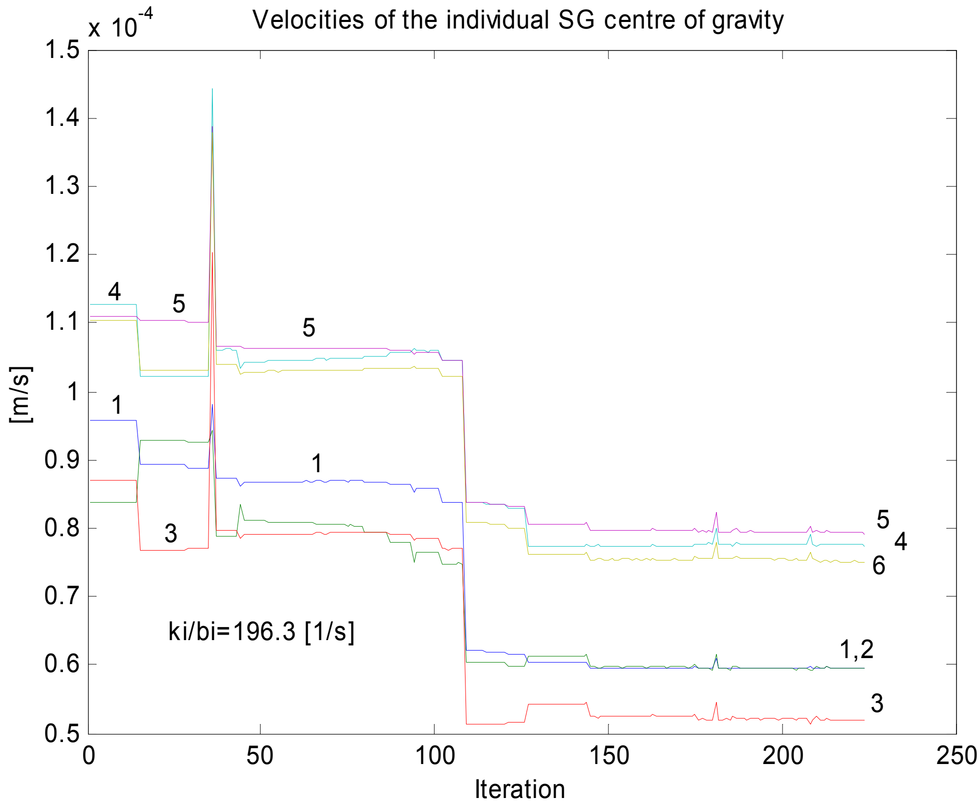

Fig. 6 Velocities of the SG centre of gravity of the individual loop $j = 1,2,....,6$ (system is excited by slightly different revolutions of MCPs)

CONCLUSION

The viscous-elastic dampers installed into the coolant loops of the PC significantly effect modal parameters corresponding to the loop mode shapes and increase corresponding eigenfrequencies. These dampers reduce the dynamic response of the loops but their influence to the response of the reactor and its components is small. The response of the coolant loops can be easily reduced by means of the optimization of the stiffness and damping GERB parameters of stiffnesses and dependent damping.

APPOLOGIES

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