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Probabilistic Procedure to Evaluate Integrity of Degraded Pipes Under Internal Pressure and Bending Moment

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ABSTRACT

The determination of critical crack sizes or permissible/allowable loading levels in pipes with degraded pipe sections (circumferential cracks) for the assurance of component integrity is usually based on deterministic approaches. Therefore along with numerical calculational methods (finite element (FE) analyses) limit load calculations, such as e.g. the "Plastic limit load concept" and the "Flow stress concept" as well as fracture mechanics approximation methods as e.g. the R-curve method or the "Ductile fracture handbook" and the R6-Method are currently used for practical application.

Numerous experimental tests on both ferritic and austenitic pipes with different pipe dimensions were investigated at MPA Stuttgart. The geometries of the pipes were comparable to actual piping systems in Nuclear Power Plants, both BWR as well as PWR. Through wall cracks and part wall through cracks on the inside surface of the pipes were considered. The results of these tests were used to determine the flow stresses used within the limit load calculations. Therefore the deterministic concepts assessing the integrity of degraded pipes are available

A new post-calculation of the above mentioned tests was performed using probabilistic approaches to assure the component integrity of degraded piping systems. As a result the calculated probability of failure was compared to experimental behaviour during the pipe test. Different reliability techniques were used for the verification of the probabilistic approaches.

KEY WORDS

Proof of integrity, small diameter piping, leak-before-break (LBB) behaviour, probabilistic procedure, integrity of piping, probability of failure, sensitivity study

INTRODUCTION

The operators of nuclear power plants are obligated, in conformity with the Nuclear Law and the "Safety criteria for nuclear power plants" [1] to guarantee particularly the required precautions against damage in accordance with current status of science and technology, in the construction and operation of the plant. This comprises, in particular, proof of integrity of the components and systems in operation by a comprehensive quality assurance during manufacture, installation and operation, a reliable monitoring of the operating conditions, the performance of in-service inspection at an appropriate level and the recording, evaluation and safety related utilisation of operating experience.

This situation is also reflected in the German Basis Safety Concept (BSC), Fig. 1, established into nuclear practice since 1979 [2, 3] as well as in development of the Guidelines of the Reactor Safety Commission (RSK) [4] and in the statements of the RSK on leak-before-break (LBB) behaviour and fracture exclusion. Moreover the requirements put forward in the RSK Guidelines have exerted some influence on the technical safety regulations of the nuclear technology committee (KTA) [5].

The procedural methods for the proof of integrity are orientated towards the basic ideas of the Basis Safety Concept (concept of fracture exclusion) [2, 3]. For the practical application of fracture exclusion under operational conditions therefore the "independent redundancies" contained in the Basis Safety Concept come to the fore, in which their weighting has to be decided from case to case [3, 6]. The independent redundancies compensate for possible uncertainties in the design, manufacture and operation (specifications, approvals, state of knowledge). Consequently for the real component to be assessed a unified total concept with a sufficient number of independent staged redundancies is available. By the chosen measures specific deficiencies in quality can be compensated for (principle of quality through production - basis safety). At this one of the most important aspects to assure the integrity for further operation is plant monitoring [7].

In connection with the proof of LBB behaviour and fracture exclusion, extensive research and development work at a national and international level has been carried out over the past 25 years for pressure bearing components in nuclear power plants, e.g. [8]. An important measure in conducting the demonstration of safety is the postulation of incipient cracks that have to be evaluated by fracture mechanics analyses on a deterministic basis. For this the minimum crack sizes that just could be found by non-destructive test methods are determined. Through this a fracture mechanics analysis already has its significance during the design in order to optimise the support concept by means of the minimum detectable defect size, - which is postulated for the future in-service inspection -, the specific loadings (operating and abnormal loadings) and the material properties of the components. Thus it is ensured that the determined critical crack sizes for subsequent operation are sufficiently large and hence no massive fracture (LBB behaviour or no leak) is to be expected. In assuring the existing quality of components in further operation the fracture mechanics evaluation is carried out using the minimum detectable crack size which is dictated by the testing method employed and the boundary conditions during the performance of the testing together with the recorded operational (by in-service monitoring) and faulted condition loadings assumed in accordance with the latest state of knowledge.

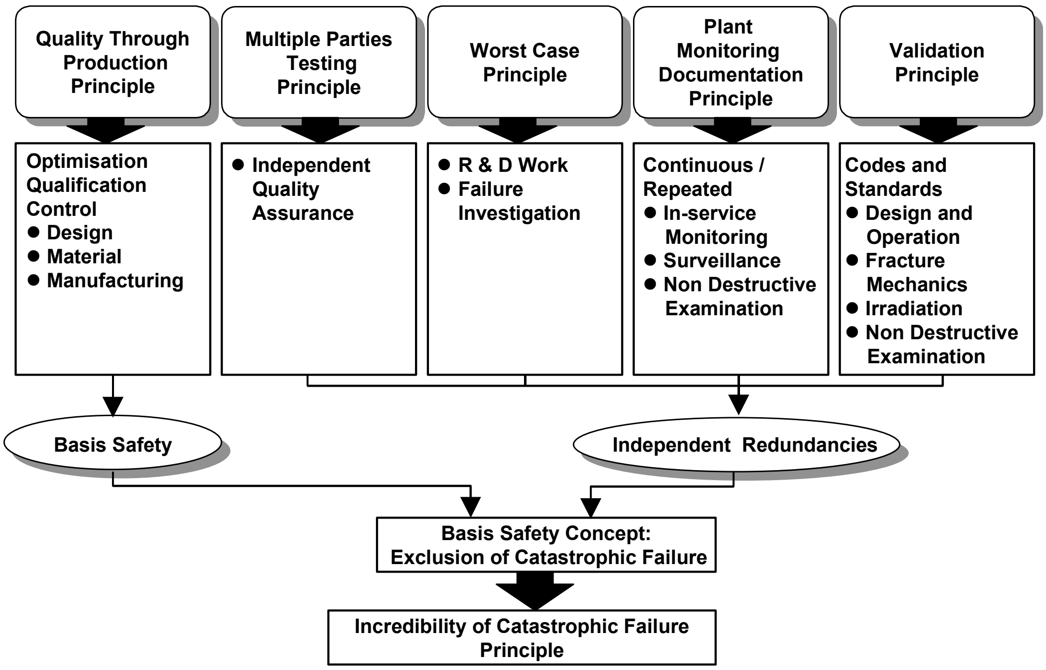

Figure 1: German Basis Safety Concept (BSC) schematically

In connection with the overall conception of the assurance of component integrity [9, 10], various research projects have been carried out at MPA Stuttgart to extend the state of knowledge of the load bearing and failure behaviour of austenitic and ferritic piping. The encompassed experimental, analytical and numerical investigations of the load bearing and failure behaviour (crack initiation, crack growth, leak-before-break behaviour) are called into play in the assessment of the integrity of pressurized piping. A new re-calculation of the above mentioned tests was performed using probabilistic approaches to assure the component integrity of degraded piping systems. As a result the calculated probability of failure was compared to experimental behaviour during the pipe test. Different reliability techniques were used for the verification of the probabilistic approaches.

POSSIBILITIES OF ANALYTICAL AND NUMERICAL ASSESSMENT OF DEGRADED PIPIING

For the determination of critical crack sizes or allowable loading levels in pipes with circumferential cracks for the assurance of component integrity, along with numerical methods (finite element (FE) analyses) in principal limit load calculations, such as e.g. the "Plastic limit load concept"(PLL) [11, 12] and the "Flow stress concept" (FSC) [13, 14] as well as fracture mechanics approximation methods as e.g. the GE/EPRI method or the "Ductile fracture handbook" [15, 16] and the R6-Method [17, 18] are currently called upon.

The PLL [19, 20] - with the assumption of a constant stress distribution over the crack bearing cross section of the pipe under consideration - is a common approximation method for the assessment of circumferential cracks in straight pipes. The FSC - according to bending theory assuming a linear stress distribution in the cracked cross section under consideration- is used mainly in Germany and some other countries [21]. For practical application usually the PLL or FSC are used to assure the integrity of pressurized components.

In both methods the maximum stress used (failure criterion) is described as the flow stress σ_f . The mechanical properties from the tensile tests are used for the determination of the yielding flow stress as the failure criterion. The stress distribution in the weakened cross section on which the methods are based are simplified model representations and are fitted to the test results by an appropriate choice of flow stress. Usually the flow stress is chosen that the maximum applied bending moment in the test is conservatively covered by the limit load calculation. The values to be adopted for the flow stress are verified by experimental investigations in the pipe bending tests as a function of the material (austenitic, ferritic), the pipe geometry and the loading [22]. For this no fracture mechanics material properties are used. Statements as to crack initiation and crack propagation are therefore not possible. Crack growth is not taken into account. A statement concerning the subsequent development of the failure is not possible. The R6-Method is based on the proposition that the failure of the component can extend from brittle fracture to fully plastic failure depending on the material condition. The transition between the two extremes is assumed to be continuous. Modified limit curves are given in [17] for various application cases, specifically for austenitic material a limit curve is given in [18]. The finite element (FE) method has established itself as a numerical procedure for the performance of two and three dimensional fracture mechanics structural analyses. Dependent on the assumption of the material behaviour (linear-elastic or elastic-plastic) the ascertaining of the stress intensity factors, the determination of J-integral values and displacement quantities (COD) as a function of the component, the crack geometries and the loading is possible. The calculations for cracked piping components under monotonically rising loading may be subdivided into FE analyses which neglect the ductile crack extension (fracture mechanics FE analysis) and those which take account of ductile crack extension (damage mechanics FE analysis). In principle with these procedures, under monotonically rising loading a statement on failure above and beyond crack initiation can also be made without explicit simulation of crack growth. This is only possible, if the crack resistance curves determined from laboratory specimens can be considered as representative of the cracked component. A precondition for this is that the pattern of multiaxiality of the stress state in the ligament of both specimen and component agree [23, 24]. If this is not the case then the application of fracture mechanics calculational methods to the determination of the load level at crack initiation is limited.

PROBABILISTIC CALCULATION

To perform the probabilistic calculations the XPiPE Code [25] was used extended by a probabilistic modul based on the Monte-Carlo method [26]. The Monte Carlo method is used in order to calculate a complex probability p to a specific condition. The calculations are performed by inserting random-generated numbers for all input parameter based on their statistical distributions. A more detailed description of the probabilistic modul used within the XPiPE Code is pointed out in [27]. The theoretical point of view on the case of multivariate statistics, taking also into account the correlations of input parameters is dealed with in [28, 29].

EXPERIMENTAL INVESTIGATIONS

Material

The pipes of nominal diameters DN50, DN80 and DN300 were fabricated of the austenitic material X 6 CrNiNb 18 10 (Material No. 1.4550) and those of nominal diameter DN200 from the material X 10 CrNiTi 18 9 (Material No. 1.4541). In addition pipes of nominal diameters DN80 with ferritic material 15 Mo 3 (Material No. 1.5415) were available. The characteristic strength, ductility and fracture mechanics properties were determined, Tab. 1. All the values are higher compared to the minimum guaranteed values called for in the KTA nuclear safety standard [5].

Table 1: Material properties at room temperature

Pipe	Material	Yield Strength $R_{p0.2}$ (MPa)	Yield Strength $R_{p1.0}$ (MPa)	Ultimate Tensile Strength R_m (MPa)	Young's Modulus E (MPa)	Reduction of Area Z (%)	Elongation A5 (%)	J-value J_i (N/mm)
DN300	X 10CrNiNb 18 9							
	Base metal	240-265	277-288	562-591	197 000	63.6-77.3	53.0-68.1	213-361
	Weld metal	440-444	471-480	631-639	170 500	46.8-57.1	30.9-42.3	80-117
DN200	X 10CrNiTi 18 9							
	Base metal	211-243	249-286	556-601	197 800	75-80	53-66	302-398
	Weld metal	437-496	488-533	669-720	196 600	38-58	23-46	73-146
DN80	X 10CrNiNb 18 9							
	Base metal	240-277	324-332	584-654	198 500	79-81	57-58.5	229-258
	Weld metal							288-317
DN80	15 Mo 3							
	Base metal	319-336		492-495	212 600	68-69	30.5-43	132-160
	Weld metal							126-128
DN50	X 10CrNiNb 18 9							
	Base metal	256-263	297-309	634-638	191 300	77-78	55.5-56	229-258
	Weld metal							288-317

Results of the Pipe Tests

Loading of the test pipes was achieved by internal pressure and a superimposed external bending moment using bending rigs specially made for these tests. The bending moment was imposed to the test pipes through a lever arm and extension pipes by means of two double acting (push-pull) hydraulic cylinders. The moment loading was applied quasi statically increasing monotonically (pipes without defects), quasi statically increasing monotonically with inserted fatigue cycling (principally on DN50 and DN80 pipes) and displacement controlled with partial unloading to determine the component J_R -curve (DN200 and DN300 pipes) up to the attainment of the of the maximum possible hydraulic ram stroke, Tab. 2.

Table 2: Test matrix

Pipe Dimensions			Material	Crack		Loading	
Nom. Diameter	Outer Diameter (mm)	Wall Thickness (mm)		Depth (a/s)	Length 2α (°)	Internal Pressure p_i (MPa)	External Bending Moment
DN300	331	32.1	X 10 CrNiNb 18 9	0.5 – 1.0	60 - 120	16	Quasi static
DN200	219.1	14.2	X 10 CrNiTi 18 9	0.5 – 1.0	120 - 270	7	Quasi static
DN80	88.9	8.8	X 10 CrNiNb 18 9	0.25 – 1.0	60 - 120	16	Quasi static
DN80	88.9	8	15 Mo 3	0.5 – 1.0	60	8	Quasi static
DN50	60.3	8.8	X 10 CrNiNb 18 9	0.25 – 1.0	60 - 120	16	Quasi static

Provided that crack initiation occurred at all this took place each time clearly before the theoretical collapse load or before the maximum loading attained in the test. Crack initiation was determined by potential probe measurements and photographic records.

- Pipes of DN50 nominal diameter (austenitic material): In tests with defect dimensions a/s=0.25 with $2\alpha=120^O$ and a/s=0.75 with $2\alpha=60^O$ no crack initiation could be determined. In the remaining tests even with circumferential slits (a/s=1) only slight stable crack growth occurred.
- Pipes of DN80 nominal diameter (austenitic material): In tests with defect dimensions a/s=0.25 with $2\alpha=120^O$ no crack initiation could be determined. In the remaining tests even with circumferential slits (a/s=1) only stable crack growth occurred.
- Pipes of DN80 nominal diameter (ferritic material): Stable crack growth occurred at times following crack initiation. At no time there were indications of unstable crack propagation present
- Pipes of DN200 and DN300 nominal diameter (austenitic material): Stable crack growth occurred at times following crack initiation. No indications of unstable crack propagation were found.

RESULTS OF THE PROBABILISTIC CALCULATION

To estimate the loading (moment) at crack initiation the R6-Method extended by a probabilistic modul was used [25, 26]. All 9 input parameters H_i ($i \leq n=9$) for the post-calculation of 15 pipe tests with DN200 and DN300 are shown for 6 out of 15 in Tab. 3 and 4.

To check the deviation of the analytical results developed by a deterministic R6-Method calculation the mean values are used. The results are most suitable for estimating the loading at initiation, Fig. 2. The maximum deviations are ±50 %. The mean deviation is only +3 %. This indicates that the prediction of the loading moment at crack initiation is unbiased.

Table 3: Values of the input parameters for tests of DN300 (material X 10CrNiNb 18 9)

Input parameter H_i	Mean value	Standard deviation	Min. guaranteed values	Distribution
Yield strength (MPa)	254	10.5	205	log. normal
Tensile strength (MPa)	608	29.5	500-750	log. normal
Fracture Toughness K_{IJ} (N/mm)	247	19,7		normal
Outer diameter (mm)	331,2	1,2		normal
Wall thickness t (mm)	32,2	0,15		normal
Crack depth a/t (Test 1)	1			normal
Crack depth a/t (Test 2)	1			normal
Crack depth a/t (Test 3)	0.5			normal
Crack depth a/t (Test 4)	0.5			normal
Crack angle α (°) (Test 1)	30	1,5 (estimate)		normal
Crack angle α (°) (Test 2)	60	3 (estimate)		normal
Crack angle α (°) (Test 3)	60	3 (estimate)		normal
Crack angle α (°) (Test 4)	30	1,5 (estimate)		normal
Internal pressare p_i (MPa)	16			constant

Then a sensitivity study was performed in order to set up a ranking of influences among the major input paramerters H_i. To describe the sensitivity of the i^{th} input parameter the amplification ratio V_i is defined as follows:

$$V_i = \frac{p_f\left(\overline{H_i} \pm 2s_i, H_j\right)}{p_f(H_i, H_j)} \quad j \neq i,\ j \leq n \text{ and } p_f(H_i, H_j) \equiv 0{,}01$$

Table 4: Values of the input parameters for tests of DN200 (material X 10CrNiTi 18 9)

Input parameters H_i	Mean value	Standard deviation	Min. guaranteed values	Distribution
Yield strength (MPa)	226	16.4	200	log. normal
Tensile strength (MPa)	581	13.2	500-730	log. normal
Fracture Toughness K_{IJ} (N/mm)	273	17,7		normal
Outer diameter (mm)	218,1	0,8		normal
Wall thickness t (mm)	14,7	0,1		normal
Crack depth a/t (Test 5)	0,46			
Crack depth a/t (Test 6)	1			
Crack length α (°) (Test 5)	60	3 (estimate)		normal
Crack length α (°) (Test 6)	30	1.5 (estimate)		normal
Internal pressare p_i (MPa)	16			constant

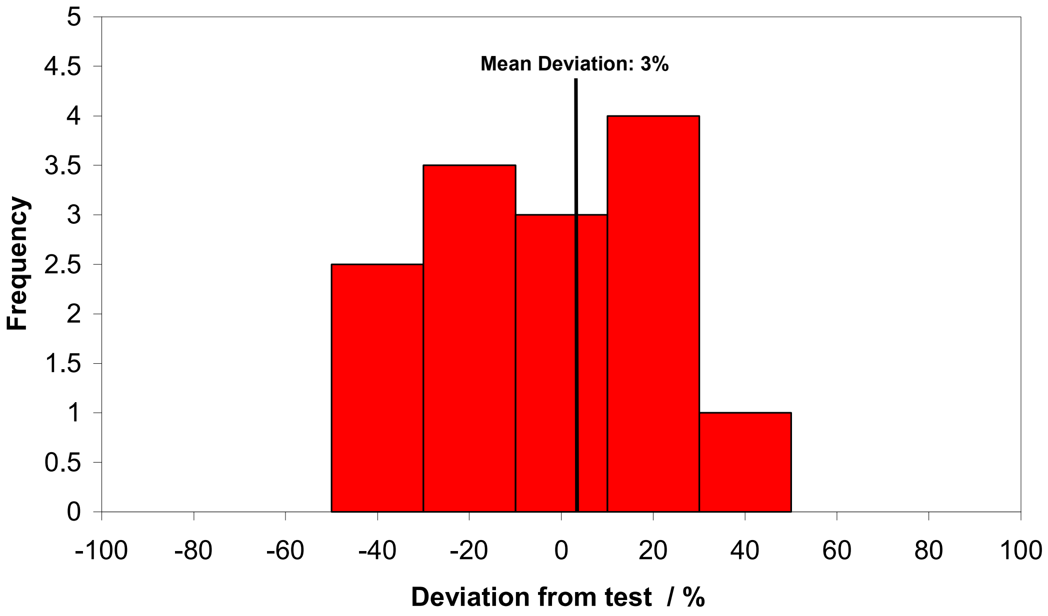

Figure 2: Comparison of moment for crack initiation calculated by the R6-Method with the experimental results of the tests (15 pipe component tests)

The denominator p_f (H_i, H_j) is the probability of failure p_f considering the input parameter H_i and the input parameters H_j to be statistically distributed. It is set to 0.01 representing a small probability of failure that should be given for the pipe being in service. The numerator p_f ($\overline{H_i}$ ±2s_i, H_j) describes the probability of failure p_f considering the parameter H_i to be the constant value of $\overline{H}_i \pm 2s_i$ (mean value $\overline{H}_i$, standard deviation s_i) and the remaining input parameters H_j considered to be statistically distributed. The mean value will be changed (addition or subtraction of $2s_i$) such that the probability of failure p_f will be raised. Based on the input parameters of the 6 pipe tests, Tab. 3 and 4, the results are shown in Fig. 3. One can state a clear influence of the bending moment M_b if it varies by 10% within a 2s-range. Furthermore the sensitivity of the crack angle depends a lot on the geometry of the crack itself as can be taken from Tab. 3. If the crack angle is about 120 degrees combined with a through wall crack, the sensitivity is significantly higher compared to cases where the crack angle is only 60 degrees or the ratio a/t is just 0.5. The sensitivities of the required material properties are more or less situated in the same range. However the influence of the yield strength is slightly greater than that of the fracture toughness whereas the tensile strength seems to have less influence. The wall thickness clearly dominates the importance of the outer diameter which has a rather little impact on the result.

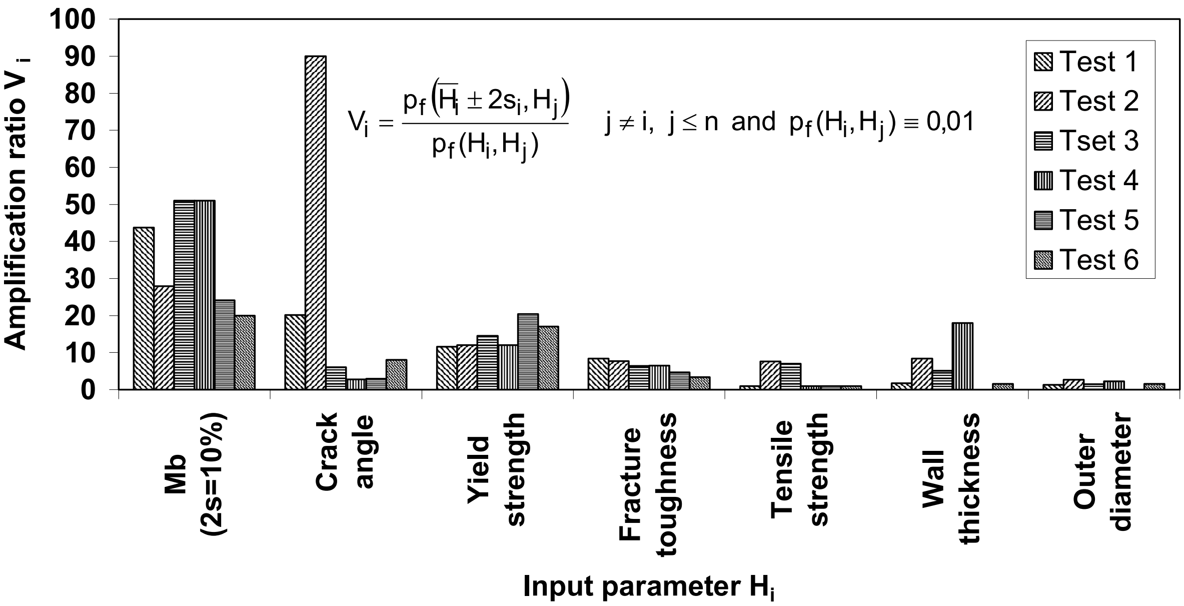

Figure 3: Influence on moment of crack initiation of piping components if the specific input parameter H_i deviates by $2s_i$ from its mean value

CONCLUSIONS

To estimate the condition of crack initiation the R6-Method extended by a probabilistic modul based on the Monte-Carlo method was used. Supported by this tool sensitivity studies of 6 pipe component tests were performed with statistically distributed data from strength and fracture laboratory specimen testing. The results of the investingations point to the fact, doing probabilistic calculations, the bending moment has the greatest influence if it shows scattering of at least 10%. With increasing crack extension in radial or circumferential direction the sensitivity increases accordingly. Material Properties as well as wall thickness do have a comparable influence whereas the outer diameter is less endangered to change results by scattering.

NOMENCLATURE

a [mm]	= crack depth	R_m	= tensile strength
A_5 [%]	= fracture elongation	$R_{p0,2}$	= yield strength
p	= probability	K_{IJ} [N/mm]	= fracture toughness
p_i [bar]	= pipe internal pressure	Z [%]	= reduction of area
t [mm]	= wall thickness	Δa, Δc	= crack growth
s, s_i	= standard deviation	α	= half circumferential crack angle
DN	= nominal pipe diameter (outside)	σ_{fl}	= flow stress
M_b [kNm]	= pipe bending moment	θ	= pipe bending angle
H	= input parameter		

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