Advanced Transport Modeling of Toroidal Plasmas with Transport Barriers

A. Fukuyama 1), S. Murakami 1), M. Honda 1), Y. Izumi 1) M. Yagi 2), N. Nakajima 3), Y. Nakamura 4), T. Ozeki 5)

1) Graduate School of Engineering, Kyoto University, Kyoto, Japan
2) Research Institute of Applied Mechanics, Kyushu University, Kasuga, Japan
3) National Institute for Fusion Science, Toki, Japan
4) Graduate School of Energy Science, Kyoto University, Uji, Japan
5) Japan Atomic Energy Research Institute, Naka, Japan

email contact of main author: fukuyama@nucleng.kyoto-u.ac.jp

Abstract. Transport modeling of toroidal plasmas is one of the most important issue to predict time evolution of burning plasmas and to develop control schemes in reactor plasmas. In order to describe the plasma rotation and rapid transition self-consistently, we have developed an advanced scheme of transport modeling based on dynamical transport equation and applied it to the analysis of transport barrier formation. First we propose a new transport model and examine its behavior by the use of conventional diffusive transport equation. This model includes the electrostatic toroidal ITG mode and the electromagnetic ballooning mode and successfully describes the formation of internal transport barriers. Then the dynamical transport equation is introduced to describe the plasma rotation and the radial electric field self-consistently. The formation of edge transport barriers is systematically studied and compared with experimental observations. The possibility of kinetic transport modeling in velocity space is also examined. Finally the modular structure of integrated modeling code for tokamaks and helical systems is discussed.

1. Introduction

Transport modeling of toroidal plasmas is one of the most important issues to predict time evolution of burning plasmas and to develop reliable and efficient control schemes in reactor plasmas. Most of present analyses on the evolution of core plasmas in a transport time scale employ a set of diffusive transport equations based on the flux-gradient relations. With an appropriate transport model, the diffusive transport analysis reproduces the formation of internal transport barriers [1], in which the plasma rotation and the radial electric field play important roles. For dynamical analysis of barrier formation, however, a self-consistent analysis of plasma rotation and radial electric field is required. In addition, strong heating and current drive deviate modify the velocity distribution function from the Maxwellian and may affect both microscopic and global instabilities and poloidal angle dependence of the radial particle and heat flux.

We have developed an advanced scheme of transport modeling based on dynamical transport equation and applied it to the analysis of transport barrier formation. First we propose a new transport model and examine its behavior by the use of conventional diffusive transport equation. Then the dynamical transport equation is introduced to describe both edge and internal transport barriers. The possibility of kinetic transport modeling in velocity space is also examined. Finally the modular structure of integrated modeling code for tokamaks and helical systems is discussed.

2. Diffusive Transport Modeling

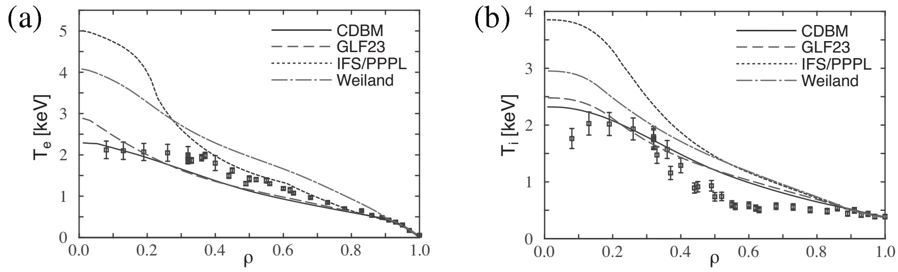

FIG. 1. Heat transport simulation results for various transport model. (a) $T_{\rm e}$ and (b) $T_{\rm i}$

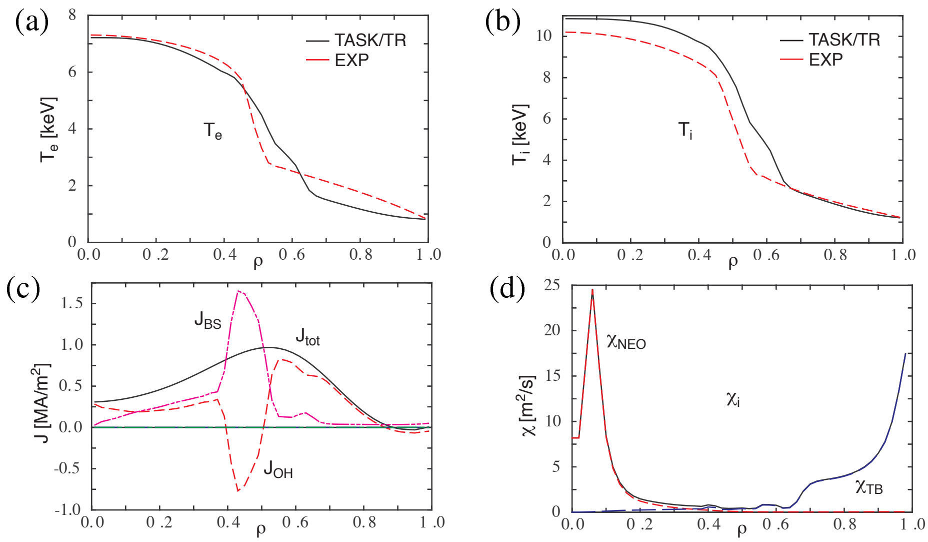

FIG. 2. Heat transport simulation results for various transport model. (a) $T_{\rm e}$ and (b) $T_{\rm i}$

In a conventional transport modeling, diffusive transport equations based on the flux-gradient relations which assume the force balance and heat flow balance in a stationary state describe the time evolution of macroscopic fluid quantities. Various turbulent transport models have been proposed to characterize the flux-gradient relations. We have implemented the transport models based on the ion temperature gradient (ITG) mode, GLF23[3], IFS-PPPL[4], and Weiland [5] models, and the ballooning mode, current diffusive ballooning mode (CDBM)[1,2] model, into the diffusive transport module TASK/TR. FIG. 1 illustrates the comparison of temperature profiles calculated with these models for the experimental data of the L-mode shot #82188 on DIII-D tokamak. The ITG and CDBM models incidentally reproduce similar profiles except near the magnetic axis. The CDBM model also reproduces the internal transport barrier observed on the shot 29728 of JT-60 tokamak as shown in FIG. 2. In this case we calculate the radial electric field from the radial force balance and evaluate the magnitude of the velocity searing rate which reduces the heat transport.

We have derived a set of model equations which describe both the electrostatic toroidal ion temperature gradient mode and the electromagnetic ballooning mode and evaluated the turbulent transport coefficients from the nonlinear marginal stability condition of the most unstable mode [6]. FIG. 3 illustrates a typical behavior of the ion thermal diffusivity $\chi_{\rm i}$ as a function of normalized pressure gradient $\alpha \equiv -q^2 R {\rm d}\beta/{\rm d}t$ for various values of magnetic shear $s \equiv (r/q)({\rm d}q/{\rm d}t)$ as. The transport coefficients strongly depend on α and decrease with the decrease of $s - \alpha$. The dependence on $s - \alpha$ comes from the ballooning structure of the two modes. The rapid increase of $\chi_{\rm i}$ for $\alpha \gtrsim 0.1$ is attributed to the ballooning mode. We have derived an approximate formula of χ for small α [7] and carried out a diffusive transport simulation with this formula

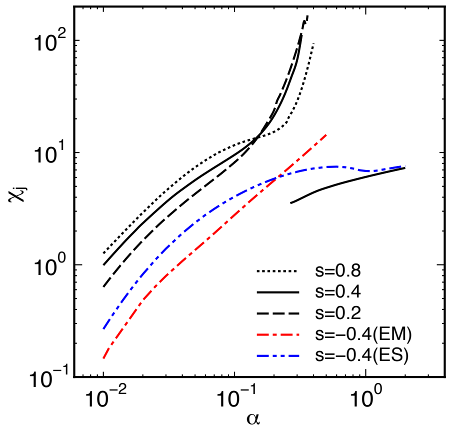

FIG. 3. Pressure gradient dependence of the transport coefficients for various values of s.

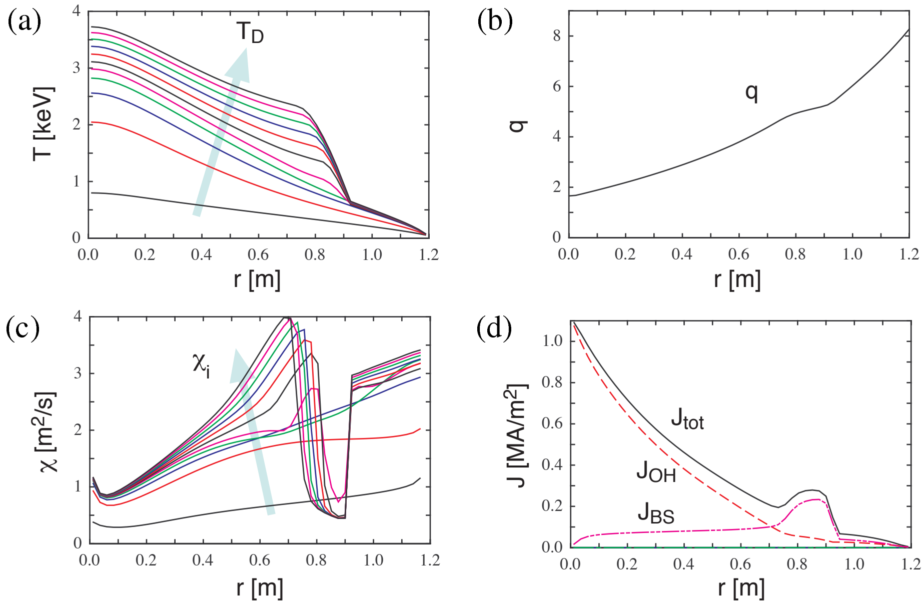

FIG. 4. Radial profiles of ion temperature, safety factor, ion thermal diffusivity, and current density in the high-β_p operation mode.

by the TASK/TR code [1]. In the case of high β_p operation, the internal transport barrier can be reproduced mainly through the $s - \alpha$ dependence of χ as shown in FIG. 4. Extension of the model for large α and comparison with turbulence simulation results, experimental results on JT-60, and ITPA profile database is under way.

3. Dynamical Transport Modeling

Recently it has been widely recognized that plasma rotation and radial electric field, E_r, strongly affect the radial transport, especially the formation of transport barriers. In order to evaluate the rotation as well as E_r self-consistently and describe the dynamics of transport barriers, we have formulated a set of dynamical transport equations [8], which consists of flux-surface-averaged fluid equations for electron and ions, diffusion equation of fast and slow neutrals and Maxwell's equations.

We consider a tokamak with a circular cross section and use the toroidal coordinates (r, θ, ϕ). The two-fluid equations for the density n_s, radial flow u_{sr}, poloidal rotation $u_{s\theta}$ and toroidal rotation $u_{s\phi}$ are solved for electrons and bulk ions:

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r n_s u_{sr} + S_s$$

$$\frac{\partial}{\partial t} m_s n_s u_{sr} = \frac{1}{r} m_s n_s u_{s\theta}^2 + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta) - \frac{\partial}{\partial r} n_s T_s$$

$$\frac{\partial}{\partial t} m_s n_s u_{s\theta} = e_s n_s (E_\theta - u_{sr} B_\phi) + \frac{1}{r^2}\frac{\partial}{\partial r} r^3 n_s m_s \mu_s \frac{\partial}{\partial r}\frac{u_{s\theta}}{r} + F_{s\theta}^{\rm NC} + F_{s\theta}^{\rm C} + F_{s\theta}^{\rm W} + F_{s\theta}^{\rm X} + F_{s\theta}^{\rm L}$$

$$\frac{\partial}{\partial t} m_s n_s u_{s\phi} = e_s n_s (E_\phi + u_{sr} B_\theta) + \frac{1}{r}\frac{\partial}{\partial r} r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi} + F_{s\phi}^{\rm NC} + F_{s\phi}^{\rm C} + F_{s\phi}^{\rm W} + F_{s\phi}^{\rm X} + F_{s\phi}^{\rm L}$$

$$\frac{\partial}{\partial t}\frac{3}{2} n_s T_s = -\frac{1}{r}\frac{\partial}{\partial r} r\left(\frac{5}{2} u_{sr} n_s T_s - n_s \chi_s \frac{\partial}{\partial r} T_{\rm e}\right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi}) + P_s^{\rm C} + P_s^{\rm L} + P_s^{\rm H}$$

where m_s and e_s are the mass and charge of particle species s. The particle source and sink, S_s, neoclassical viscosity force, $F^{\rm NC}$, collisional momentum transfer, $F^{\rm C}$, force due to the interaction with turbulent electric field, $F^{\rm W}$, and charge exchange force on ions, $F^{\rm X}$, are calculated from local quantities. The detail of each term was discussed in [8]. The perpendicular viscosity, μ_s, and thermal conductivity, χ_s, represent anomalous transport due to the turbulence. The diffusion equation for the neutral density, n_{0s},

$$\frac{\partial n_{0s}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} - r D_{0s}\frac{\partial n_{0s}}{\partial r} + S_0$$

is also solved simultaneously. These equations couple with the Poisson's equations for the radial electric field, E_r and Maxwell's equations for the poloidal and toroidal components of the magnetic field, B_θ and B_ϕ, and the electric field, E_θ and E_ϕ,

$$\frac{1}{r}\frac{\partial}{\partial r}(r E_r) = \frac{1}{\epsilon_0}\sum_s e_s n_s, \qquad \frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \qquad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r E_\phi$$

$$\frac{1}{c^2}\frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r} B_\phi - \mu_0 \sum_s n_s e_s u_{s\theta}, \qquad \frac{1}{c^2}\frac{\partial E_\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r B_\theta - \mu_0 \sum_s n_s e_s u_{s\phi}$$

Taking account of the poloidal momentum conservation between electrons and ions, we use a model formula for the interaction with turbulent electric field

$$F_{{\rm e}\theta}^{\rm W} = -F_{{\rm i}\theta}^{\rm W} = \frac{e^2 B_\phi^2}{T_{\rm e}} n_{\rm e} D \left(u_{{\rm e}\theta} - \left\langle \frac{\omega}{m} \right\rangle r\right) \tag{1}$$

As a turbulent diffusion coefficient D, we employ the self-sustained turbulence model for the current-diffusive high-n ballooning mode [2]. We use an expression for D including the reduction due to poloidal rotation shear [10],

$$D = \frac{f(s,\alpha)}{1 + G_1 h^2}\alpha^{3/2}\frac{c^2}{\omega_{\rm pe}^2}\frac{v_{\rm A}}{qR} \tag{2}$$

where h is the rotation shear $(qR/v_{\rm A})(1/sB)dE_r/dr$ and f is a function of the pressure gradient α and the magnetic shear s. We employ an interpolation formula of $f(s,\alpha)$ [1]. Though G_1 is also a function of s and α as given in Ref. [10], we take it constant in the following calculation

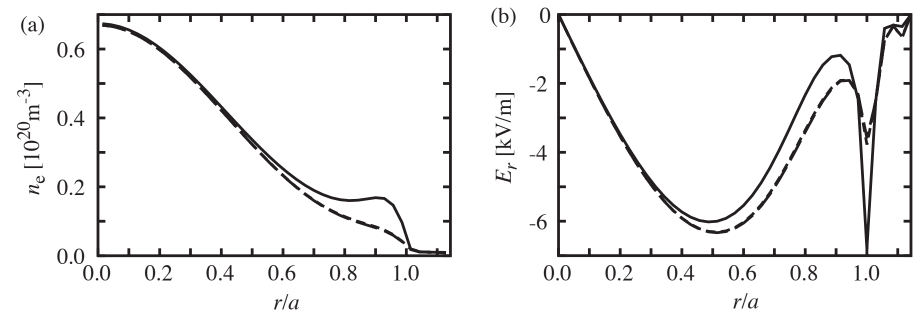

FIG. 5. Radial profiles of the density and E_r with (dashed lines) and without (solid lines) turbulent transport.

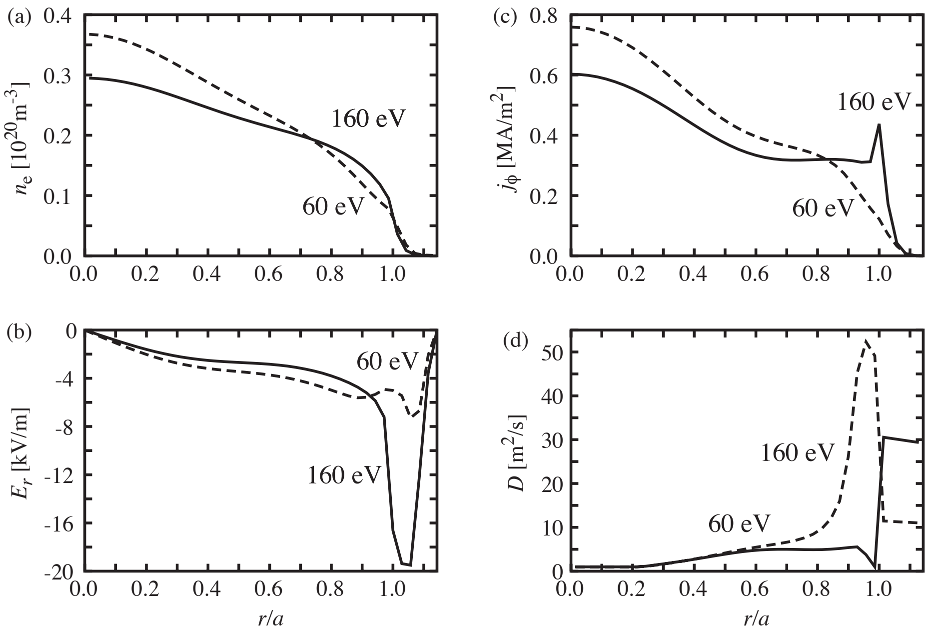

FIG. 6. Edge temperature dependence of radial profile

for simplicity; a typical value $G_1 = 24$ for $r/R = 0.3$, $q = 3$, $s = 0.5$ and $\alpha = 0.3$[10]. The perpendicular viscosity μ_s and the thermal diffusivity χ are assumed to be proportional to D. The spectrum averaged frequency $\langle\omega/m\rangle$ is taken to be 0 for the ballooning mode. The neoclassical effect is included as a poloidal viscosity in tokamaks and both poloidal and toroidal viscosities in helical systems. Enhanced loss along the field line dominates in the SOL region ($r/a > 1$).

This model was implemented as a transport module TASK/TX. The analysis without turbulent transport has revealed that large E_r is generated near the separatrix owing to the difference of transport mechanism in the regions of nested and open magnetic surfaces (FIG. 5). If the suppression of turbulent transport due to the poloidal $E \times B$ rotation shear is included, edge transport barrier formation is reproduced. FIG 6 shows the radial profiles of the density, radial electric field, current density and diffusion coefficient for the cases of $T_{\rm edge} = 60\,{\rm eV}$ (dashed lines) and 160 eV (solid lines). The central temperature is fixed to 700 eV. With the increase of $T_{\rm edge}$, E_r builds up near the plasma edge and the $E \times B$ rotation shear reduces the diffusion coefficient D. The density gradient inside the separatrix ($r/a = 1$) is enhanced. We should note that the central density cannot be sustained in the present model. The increase of the pressure gradient induces the bootstrap current near the edge. The edge plasma current weakens the

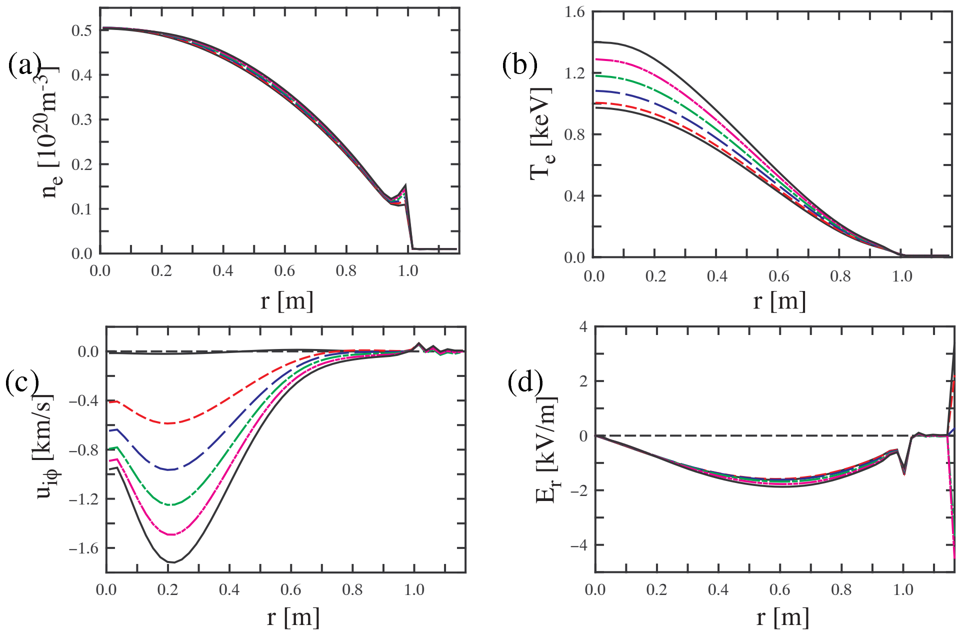

FIG. 7. Heat transport simulation results for various transport model. (a) T_e and (b) T_i

magnetic shear and also contributes to the reduction of D.

In a non-axisymmetric devices, the neoclassical toroidal viscosity leads to the bifurcation between the electron root and the ion root. Typical profiles with neutral beam injection are shown in FIG. 7. The toroidal rotation speed is one order of magnitude lower than that of tokamaks.

4. Kinetic Transport Modeling

The velocity distributions in a fusion plasma will not be necessarily close to the Maxwellian, especially in the initial auxiliary heating phase, and affect the transport and the heating power requirement for startup. The TASK code includes a three-dimensional bounce-averaged Fokker-Planck module TASK/FP which has been used for the analysis of electron cyclotron heating and current drive. This module can be enhanced to describe the time evolution of non-Maxwellian velocity distribution in a transport time scale. We are formulating the neoclassical radial diffusion and parallel force driven by the spatial gradient. Turbulent diffusion in velocity space and radius will be also implemented. The kinetic transport analysis may be a long-range task, but the framework of the integrated modeling code should be prepared for such advanced analyses.

5. Integrated Modeling of Toroidal Plasmas

We are proposing a framework of integrated modeling of toroidal plasmas. The central part of the framework is a set of data interface for various numerical codes and experimental profile databases. As a sample implementation and for verification of the interface, the TASK code is being renovated. It comprises the modules for equilibrium, transport, velocity distribution, ray tracing, full wave and data conversion. The state of plasma is described by fluid quantiles calculated by the transport module and/or kinetic quantities by the Fokker-Planck velocity distribution module, as well as quasi-static magnetic field and metric quantifies by equilibrium module. The propagation and absorption of the waves and neutral beams are calculated with the state variables of the plasma. The macroscopic and microscopic instabilities will be also

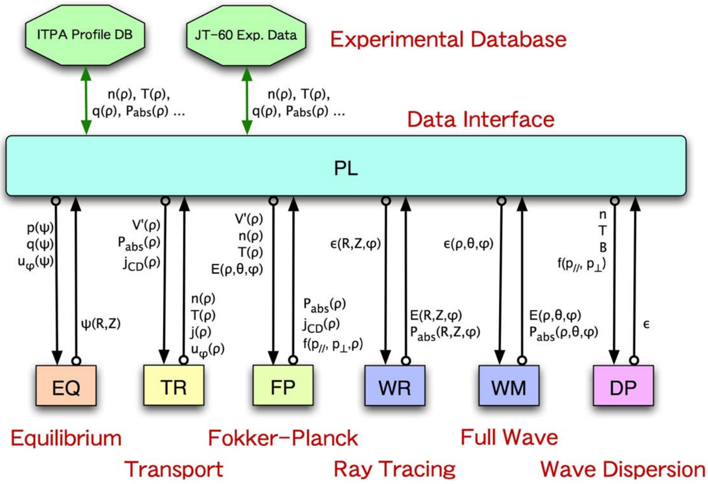

FIG. 8. Modular structure of TASK code system

examined in additional modules using the state variables. Experimental profile database can be used for initial and/or boundary conditions and for systematic quantitative comparison with simulation results. Most of modules are applicable for non-axisymmetric configuration, but the interaction between the equilibrium analysis and the transport analysis requires careful consideration in extending to the three-dimensional helical configuration. Comparison of integrated analyses and experimental results in transport barrier formation will be reported in near future for both tokamaks and helical devices.

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