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XA0501005

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preprint

Available at: http://www.ictp.it/~pub_off IC/2005/005

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# COLLIDING BLACK HOLE SOLUTION

Mainuddin Ahmed
*Department of Mathematics, Rajshahi University, Rajshahi 6205, Bangladesh* [1]
*and*
*The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.*

## Abstract

A new solution of Einstein equation in general relativity is found. This solution solves an outstanding problem of thermodynamics and black hole physics. Also this work appears to conclude the interpretation of NUT spacetime.

MIRAMARE – TRIESTE

January 2005

[1] Address for correspondence. amainuddin@yahoo.com

# 1 Introduction

In 1993 Kastor and Transchen discovered a multi-black hole solution [1] in the de-Sitter Universe. The solution is interesting in that it provides an example of colliding black holes [2].

In this paper, we consider a solution of Einstein equation. From this solution, under some special considerations we obtain a colliding black hole solution.

We consider the following solution:

$$\begin{aligned} ds^2 &= \rho^2(\Delta dr^2 + d\theta^2) + \rho^{-2}a^{-2}\sin^2\theta \left[dt - \frac{(r^2+a^2)^2}{a}d\varphi\right]^2 \\ &\quad -\Delta\rho^{-2}\left[dt - \left(a - \frac{(n-a\cos\theta)^2}{a}\right)d\varphi\right]^2 \end{aligned} \tag{1}$$

where

$$\begin{aligned} \rho^2 &= r^2 + (n - a\cos\theta)^2 \\ \Delta &= (r^2 + a^2 - n^2) - 2Mr \end{aligned}$$

The solution (1) possesses three parameters: $M$ the mass parameter, $a$ the angular momentum per unit mass parameter, $n$ the NUT parameter. We can call the solution (1) as a NUT-Kerr solution.

The solution (1) includes as special cases (i) the well-known Kerr solution for $n = 0$ and (ii) the NUT (Newman-Urti-Tamburino) solution for $a = 0$ [3]. It is interesting to note that both the Kerr solution and the NUT solution were discovered in the same year, in 1963, but their position is quite opposite from the physical interpretation point of view. The Kerr solution possesses a deep physical interpretation. The discovery of the Kerr solution as well as the discovery of quasars (quasi-stellar source) in the same year (1963) [4] attracted people in the scientific community to work on the general theory of relativity which was neglected before on the consideration that it has no physical importance except in the Big Bang theory.

The NUT solution, like the Kerr solution, is a stationary and axisymmetric solution of Einstein's empty space equation $R_{\alpha\beta} = 0$, but it created a lot of problems from the physical interpretation point of view. An ingenious interpretation was given by Misner [5, 6]. According to Misner's interpretation, an observer in the NUT spacetime moves forward in time only to find himself in his own past. According to Bonnor's remark [7], this interpretation of Misner is physically puzzling and resembles, to some extent, the world of Dr. WHO.

Bonnor [7] gave a quite different interpretation of the NUT solution. According to Bonnor, the NUT solution is due to the field of a spherically symmetric mass together with a semi-infinite massless source of angular momentum along the axis of symmetry. The main purpose of Bonnor was to give an interpretation of the NUT parameter. According to Bonnor, the NUT parameter arises due to the strength of the physical singularity on $\theta = \pi$. The NUT solution is singular along the axis of symmetry $\theta = 0$ and $\theta = \pi$. The singularity along $\theta = 0$ is removable by a co-ordinate transformation. But this transformation cannot remove the singularity on $\theta = \pi$. Although the singularity on $\theta = \pi$ may have been removed by other transformations, Bonnor assumed that $\theta = \pi$ is a physical singularity representing the source of the field and he, of course, gave justification for his assumption. Here we should note that Misner [5, 6] introduced a periodic time co-ordinate to remove the singularity.

Now according to Bonnor's interpretation, if we consider that the NUT parameter $n$, is due to the strength of the physical singularity on $\theta = \pi$ and if we further assume that $n = a$, then the solution (1) reduces to the form

$$ds^2 = r^2(\Delta dr^2 + d\theta^2) - \Delta r^2(dt - ad\varphi)^2 \tag{2}$$

where

$$\Delta = r^2 - 2Mr$$

That means, when the angular momentum of the mass $M$ and the angular momentum of massless rotating rod collide, the solution (2) arises.

## 2 Physical properties

The surface gravity of the event horizon $r = 2M$ of the solution (2) can be calculated as

$$\kappa = \frac{M}{2(2M^2 + a^2)} \tag{3}$$

The Hawking temperature of the event horizon will be given by

$$T_H = \frac{M}{4\pi k_B(2M^2 + a^2)} \tag{4}$$

where $k_B$ is the Boltzmann constant.

The entropy of the spacetime given by (2) is given by

$$S = 4\pi k_B M^2$$

when $M \to 0$, $T_H \to 0$ and $S \to 0$. Thus at zero temperature, the entropy becomes zero. Thus the black hole given by (2) in the extreme case ($M \to 0$) does not violate the black hole analog of the "Nerst Theorem" which states that the entropy of a system will be zero as its temperature goes to zero. Usually the entropy of a black hole does not become zero as its temperature goes to zero. This fact creates an awkward situation between thermodynamics and black hole physics although there is a striking similarity between the laws of thermodynamics and those of black hole physics.

Recently Bini and Cherubini [8] generalized the results of the algebraically special (AS) gravitational perturbation of Schwarzchild and Kerr black holes in the case of NUT and NUT-Kerr spacetimes. They found that the spectrum of the associated frequencies in the case of NUT spacetime is purely imaginary as obtained in the case of Schwarzschild black hole. However, because of the quantization condition required by the periodicity of the time co-ordinate as introduced by Misner [5, 6] to remove the singularity at $\theta = \pi$ of the NUT spacetime, purely imaginary values of frequencies must be rejected as unphysical [8]. That means, NUT spacetime does not possess AS solution.

In the case of NUT-Kerr spacetime they [8] did not complete their study. But we think, the same problem will exist in the case of NUT-Kerr spacetime as in the case of NUT spacetime, since both spacetimes are endowed with NUT parameter. We find that there is a problem of interpreting the NUT solution. Like the NUT solution, any other metric endowed with NUT parameter does not have any direct physical interpretation. But our special consideration made in this paper to obtain the solution (2), has made problem easy to access. The spacetime given by (2) has horizon $\Delta \equiv r^2 - 2mr = 0$ which is the same as the horizon of Schwarzschild black hole. So the AS solution is possible for the NUT-Kerr spacetime under the special consideration as made in this paper as in the case of Schwarzschild black hole.

## 3 Remarks

So, we observe, our work seems to solve some problems.

If the mass of the colliding black hole given by (2) results into zero, then we find complete analog between the laws of thermodynamics and those of black hole physics. Thus we find a solution of an outstanding problem of thermodynamics and black hole physics.

It seems to be easier to obtain the AS solution of the NUT-Kerr spacetime if we follow the interpretation of Bonnor [7] for the NUT metric as well as our consideration that $n = a$.

Besides the solutions of these problems, we should observe that this work appears to solve another serious problem of interpreting the NUT solution.

As far as our work is concerned, the interpretation of Bonnor is getting more support (at least from the work of Bini and Cherubini [8]) than that of Misner.

**Acknowledgments.** I would like to thank the Director of ICTP as well as Professor Randjbar-Daemi, Head of the High Energy section, ICTP, for providing me an opportunity to visit the Centre. This paper was prepared during my visit.

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