DVCS in the fragmentation region of polarized electron

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For the kinematical region when a hard photon is emitted predominantly close to the direction of motion of a longitudinally polarized initial electron and relatively small momentum transfer to a proton we calculate the asimuthal asymmetry of a photon emission. It arises from the interference of the Bethe-Heitler amplitude and those which are described by a heavy photon impact factor. The asimuthal asymmetry does not decrease in the limit of infinite cms energy. The lowest order expression for the impact factor of a heavy photon is presented.

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I. INTRODUCTION

Recently a lot of attention has been paid to deeply virtual Compton scattering (DVCS) [1–5]. It was realised that there exists a close relation with the problem of a proton spin carried by gluons and quarks. Indeed the decomposition of the nonforward Compton scattering amplitude for the case when one of photons on-mass shell and another one is off-mass shell contains fifteen structure functions and the four out of them could be put to the test [4,5,1,3,2]. Their first moments determine the Dirac, Pauli, axial-vector and pseudoscalar formfactors of a proton while their second moments are related with a proton spin fraction carried out by quarks and gluons and the orbital momenta of the latter. These structure functions can be tested in DIS experiments with longitudinally polarised initial lepton aimed at measuring the asimuthal correlation between a real photon and a scattered lepton. There are two mechanisms of photon emission, namely the emission from lepton (Bethe-Hietler (BH)) and quark lines. The last option may be split up into two gauge-invariant mechanisms: emission off a valence quark of a proton and emission from a quark-antiquark pair created by a virtual photon and a gluon out of a proton. For the case of small angle scattering (the limit of small Bjorken variable) the main nonvanishing in the limit of high cms energies contribution arises from the last mechanism which we call the impact factor (IF) mechanism which is thought of as the amplitude of virtual photon conversion into real photon in the gluonic field of proton. The effect of asimuthal correlation appears as the interference of the real BH amplitude with the pure imaginary one of IF mechanism of real photon creation. The interference is not zero due to pure imaginary spin density matrix of the polarised lepton.

The asimuthal asymmetry we obtain has the simplest form $A = \Delta|M|^2/|M_{BH}|^2 \sim \sin\phi$, where ϕ is the asimuthal angle between the planes formed by the momenta of initial and scattered leptons and an initial lepton and photon. It was shown that the higher harmonics in the Fourier decomposition of the asymmetry are related with the structure functions mentioned above. The contribution derived here is sensitive only to the gluon density $xg(x,Q^2)$ inside a proton. For small values of energy fraction x carried by sea gluons, one has $xg(x,Q) \approx 6Q^2\ (GeV), Q^2 \sim 1\ GeV^2$.

We consider below the case when the initial proton is unpolarised and the final state is a scattered lepton, a recoil proton and a hard photon from the fragmentation region of initial lepton. Moreover the contribution of the lowest order ($\sim \alpha_e^2$) to the asymmetry is dealt with. The higher PT effects were considered in the paper [3] and took into account the BFKL ladder.

Let's consider the radiative electron-proton scattering,

$$e(p_1,\xi) + P(p) \to e(p_2) + P(p') + \gamma(k_1),$$

where we indicate in parenthesis the 4-momenta of particles, ξ is the degree of the longitudinal polarisation of electron. We will restrict ourselves to the kinematics when the absolute magnitude of a square of transfer momentum between initial and scattered electrons is small with respect to a cms energy squared,

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$$s = (p_1 + p)^2 \gg Q_1^2 = -(p_1 - p_2)^2 \qquad (1)$$
$$\sim Q^2 = -q^2 \gg p_1^2 = p_2^2 = m_e^2,$$
$$p^2 = p'^2 = M^2, \quad q = p' - p.$$

The main contribution non-vanishing in the limit of large s arises from the two Feynman amplitudes. One of them, describing the hard photon emission by the electron blob, the so called Bethe-Heitler amplitude, has the following form,

$$M^{BH} = \frac{(4\pi\alpha)^{3/2}}{q^2}\bar{u}(p_2)O_{\mu\sigma}u(p_1,\xi)\bar{u}^\lambda(p')\gamma_\nu u^\lambda(p)F_1(q^2)$$
$$\times g^{\mu\nu}e^\sigma(k_1) = -\frac{2x_1}{sdd_1}\bar{u}(p_2)v_\sigma u(p_1,\xi)e^\sigma(k_1)sN_\lambda, \qquad (2)$$

with

$$N_\lambda = \frac{1}{s}\bar{u}^\lambda(p')p_1u^\lambda(p)_\lambda, \qquad |N_\lambda| = 1.$$

Here $\lambda = \pm 1$ describes the chiral state of proton, $e(k_1)$ is the polarisation vector of photon and

$$v_\sigma = sx(d - d_1)\gamma_\sigma + xd_1\gamma_\sigma\hat{q}\hat{p} + d\hat{p}\hat{q}\gamma_\sigma,$$

the effective vertex describing the Compton scattering [6] and the quantities

$$d = xx_1[(p_1 - q)^2 - m_e^2], \quad d_1 = -x_1[(p_1 - k_1)^2 - m_e^2]$$

are expressed via Sudakov variables

$$d = x_1^2m_e^2 + (k_1 + x_1q)^2, \quad d_1 = x_1^2m_e^2 + k_1^2,$$

where k_1, p_2, q are the two-dimensional euclidean vectors, transverse to the beam axis ($k_1 + p_2 + q = 0$) and x, x_1 are the energy fractions of the scattered electron and photon obeying $x + x_1 = 1$. The corresponding modulus of the matrix element squared and summed over polarisation states and the cross section can be brought to the form [6],

$$\sum |M^{BH}|^2 = 2^{11}\pi^3\alpha^3\frac{s^2}{q^2}\frac{x_1^2x(1+x^2)}{dd_1}F_1^2(Q^2), \qquad (3)$$
$$d\sigma^{eP\to(e\gamma)P} = \frac{2\alpha^3x_1(1+x^2)}{\pi^2q^2dd_1}F_1^2(Q^2)d^2k_1d^2qdx,$$

with $F_1(Q^2)$ is the Dirac form factor of proton. It is important to note that the amplitude A^{BH} is real. Consider now the 2-loop level correction to the amplitude considered above, describing the emission of a hard photon from the intermediate state of a pair of charged quarks created by virtual photon and converted to the real one through the two gluons-exchange. The corresponding amplitude differs from the QED ones only by the factor $C = \sum Q_q^2$, on the squares of quark charges (in units of e) and the gluon density factor $G(x,k,Q) = xdg(x,k,Q)/d\ln Q^2, k^2 \sim Q^2 \ll s$. The amplitude of IF mechanism is pure imaginary and may be expressed in terms of impact factor (IF) of photon,

$$M^{IF} = C\frac{is(4\pi\alpha)^{1/2}}{q_1^2}\bar{u}(p_2)\gamma_\mu u(p_1,\xi)\frac{4\pi\alpha_sN_\lambda}{(2\pi)^2} \qquad (4)$$
$$\times \int\frac{d^2kG(k,Q,x)}{k^2(q-k)^2}\tau_{\mu\sigma}^{\gamma\gamma}e^\sigma(k_1), \quad q_1^2 = -\frac{1}{x}p_2^2,$$

where

$$\tau_{\mu\sigma}^{\gamma\gamma} = \alpha\alpha_s\int\frac{d^2q_+dxdy}{\pi x_+x_-}I_{\mu\sigma},$$

with the tensor $I_{\mu\sigma}$ given in the Appendix A

In Appendix C we give the explicite expression for IF of heavy photon for the case when both photons off mass shell.

The relevant expression for the contribution to the cross section looks

$$\Delta|M|^2 = 2M^{IF}(M^{BH})^*$$
$$= s^2\xi\frac{8Cxx_1}{q^2p_2^2dd_1}\frac{\alpha^2\alpha_s^2}{\pi}\int\frac{d^2kG}{k^2(q-k)^2}$$
$$\times\int\frac{d^2q_+dx_+dy}{x_+x_-\pi}IF_1(Q^2), \qquad (5)$$
$$I = \frac{i}{s}I_{\mu\nu}\cdot L_{\mu\nu}, \quad L_{\mu\nu} = Tr[\hat{p}_2v_\nu\hat{p}_1\gamma_\mu\gamma_5].$$

Performing the integrations over d^2k, d^2q_-, dx_- (for details see appendices A,B), we obtain for the asymmetry,

$$A = \xi\frac{\alpha_s^2}{\pi}\frac{xg(x,Q/2)_{x\to 0}}{F_1(Q^2)}\frac{Q}{p_2}\frac{x_1^2}{x(1+x^2)}$$
$$\times\sin\phi\left[xx_1P(x)\frac{k_1^2}{Q_1^2} + \frac{2}{x_1}\frac{(k_1+x_1q)^2}{Q_1^2}\right], \qquad (6)$$

where $Q = |q|$ is the momentum transferred to a proton Then we have,

$$Q_1^2 = \left(p_2 + \frac{1}{2}q\right)^2, \quad P(x) = P_1(x)\ln\frac{Q_1^2}{m_q^2} + P_2(x),$$
$$P_1(x) = -1 + \frac{2}{3}x_1 - 2x_1^2 - \frac{x}{x_1} + \frac{2}{3}xx_1 - 2\frac{x}{x_1^3},$$
$$P_2(x) = 1 - \frac{1}{2}x_1 + 2\frac{x}{x_1} + \frac{1}{x_1^2} - \frac{2}{3}xx_1 + \frac{x_1}{3x} - \frac{1}{2x_1} \qquad (7)$$

Here $q + k_1 + p_2 = 0$; $m = 0.3\,GeV$ is a quark constituent mass. Besides it has been assumed that $Q_1^2 \gg m^2$

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APPENDIX A: INTEGRATION OVER VIRTUAL GLUON MOMENTUM

To calculate the contribution of FD containing the impact factor of the heavy photon we need the trace,

$$I_{\mu\nu} = \frac{1}{s^2} Tr[(q_- + m)B_\mu(q_+ - m)R_\nu], \qquad (A.1)$$

$$B_\mu = \frac{1}{d_-}\gamma_\mu(k - q_+ + m)p_2 + \frac{1}{d_+}p_2(q_- - k + m)\gamma_\mu,$$

$$R_\nu = \frac{1}{d'_+}\gamma_\nu(q_- - k' + m)p_2 + \frac{1}{d'_-}p_2(k' - q_+ + m)\gamma_\nu,$$

where m is a quark mass and

$$\begin{aligned} d_\pm &= k^2 - 2kq_\pm \approx -sx_\pm\alpha_k, \\ d'_\pm &= k'^2 - 2k'q_\pm \approx -sx_\pm\alpha_{k'}, \\ s\alpha_k &= s\alpha_{k'} = s_t = \frac{1}{x_1}[s_1 + (k+p_2)^2], \qquad (A\ 2) \\ s_1 &= (q_+ + q_-)^2 = \frac{1}{x_+x_-}[m^2x_1^2 + r^2], \\ r &= x_-q_+ - x_+q_-. \end{aligned}$$

Here $x_\pm$ are the energy fractions of a pair particles, $q_\pm$ — their components of momentum transverse to the beam axis; $q_1 = p_1 - p_2$ and k_1 are the 4-momenta of the initial (virtual) and final (real) photons respectively. Using the gauge invariance condition $k^\mu I_{\mu\nu} = k'^\nu I_{\mu\nu} = 0$, $k' = q - k$ we may replace 4-vector p_2 in B_μ (R_ν) by

$$\frac{-sk_{\perp\mu}}{s_t}\left(\frac{-sk'_{\perp\nu}}{s_t}\right).$$

Using as well the Dirac equation for on-mass shell quarks we bring $I_{\mu\nu}$ to the form,

$$I_{\mu\nu} = \frac{1}{4s_t^4} Tr(\hat{q}_- + m)B_{1\mu}(q_+ - m)R_{1\nu}, \qquad (A.3)$$

$$B_{1\mu} = \frac{s_t}{s}\left(\frac{1}{x_+}\gamma_\mu\hat{p}\hat{k} - \frac{1}{x_-}\hat{k}\hat{p}\gamma_\mu\right) + \gamma_\mu Z,$$

$$R_{1\nu} = \frac{s_t}{s}\left(-\frac{1}{x_-}\gamma_\nu\hat{p}\hat{k}' + \frac{1}{x_+}\hat{k}'\hat{p}\gamma_\nu\right) + \gamma_\mu Z',$$

$$Z = \frac{1}{x_+x_-}rk, \quad Z' = \frac{1}{x_+x_-}rk'.$$

Here the vectors k, k' are pure 2-dimensional transverse to the beam axis.

The next step is to perform the d^2k integration. We suppose that small values of $|k|$ dominate as this region is enhanced by the factor $xg(x, |k|)$. The integration may be carried out using the following equalities,

$$\int \frac{d^2k}{\pi k^2 k'^2} k^i k'^j G(x, k, k')$$

$$= \int_0^1 dx \int \frac{d^2k\, k^i(q-k)^j G(x, k, q-k)}{\pi[(k - xq)^2 + Q^2x(1-x)]^2}, \qquad (A\ 4)$$

$$\approx \frac{1}{2}\delta^{ij}\int_0^1 dx\, xg(x, xQ, (1-x)Q) \approx \frac{1}{2}\delta^{ij} xg(x, Q/2)$$

Thus to the accuracy of approximately 10% (for $Q^2 \sim 1 \div 2GeV^2$) we may put $k = k' = q/2$ in the nonsingular part of integrand.

APPENDIX B: INTEGRATION OVER QUARK PAIR MOMENTA

Using the Sudakov parametrization of 4-vectors,

$$\begin{aligned} p_1 &= \tilde{p}_1, \quad p_2 = x\tilde{p}_1 + \frac{p_{2\perp}^2}{xs}\tilde{p} + p_\perp, \\ q_\pm &= x_\pm\tilde{p}_1 + \frac{q_{\pm}^2 + m^2}{sx_\pm}\tilde{p} + q_{\pm\perp}, \qquad (B\ 1) \\ k_1 &= x_1\tilde{p}_1 + \frac{k_\perp^2}{sx_1} + k_{1\perp}, \quad p_1^2 = \tilde{p}^2 = 0, \\ \tilde{p} &= p - \frac{M^2}{s}p_1, \quad 2\tilde{p}p_1 = s, \quad a_\perp p_1 = a_\perp\tilde{p} = 0 \end{aligned}$$

It follows from the conservation laws read,

$$x_1 + x = 1, \quad x_+ + x_- = x_1, \quad p_2 + k_1 + q = 0.$$

The scalar products may be written in the form,

$$\begin{aligned} 2p_1q_\pm &= \frac{1}{x_\pm}[q_\pm^2 + m^2], \\ 2p_2q_\pm &= \frac{1}{xx_\pm}[m^2x^2 + (xq_\pm - p_2x_\pm)^2], \qquad (B.2) \\ s_t\frac{x_+x_-}{x_1} &= \sigma = m_*^2 + q_t^2, \quad q_t = q_+ + y_+p_2, \\ m_*^2 &= m^2 + Q_1^2y_+y_-, \quad y_+ + y_- = 1, \quad y_\pm = \frac{x_\pm}{x_1}. \end{aligned}$$

The relevant integrals look

$$\int \frac{d^2q_+}{\pi}\left\{\frac{1}{\sigma^2}, \frac{r^2}{\sigma^4}, \frac{q_-^j r^j}{\sigma^3}, \frac{r^i}{\sigma^3}, \frac{r^2q_+^iq_-^j}{\sigma^4}\right\} \qquad (B\ 3)$$

$$= \left\{\frac{x_1^2}{6m_*^2}, -\delta^{ij}\frac{x_1}{4m_*^2}, 0, -x_1\left[\delta_{ij}\frac{1}{6m_*^2} - p_2^ip_2^j\frac{y_+y_-}{6m_*^4}\right]\right\}.$$

The integration over $x_\pm$ becomes almost trivial in the limit $Q_1^2 \gg m^2$:

$$\int_0^{x_1} dx_+\left[1, \frac{1}{m_*^2}\right] = \left[x_1, \frac{2x_1}{Q_1^2}\ln\frac{Q_1^2}{m^2}\right]. \qquad (B.4)$$

APPENDIX C: IMPACT FACTOR OF HEAVY PHOTON

To obtain IF of heavy photon we consider the s-channel discontinuity of the heavy photon amplitude in an external field,

$$\begin{aligned}\gamma_\mu(P_1) + A(p) &\to q(q_+) + \bar{q}(q_-) + A(p'') \\ &\to \gamma_\nu(P_2) + A(p'), \qquad \text{(C 1)}\\ P_1^2 = -Q^2, &\quad P_2^2 = -Q'^2,\end{aligned}$$

which is described by the tensor

$$\Delta A_{\mu\nu}(s,t) = \frac{(4\pi\alpha)^2}{k^2 k'^2}\left(\frac{2}{s}\right)^2 N_2 s^4 I_{\mu\nu}\mathrm{d}\Gamma_3, \qquad \text{(C.2)}$$

with

$$\begin{aligned}\mathrm{d}\Gamma_3 &= \frac{1}{(2\pi)^5}\frac{\mathrm{d}^3\vec{q}_+}{2\epsilon_+}\frac{\mathrm{d}^3\vec{q}_-}{2\epsilon_-}\frac{\mathrm{d}^3\vec{p}''}{2E''}\delta^4(P_1 + p - q_+ - q_- - p'') \\ &= \frac{\mathrm{d}^2k\mathrm{d}^2q_+\mathrm{d}x_+}{4s(2\pi)^5 x_+ x_-}. \qquad \text{(C.3)}\end{aligned}$$

The tensor $I_{\mu\nu}$ has the form (see Eq. (A.1)),

$$\begin{aligned}\frac{1}{x_+x_-}I_{\mu\nu} = (1+\mathcal{P}_\pm)\Big\{&\frac{x_+}{a_+a'_+}(2q_\mu q_{+\nu} + (2-4x_+)q_\nu q_{+\mu} \\ &- 8x_-q_{+\mu}q_{-\nu}) + \frac{1}{a_-a'_+}(2x_+q_\mu q_{-\nu} + (-2x_+ \\ &+ 4x_+x_-)q_\nu q_{-\mu} - 2q_{-\nu}q_{+\mu} + (2-8x_+x_-) \\ &\times q_{+\nu}q_{-\mu}) + g_{\mu\nu}\big(\frac{1}{a_-a'_+}(-x_-k^2 - x_+k'^2 \\ &+ x_+x_-(q^2 - Q^2 - Q'^2)) + \frac{x_-}{a_-a'_-}(x_+(Q^2 \\ &+ Q'^2) + x_-q^2))\Big\}, \qquad \text{(C.4)}\end{aligned}$$

with

$$\begin{aligned}a_\pm = a + q_\pm^2, &\quad a'_\pm = b + (q_\pm - x_\pm q)^2, \\ a = m^2 + x_+x_-Q^2, &\quad b = m^2 + x_+x_-Q'^2,\end{aligned}$$

and $\mathcal{P}_\pm$ is the permutation operator. One can argue that the gauge condition $I^{\mu\nu} = 0$ for $k = 0, k' = 0$ is satisfied. Joining the denominators with the use of the Feynman trick and performing an integration over the transverse to the beam axis components of the quark pair momenta we get,

$$\begin{aligned}\int\frac{\mathrm{d}^2q_+}{\pi a_+ a'_+} = \int_0^1\frac{\mathrm{d}y}{D_{++}}, &\quad \int\frac{\mathrm{d}^2q_+}{\pi a_+ a'_-} = \int_0^1\frac{\mathrm{d}y}{D_{-+}}, \qquad \text{(C.5)}\\ D_{++} = A + \vec{q}^2x_+^2y^2, &\quad D_{-+} = A + y^2b^2, \\ b = k - x_+q&, \\ A = m^2 + x_+x_-(yQ'^2 &+ (1-y)Q^2).\end{aligned}$$

The result for IF takes the following form (we choose only the transverse polarizations of photons $\mu \equiv i, \nu \equiv j$).

$$\begin{aligned}\tau_{ij}^\gamma = 2\alpha^2\int_0^1 \mathrm{d}x_+\mathrm{d}x_-\delta(x_+ + x_- - 1)\int_0^1\mathrm{d}y\Big[\frac{x_+^2}{D_{++}} \\ \times (8x_+x_-y(1-y)q_iq_j - q^2\delta_{ij}(1 + 4x_+x_-y(1-2y))) \\ - \frac{1}{D_{-+}}(8x_+x_-y(1-y)b_ib_j - b^2\delta_{ij}(1 + 4x_+x_-y(1 \\ - 2y))) + 4x_+x_-y(x_+ - x_-)q_j\left(\frac{x_+q_i}{D_{++}} + \frac{b_i}{D_{-+}}\right) \\ + \delta_{ij}x_+x_-(Q^2 + Q'^2 + 4x_+x_-y(Q'^2 - Q^2)) \\ \times\left(\frac{1}{D_{-+}} - \frac{1}{D_{++}}\right)\Big], \qquad \text{(C.6)}\end{aligned}$$

with $D\pm\pm$ defined as above and $b = k - x_+q$. Once again it is clearly seen that the gauge conditions are satisfied,

$$\tau|_{k=0} = \tau|_{k=q} = 0.$$

It is important to note that even for the on-mass shell photons $Q^2 = Q'^2 = 0$ this expression differs from the one derived by Cheng and Wu [7]. The difference is found to be

$$\begin{aligned}\Delta\tau_{ij}^\gamma = \tau - \tau_{CW} = 4\alpha^2\int\mathrm{d}x_+\mathrm{d}y\; x_+x_-(1-2x_+)q_j \\ \times\left(\frac{x_+q_i}{D_{++}^0} + \frac{b_i}{D_{-+}^0}\right), \qquad \text{(C.7)}\\ D_{++}^0 = m^2 + x_+^2y(1-y)q^2,\; D_{-+}^0 = m^2 + x_+^2y(1-y)b^2.\end{aligned}$$

The reason for this difference is in a different definition of the initial and final photons' 4-momenta. The similar results were obtained in the Ref [8]

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