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Gamma Band Odd-Even Staggering in Some Deformed Nuclei

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ABSTRACT

A complete investigation was carried out in studying the odd-even staggering (OES) of gamma bands energy levels in some deformed nuclei up to angular momentum L=13$\hbar$. With the help of Minkov treatment in the framework of a collective Vector Boson Model (VBM) with broken SU (3) symmetry. The OES behavior of deformed isotopes ^{162}Er, ^{164}Er, ^{166}Er, ^{156}Gd, ^{170}Yb and ^{232}Th was studied and discussed.

Key Words:** γ **Bands/ (OES) Staggering/ Deformed Nuclei.

INTRODUCTION

In their early investigations on nuclear rotational motion Bohr and Mottelson[1,2] referred that there are various types of deviation of nuclear collective rotations causing some order of effects in the structure of nuclear rotational spectra such as squeezing, back-bending and odd-even staggering (OES). A bifurcation of rotational bands into sequences of states differing by several units of angular momentum was caused due to the staggering effects.

The odd-even staggering was observed in the collective γ bands[1]. Flibotte[3], Cederwall[4] and Bonatsos et al [5], suggested that the ΔL=1, ΔL=2 and ΔL=4 staggering in super deformed bands and ΔL=2 staggering in the ground –state band of normally deformed nuclei [6] was also observed.

In even-even nuclei [1] the odd-even staggering represents a relative displacement of the odd angular momentum levels of the γ band with respect to their neighboring levels with even angular momentum. Also Bonatsos[7] and Fields[8] stated that the OES has been interpreted as a result of the interaction between the even angular momentum of the γ band and corresponding levels of the β band, such suggestion has been addressed to the SU(3) limit of the Interaction Boson Model (IBM)[9] in which the lowest β and γ rotational bands interact in the frame work of the same irreducible representation[9] (λ, μ=2) of the group SU(3).

From the above review Bonatsos [5] suggested that in some nuclei the β band, which should be responsible for the OES in the γ band is not observed experimentally[10]. While Menkov[11] and others suggested that the OES in the gamma vibrational bands of heavy deformed nuclei can be reasonably characterized by a discrete approximation of the fourth derivative of the odd-even energy difference as a function of angular momentum L.

They refer that such OES can be interpreted as a result of the interaction of the γ band with the ground state band in the frame work of a Vector Boson Model(VBM) with SU(3) dynamical symmetry. In their new work, they discuss such staggering in some heavy elements of mass number around A=180.

In the present study we follow such suggestion in some heavy elements of A between 150-180 and A>200 showing to what extent the staggering phenomenon takes place in the nuclei under investigations.

Basic Method of Calculation

Minkov et al[11] proposed the following parameters and numerical calculations used in determination of the reduced transition probability B(E2) inter band transition ratios as

$$R_1(L) = \frac{B[E2; L_\gamma \to L_g]}{B[E2; L_\gamma \to (L-2)_g]} \qquad , \qquad L\ (\text{even})$$

$$R_2(L) = \frac{B[E2; L_\gamma \to (L+2)_g]}{B[E2; L_\gamma \to L_g]} \qquad , \qquad L\ (\text{even})$$

$$R_3(L) = \frac{B[E2; L_\gamma \to (L+1)_g]}{B[E2; L_\gamma \to (L-1)_g]} \qquad , \qquad L\ (\text{odd}) \qquad (1)$$

and the ground state band (gsb) inter band transition ratio as

$$R_4(L) = \frac{B[E2; L_g \to (L-2)_g]}{B[E2; (L-2)_g \to (L-4)_g]} \qquad , \qquad L(\text{even}) \qquad (2)$$

and making use of the Hamiltonian matrix elements, the ground state band (gsb) and the gamma band energy levels can be obtained in the form

$$E_g(L) = AL(L+1) - B[\sqrt{1 + CL(L+1)^2 + DF(L)} - 1] \qquad (3)$$

$$E_\gamma(L) = 2B + AL(L+1) + B[\sqrt{1 + CL(L+1)^2 + DF(L)} - 1] \qquad , \qquad L(even) \qquad (4)$$

$$E_\gamma(L) = 2B + AL(L+1) \qquad , \qquad L(odd) \qquad (5)$$

Where

$$A = g_1 - (2\lambda + 5)g_2 - g_3 \qquad (6)$$

$$B = 6(2\lambda + 5)g_2 - 2(\lambda + 3)^2 g_3 \qquad (7)$$

$$C = \frac{1}{6(2\lambda + 5)} \; \frac{g_3}{g_2} \qquad (8)$$

$$D = \frac{12}{B^2}\left(3g_2^2 - g_2 g_3\right) \qquad (9)$$

and

$$F(L) = L(L-1)(L+1)(L+2) \qquad (10)$$

Where A could be interpreted as the inertia term, corresponding to the none mixed part of the energy levels.
2B has the meaning of the γ head band.
C and D contribute to the mixed of the energy levels.

and F(L) coincides with the square of the ΔK=2 band mixing term of the Bohr –Mottelson model [1].

Minkov [11]rewrite equations (3), (4) as follows

$$E_\nu(L)=\left(\frac{1}{2\vartheta_0}+\frac{1}{2\vartheta_L}\right)L(L+1)+\frac{1}{2\vartheta_L}\frac{D}{C}F(L)+\frac{C}{2\vartheta_L}L^2(L+1)^2 \qquad (11)$$

where L is even and stands for gb or γb

$$\text{and}\quad \vartheta_0=\frac{1}{2A} \qquad \text{and}\quad \vartheta_L=\frac{1}{BC}\left(1+\frac{1}{2B}\Delta E(L)\right) \qquad (12)$$

$$\text{With}\quad \Delta E(L)=E(L)-\frac{1}{2\vartheta_0}L(L+1) \qquad (13)$$

Where ϑ_0 and ϑ_L represented ground state and excited states moment of inertia. The first term is the moment of inertia, which is angular momentum dependent which is similar to (VMI) model[12].

Making use of Powell's method [13], the quality of the energy fits is measured by

$$\sigma_E=\sqrt{\frac{1}{n_E}\sum_\nu\left[E_\nu^{th}-E_\nu^{exp}\right]^2}$$

This is the standard energy rms deviation with n_E being equal to the number of levels used in fitting.

Table (1):Parameters of energy levels fitting of the ground state band & gamma band.

Nuclei	A	B	C	a	b	g1	g2	g3	λ	σ (E)
Er^{162}	13.76	434.4	-1.4 E-4	-4.83 E-3	2.07 E-5	-1.74	-0.503	-1.04	13	37.72
Er^{164}	12.76	406.15	-8.44E-4	-4.46E-3	1.40E-5	-0.044	-0.404	- 0.903	13	24.42
Er^{166}	11.73	359.85	-1.43E-3	-2.9E-3	596E-6	2.759	-0.233	-0.52	16	31.75
Gd^{156}	12.80	555.97	4.98E-4	-2.76E-3	1.35E-5	141.91	3.332	0.277	17	21.8
Yb^{170}	11.5	540	-1.23E-3	-2.02345	9.385E-7	3.71	-0.19	- 0.665	18	43.49

Th^{232}	6.5	382.57	-1.1E-3	-1.356E-3	8.13E-7	1.125	-0.125	-.042	19	18.3

Table(2): The experimental and theoretical values of γ band energies in KeV

L	^{162}Er		^{164}Er		^{166}Er		^{156}Gd		^{170}Yb		^{232}Th	
	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp	Theo.	Exp.	Theo	Exp
2	945.6	900.7	885.6	860.3	790.1	785.9	1182.6	1154	1149.1	1145.7	798.9	785
3	1033.9	1001.9	965.5	946.4	860.5	859.4	1265.5	1247	1218.0	1225.4	836.9	829
4	1125.4	1128.2	1057.1	1058.3	954.5	956.2	1348.3	1355	1310.3	1329.8	892.3	890
5	1281.7	1286.3	1195.2	1197.6	1071.6	1075.3	1495.9	1506	1434.0	1459.8	953.9	960
6	1411.6	1459.7	1328.2	1358.6	1213.6	1216.0	1611.6	1644	1563.5	1601.5	1038.9	1050
7	1639.5	1669.2	1526.9	1544.9	1376.6	1376.0	1828.6	1849	1724.0	1780.6	1122.9	1142
8	1808.3	1872.8	1701.7	1744.7	1568.1	1556.0	1976.4	2010	1908.3	1954.6	1239.1	1260
9	2107.4	2133.9	1960.9	1977.1	1775.4	1751.4	2263.8	2249	2115.0	2214.4	1343.9	1370
10	2323.4	2346.7	2182.2	2184.1	2019.6	1964.1	2448.8	2442	2344.3	2373.1	1492.7	1512
11	2685.4	2656.6	2496.8	2479.4	2268.1	2189.1	—	—	2598.0	2677.5	1616.9	1639
12	2967.2	2911.1	2776.6	2733.1	2570.3	2429.0	—	—	2870.9	2827.1	1800.2	1800
13	—	—	—	—	2854.5	2654.0	—	—	3173.0	3168.5	1941.9	1947
14	—	—	—	—	3223.3	2880.0	—	—	3487.0	3307.5	2161.5	2123

On the above basis the quantity stg (L) was deduced by Minkov[11] to study the (OES) in the gamma bands of heavy deformed nuclei as follows:
In case of experimental data

$$Stg(L)=10E(L+1)+5E(L-1)+E(L+3)-[10E(L)+5E(L-1)+E(L-2)]$$

But in case of calculated theoretical value using the parameters deduced in Table (1) it is convenient to use the following equations

$$\begin{aligned}\mathrm{Stg}(L)=&\frac{B}{2}\{10R(L+1)+5R(L-1)+R(L+3)\}\left[1+(-1)^{L+1}\right]\\&-\frac{B}{2}\{10R(L)+5R(L+2)+R(L-2)\}\left[1+(-1)^{L}\right]\end{aligned}$$

where

$$R(L) = \sqrt{1 + aL(L+1) + bL^2(L+1)^2} - CL(L+1) - 1$$

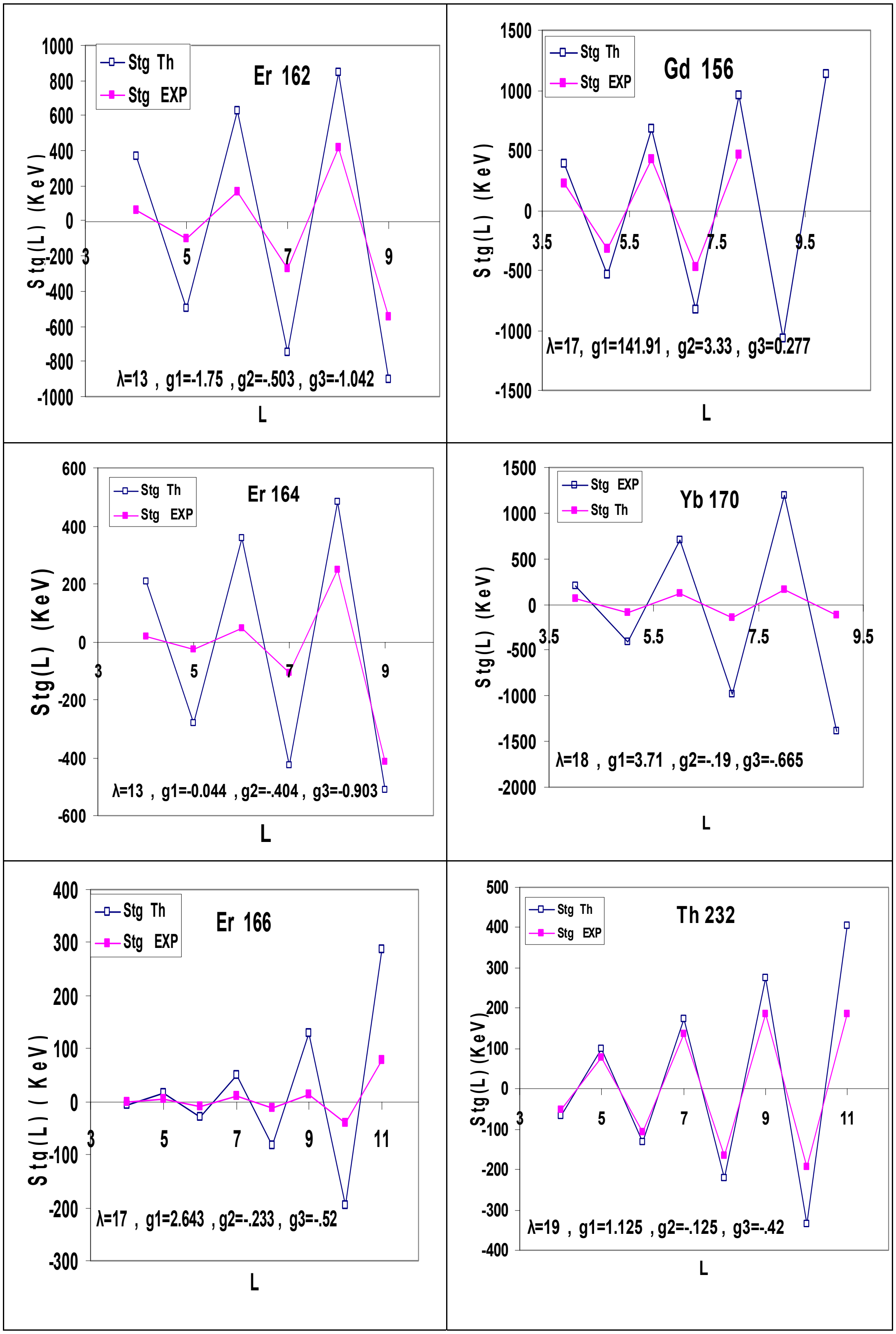

Figure (1): Experimental and theoretical representation of stg(L) in case of ^{162}Er, ^{164}Er, ^{166}Er, ^{156}Gd, ^{170}Yb and ^{232}Th.

RESULTS AND DISCUSSION

Making use of the experimental values [14] 0f both the ground state band and gamma band energies, the parameters of Minkov[11] frame work equations were calculated and shown in Table(1), together with the standard deviation values (σ_E) for all isotopes under study with the aid of such parameters Table(2) shows the calculated values of gamma band level energies.

Figure(1) represented the staggering pattern (zigzagging behavior) of the function stg.(L) with respect to the angular momentum (L) of all the gamma bands observed in isotopes under investigations. Both experimental and calculated values of stg.(L) were represented as a comparison. From Figure (1) it is clear that there is a regular change in the sign of stg. (L) between the odd and even angular momentum levels with increasing in amplitude as L increasing up to L= $9 \rightarrow 13$.

It is also clear that the difference between the theoretical and the experimental staggering magnitude is larger in case of ^{162}Er, ^{164}Er than in case of ^{166}Er, ^{170}Yb and ^{232}Th. One can attributed such differences to the rotational structure of both ground bands and gamma bands or/and to the increase of neutron number.

Also the situation of the investigated isotopes in the beginning or the end of deformed region $150<A<180$ may be responsible for such showings. Such conclusion may be correct according to Minkov[11,15] remarks about the interaction between ground gamma band interaction with VBM, where such interaction systematically decreases towards the middle of rotational regions.

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