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Time-Dependent Radiation Transfer with Rayleigh Scattering in Finite Plane-Parallel Media using Pomraning-Eddington Approximation

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ABSTRACT

The time-dependent radiation transfer equation in plane geometry with Rayleigh scattering is studied. The traveling wave transformation is used to obtain the corresponding stationary-like equation. Pomraning-Eddington approximation is then used to calculate the radiation intensity in finite plane-parallel media. Numerical results and shielding calculations are shown for reflectivity and transmissivity at different times. The medium is assumed to have specular-reflecting boundaries. For the sake of comparison, two different weight functions are introduced and to force the boundary conditions to be fulfilled.

Keywords: Radiation Transfer, Time-Dependent, Rayleigh Scattering

INTRODUCTION

Analytical and numerical benchmark solutions to time-dependent transport problems have been of interest to the scientific community both from the standpoint of purely mathematical interest and for the purpose of providing benchmark solutions [1-10].

Several mathematical and numerical techniques and procedures [1-6] were employed to arrive at solutions for the time-dependent angular and scalar fluxes. Among them were Fourier transforms, Laplace transforms, Bromwich contour integrations for inversion of the Laplace transforms, analytical or numerical inversion in the complex plane, polynomial expansions, the Neumann series expansion (which is also known as the collision by collision method, the multiple collision method, the method of successive approximation and the orders-of-scattering technique), the method of characteristics and numerical integration.

In this paper, the monoenergetic time-dependent radiation transfer equation in a finite slab medium is solved. The medium is considered to has specular-reflecting boundaries with angular-dependent externally-incident flux. The problem is transformed into a stationary-like problem via the traveling wave transformation. Pomraning-Eddington approximation is then used to find the solution. Numerical results are done for reflectivity and transmissivity of finite slabs of different thicknesses. Two weight functions are introduced to force the assumed boundary conditions to fulfill and for the aim of comparison.

ANALYSIS

Consider the time-dependent, monoenergetic, radiative transfer equation with anisotropic scattering as [11,12]

$$\left[\frac{1}{c}\frac{\partial}{\partial t}+\mu\frac{\partial}{\partial z}+\sigma(z)\right]\Psi(t,z,\mu)=\frac{\sigma(z)}{2}\int_{-1}^{1}P(\mu,\mu')\Psi(t,z,\mu')d\mu'+Q(t,z)$$

$$0\le z\le b,\quad -1\le\mu\le 1 \qquad \textbf{(1)}$$

where $\Psi(t,z,\mu)$ is the radiation intensity with temporal variable *t*, geometrical space variable *z*, and the angular variable μ, c is the radiation speed,

$\sigma(z)$ is the total cross-section, $\sigma_s(z)$ is the scattering cross-section, and *Q(t,z)* represents an internal source of radiation.
The anisotropic scattering phase function $P(\mu,\mu')$ is given by [13]

$$P(\mu,\mu')=\sum_{n=0}^{\infty}a_n P_n(\mu)P_n(\mu') \qquad \textbf{(2)}$$

where $P_n(\mu)$ is the Legendre polynomial functions, with $P_0(\mu)=1$, and a_0=1.

In this paper Rayleigh scattering is considered, so the scattering phase function has the form

$$P(\mu,\mu')=1+\frac{1}{2}P_2(\mu)P_2(\mu')=\frac{3}{8}\left[\left(3-\mu^2\right)+\left(3\mu^2-1\right)\mu^2\right] \qquad \textbf{(3)}$$

It is convenient to write Eq.(1) in terms of the optical depth space variable

$$x(z)=\int_0^z \sigma(z)dz \qquad ,0\le x\le d \qquad \textbf{(4.a)}$$

where the optical thickness of the medium is

$$d(b)=\int_0^b \sigma(z)dz \qquad \textbf{(4.b)}$$

In terms of *x* Eq.(1) becomes

$$\left[\frac{1}{u}\frac{\partial}{\partial t}+\mu\frac{\partial}{\partial x}+1\right]\Phi(t,x,\mu)=\frac{\omega}{2}\int_{-1}^{1}P(\mu,\mu')\Phi(t,x,\mu')d\mu'+S(t,x) \qquad \textbf{(5)}$$

where

$$\Phi(t,x,\mu)\equiv\Psi(t,z,\mu) \qquad \textbf{(6.a)}$$

$$u=c\sigma,\quad \omega=\sigma_s/\sigma \qquad \textbf{(6.b)}$$

$$S(t,x)=Q(t,z)/\sigma \qquad \textbf{(6.c)}$$

Equation (5) is assumed to subject to boundary conditions

$$\Phi(t,0,\mu)=\Lambda(\mu)+\rho_1^s\Phi(t,0,-\mu), \qquad \text{for } t=0,\mu>0 \qquad \textbf{(7.a)}$$

$$\Phi(t,d,-\mu)=\rho_1^s\Phi(t,d,\mu), \qquad \text{for } t>0,\mu>0 \qquad \textbf{(7.b)}$$

where Λ(μ) is the angular-dependent externally-incident flux on the left boundary, and ρ_i^s is the specular reflectivity of the boundaries, *(i=1,2).*
Using the transformation [7,8]

$$\eta=x+ut \qquad \textbf{(8)}$$

to have

$$\left[(1+\mu)\frac{\partial}{\partial\eta}+1\right]\Phi(\eta,\mu)=\frac{\omega}{2}\int_{-1}^{1}P(\mu,\mu')\Phi(\eta,\mu')d\mu'+S(\eta) \qquad \textbf{(9)}$$

For source-free problem, Eq.(3) in Eq.(9) will give

$$\left[(1+\mu)\frac{\partial}{\partial\eta}+1\right]\Phi(\eta,\mu)=\frac{3\omega}{16}\left[\left(3-\mu^2\right)E(\eta)+\left(3\mu^2-1\right)\int_{-1}^{1}\mu^2\Phi(\eta,\mu')d\mu'\right] \qquad \textbf{(10)}$$

Using **Pomraning-Eddington approximation**[14,15] as

$$\Phi(\eta,\mu)=\Sigma(\eta,\mu)E(\eta)+O(\eta,\mu)F(\eta) \qquad \textbf{(11)}$$

where *E(η)* is the radiant energy, and *F(η)* is the net radiant heat flux, which are defined by

$$E(\eta) = \int_{-1}^{1} \Phi(\eta,\mu)d\mu, \text{ and } F(\eta) = \int_{-1}^{1} \mu\Phi(\eta,\mu)d\mu \qquad \textbf{(12)}$$

and Σ(η,μ) and O(η,μ) are even and odd functions in μ and are slowly varying functions in η which satisfied the normalization conditions

$$\int_{-1}^{1} d\mu\Sigma(\eta,\mu) = 1 \qquad \textbf{(13)}$$

$$\int_{-1}^{1} d\mu\mu O(\eta,\mu) = 1 \qquad \textbf{(14)}$$

Substituting Eq.(11) in Eq.(10), integrating over $\mu \in$[-1,1] and using Eqs.(13, 14) one gets

$$\frac{dE(\eta)}{d\eta} + \frac{dF(\eta)}{d\eta} + \alpha E(\eta) = 0 \qquad \textbf{(15)}$$

Multiplying of Eq.(10) by μ and integrate over $\mu \in$[-1,1] gives

$$D\frac{dE(\eta)}{d\eta} + \frac{dF(\eta)}{d\eta} + E(\eta) = 0 \qquad \textbf{(16)}$$

where

$$\alpha = 1 - \omega \qquad \textbf{(17)}$$

$$\mathrm{D} = \int_{-1}^{1} \mu^2\Sigma(\eta,\mu)d\mu \qquad \textbf{(18)}$$

Equations (15) and (16) leads to the second-order differential equation satisfied by *E(η)* as

$$(1-D)\frac{d^2E(\eta)}{d\eta^2} + (1+\alpha)\frac{dF(\eta)}{d\eta} + \alpha E(\eta) = 0 \qquad \textbf{(19)}$$

whose solution is

$$E(\eta) = Ae^{-v^+\eta} + Be^{-v^-\eta} \qquad \textbf{(20)}$$

Moreover

$$F(\eta) = \gamma^+ Ae^{-v^+\eta} + \gamma^- Be^{-v^-\eta} \qquad \textbf{(21)}$$

where *A* and *B* are constants to be determined, and

$$2v^{\pm} = \frac{1+\alpha}{1-D} \mp \sqrt{\left(\frac{1+\alpha}{1-D}\right)^2 - \frac{4\alpha}{1-D}} \qquad \textbf{(22)}$$

$$\gamma^{\pm} = \alpha - (1-D)v^{\pm} \qquad \textbf{(23)}$$

Even and Odd functions are obtained by substituting Eq.(11) in Eq.(10), as

$$\Sigma(\mu) = \left(\frac{3\omega}{16}\right)\frac{3-D+(3D-1)\mu^2}{1-R-(\alpha-R)\mu^2/D} \qquad \textbf{(24)}$$

and

$$O(\mu) = \left(\frac{3\omega}{16}\right)\frac{\left[3-D+(3D-1)\mu^2\right]\mu}{D(1-R)-(\alpha-R)\mu^2/D} \qquad \textbf{(25)}$$

The parameter *R* is defined by

$$R = -\frac{1}{E(\eta)}\frac{dE(\eta)}{dE(\eta)} \qquad \textbf{(26)}$$

The two unknown parameters R and D can be determined by solving the two coupled transcendental equations obtained from substituting Eq.(24) into Eqs.(13) and (18).

Substituting Eqs.(20, 21, 24, and 25) into Eq.(11) give the **solution** in the form of

$$\Phi(\eta,\mu) = Ah_+(\mu)e^{-v^+\eta} + Bh\_e^{-v^-\eta} \qquad \textbf{(27)}$$

or

$$\Phi(t,x,\mu) = Ah_+(\mu)e^{-v^+x}e^{-v^+ut} + Bh\_e^{-v^-x}e^{-v^-ut} \qquad \textbf{(28)}$$

with

$$h_\pm(\mu) = \left(\frac{3\omega}{16}\right)\frac{\left(\gamma^\pm\mu + D\right)\left[3 - D + (3D-1)\mu^2\right]}{D\left(1-R\right) - (\alpha - R)\mu^2} \qquad \textbf{(29)}$$

To determine the constants *A* and *B*, a weight function *W(μ)* is introduced in order to force the boundary conditions to be fulfilled, as

$$\int_0^1 d\mu W(\mu)\left[\Phi(t,0,\mu) - \Lambda(\mu) + \rho_1^s\Phi(t,0,-\mu)\right] = 0, \quad \text{at } t = 0 \qquad \textbf{(30.a)}$$

$$\int_0^1 d\mu W(\mu)\left[\Phi(t,d,-\mu) - \rho_1^s\Phi(t,d,\mu)\right] = 0, \quad \text{at } t > 0 \qquad \textbf{(30.b)}$$

Using Eq.(28) we obtain

$$A = -J_0(J_-^- - \rho_2^s J_-^+)\exp\left[-\xi\left(d+ut\right)\right]/Q \qquad \textbf{(31)}$$

$$B = J_0\left(J_+^- - \rho J_+^+\right)/Q \qquad \textbf{(32)}$$

with

$$Q = (J_-^+ - \rho_1^s J_-^-)(J_+^- - \rho_2^s J_+^+) - (J_+^+ - \rho_1^s J_+^-)(J_-^- - \rho_2^s J_-^+)\exp\left[-\xi\left(d+ut\right)\right] \qquad \textbf{(33)}$$

where

$$\xi = v^- - v^+ \qquad \textbf{(34.a)}$$

$$J_0 = \int_0^1 d\mu W(\mu)\Lambda(\mu), \qquad \textbf{(34.b)}$$

$$J_\pm^\mp = \int_0^1 d\mu W(\mu)h_\pm(\mp\mu), \qquad \textbf{(34.c)}$$

NUMERICAL RESULTS AND CALCULATIONS

In this section numerical calculations are done to compute the reflectivity R_r and transmissivity T_r for a slab of thickness *d*. The reflectivity R_r and transmissivity T_r functions are defined as

$$R_r = \int_0^1 d\mu\mu I(t,0,-\mu), \qquad \text{at } t = 0, \mu > 0 \qquad \textbf{(35)}$$

$$T_r = \int_0^1 d\mu\mu I(t,d,-\mu), \qquad \text{at } t \geq 0, \mu > 0 \qquad \textbf{(36)}$$

Using Eq.(28) in Eqs.(35) and (36) one gets

$$R_r = AG_1^r + BG_2^r \qquad \textbf{(37)}$$

$$T_r = AG_1^t e^{-v^+(d+ut)} + BG_2^t e^{-v^-(d+ut)} \qquad \textbf{(38)}$$

where

$$G_1^r = \int_0^1 d\mu\mu h_+(-\mu), G_2^r = \int_0^1 d\mu\mu h_-(-\mu) \quad \textbf{(39)}$$

and

$$G_1^t = \int_0^1 d\mu\mu h_+(\mu), G_2^t = \int_0^1 d\mu\mu h_-(\mu) \quad \textbf{(40)}$$

The angular-dependent externally-incident flux $\Lambda(\mu)$ is assumed to have the form

$$\Lambda(\mu) = \mu^l, \quad 1 = 0,1,2,.... \quad \textbf{(41)}$$

Two different weight functions are suggested to do the calculations and for the aim of comparison, namely [14,15]

$$W_1(\mu) = \mu \quad \textbf{(42.a)}$$

$$W_2(\mu) = \frac{\sqrt{3}}{2}\mu(1+\frac{3}{2}\mu) \quad \textbf{(42.b)}$$

Table (1) gives the results of the reflectivity R_r at the left boundary and the transmissivity T_r from the right boundary for the case of transparent boundaries, ρ_i^s = 0.0, and angular-independent externally-incident flux in the form $\Lambda(\mu)$=1. In this table we show the variation of R_r and T_r with the scattering albedo ω for different thicknesses *d* of the slab at some instants *t*.

Table (2) shows the data of the reflectivity R_r and the transmissivity T_r for transparent left boundary, ρ_1^s = 0.0, specular-reflecting right boundary, ρ_2^s = 0.25, and the externally-incident flux in the form $\Lambda(\mu)$=μ.

The results of R_r and T_r in table (3) are done for transparent left boundary, specular-reflecting right boundary, ρ_2^s =0.5, and the externally-incident flux in the form $\Lambda(\mu)=\mu^2$

CONCLUSION

The reflectivity at the left boundary and transmissivity from the right boundary of a finite slab medium of thickness *d* are calculated for the time-dependent monoenergetic neutral particle transport equation with Rayleigh scattering.

The medium is considered to have specular-reflecting boundaries and angular-dependent externally-incident flux.

The time-dependent problem is transformed into a stationary-like problem using the traveling wave transformation, and then Pomraning-Eddington approximation is used to solve it. A weight function is introduced to force the assumed boundary conditions to be fulfilled. Because of the lake of the corresponding data in the literatures we use two different weight functions for the aim of comparison.

Table(1) The reflectivity R_r and The transmissivity T_r for $\Lambda(\mu) = 1$ and $\rho_j^s=0.0$

ω	R_r		T_r		R_r		T_r	
	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$
	t = 0.0				t = 0.1			
	d = 0.1							
0.70	0.33265	0.32058	0.47743	0.46249	0.33628	0.32313	0.45474	0.43986
0.80	0.39270	0.37892	0.47848	0.46335	0.39572	0.38064	0.45645	0.44122
0.90	0.44896	0.43364	0.47951	0.46420	0.45077	0.43391	0.45812	0.44255
	d = 1.0							
0.70	0.37835	0.35222	0.27280	0.25947	0.38498	0.35672	0.25112	0.23823
0.80	0.42815	0.39866	0.27106	0.25704	0.43286	0.40122	0.24863	0.23513
0.90	0.46875	0.43646	0.26915	0.25704	0.47115	0.43680	0.24608	0.23216
	d = 3.0							
0.70	0.48274	0.42037	0.02267	0.02071	0.48484	0.42169	0.01939	0.01769
0.80	0.49154	0.43199	0.02202	0.02013	0.49260	0.43253	0.01889	0.01727
0.90	0.49695	0.44027	0.02165	0.01984	0.49735	0.44032	0.01865	0.01708
0.95	0.49872	0.44347	0.02154	0.01977	0.49889	0.44336	0.01859	0.01705
	d = 6.0							
0.70	0.49973	0.43095	0.00016	0.00015	0.49976	0.43097	0.00014	0.00012
0.80	0.49988	0.43620	0.00019	0.00017	0.49989	0.43621	0.00016	0.00014
0.90	0.49996	0.44066	0.00022	0.00020	0.49997	0.44066	0.00018	0.00017
	t = 1.0				t = 4.0			
	d = 0.1							
0.70	0.38498	0.35672	0.25112	0.23823	0.49609	0.42869	0.00383	0.00348
0.80	0.43286	0.40122	0.24863	0.23513	0.49815	0.43533	0.00393	0.00358
0.90	0.47115	0.43680	0.24608	0.23216	0.49936	0.44058	0.00407	0.00372
	d = 1.0							
0.70	0.44425	0.39588	0.09568	0.08869	0.49889	0.43043	0.00085	0.00078
0.80	0.47065	0.42128	0.09194	0.08507	0.49948	0.43600	0.00093	0.00085
0.90	0.48863	0.43917	0.08894	0.08226	0.49983	0.44064	0.00101	0.00093
	d = 3.0							
0.70	0.49551	0.42833	0.00452	0.00410	0.49993	0.43108	0.00003	0.00003
0.80	0.49787	0.43519	0.00461	0.00419	0.49997	0.43625	0.00004	0.00003
0.90	0.49926	0.44057	0.00475	0.00434	0.49999	0.44066	0.00005	0.00004
	d = 6.0							
0.70	0.49993	0.43108	0.00003	0.00003	0.50000	0.43112	2×10^{-7}	2×10^{-7}

0.80	0.49997	0.43625	0.00004	0.00003	0.50000	0.43626	3×10^{-7}	3×10^{-7}
0.90	0.49999	0.44066	0.00005	0.00004	0.50000	0.44066	4×10^{-7}	4×10^{-7}

Table(2) The reflectivity R_r and The transmissivity T_r for $\Lambda(\mu)=\mu$ and $\rho_1^s=0.0$, $\rho_2^s=0.25$

d	R_r		T_r		R_r		T_r	
	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$
	t = 0.0				t = 0.1			
				ω=0.7				
0.01	0.21276	0.22009	0.33208	0.34482	0.21416	0.22112	0.31957	0.33166
0.10	0.21401	0.22101	0.32083	0.33298	0.21556	0.22215	0.30830	0.31980
1.00	0.23577	0.23694	0.20621	0.21232	0.23939	0.23956	0.19332	0.19879
3.00	0.31397	0.29217	0.02397	0.02399	0.31619	0.29369	0.02066	0.02067
6.00	0.33301	0.30516	0.00018	0.00018	0.33305	0.30518	0.00015	0.00015
				ω=0.9				
0.01	0.29561	0.30658	0.33229	0.34501	0.29648	0.30671	0.32152	0.33338
0.10	0.28639	0.30670	0.32262	0.33457	0.29733	0.30685	0.31133	0.32239
1.00	0.30794	0.30846	0.20330	0.20715	0.30953	0.30870	0.18869	0.19179
3.00	0.33033	0.31172	0.01982	0.01953	0.33071	0.31177	0.01712	0.01687
6.00	0.33329	0.31213	0.00020	0.00020	0.33330	0.31213	0.00017	0.00017
	t = 1.0				t = 4.0			
				ω=0.7				
0.01	0.23612	0.23719	0.20492	0.21096	0.32811	0.30183	0.00494	0.00492
0.10	0.23939	0.23956	0.19332	0.19879	0.32871	0.30224	0.00427	0.00425
1.00	0.27880	0.26772	0.08820	0.08942	0.33201	0.30448	0.00096	0.00096
3.00	0.32804	0.30178	0.00502	0.00500	0.33325	0.30532	0.00003	0.00003
6.00	0.33325	0.30532	0.00003	0.00003	0.33333	0.30537	2×10^{-7}	2×10^{-7}
				ω=0.9				
0.01	0.30810	0.30849	0.20184	0.20561	0.33261	0.31203	0.00435	0.00428
0.10	0.30953	0.30870	0.18869	0.19179	0.33269	0.31205	0.00379	0.00373
1.00	0.32279	0.31065	0.07662	0.07636	0.33316	0.31211	0.00095	0.00093
3.00	0.33260	0.31203	0.00442	0.00434	0.33332	0.31213	0.00004	0.00004
6.00	0.33332	0.31213	0.00004	0.00004	0.33333	0.31214	4×10^{-7}	4×10^{-7}

Table(3) The reflectivity R_r and The transmissivity T_r for $\Lambda(\mu) = \mu^2$ and $\rho_1^s = 0.0$, $\rho_2^s = 0.5$

t	R_r		T_r		R_r		T_r	
	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$	$W_1(\mu)$	$W_2(\mu)$
	ω=0.7				ω=0.8			
	d = 0.1							
0.0	0.15460	0.16681	0.24256	0.26281	0.18795	0.20297	0.24339	0.26358
0.1	0.15498	0.16707	0.23521	0.25494	0.18853	0.20332	0.23668	0.25626
1.0	0.16185	0.17168	0.17154	0.18719	0.19807	0.20910	0.16915	0.18256
4.0	0.23884	0.22815	0.00974	0.01150	0.24682	0.23809	0.00592	0.00629
	d = 1.0							
0.0	0.16066	0.17088	0.17855	0.19462	0.19654	0.20818	0.17739	0.19155
0.1	0.16185	0.17168	0.17154	0.18719	0.19807	0.20910	0.16915	0.18256
1.0	0.17993	0.18416	0.10718	0.11909	0.21673	0.022031	0.09166	0.09831
4.0	0.24660	0.23436	0.00233	0.00278	0.24909	0.23941	0.00144	0.00152
	d = 3.0							
0.0	0.21289	0.20810	0.04345	0.04994	0.23708	0.23237	0.02955	0.03148
0.1	0.21609	0.21052	0.03865	0.04457	0.23850	0.23321	0.02582	0.02749
1.0	0.23736	0.22698	0.01132	0.01336	0.24636	0.23782	0.00691	0.00734
4.0	0.24979	0.23695	0.00008	0.00009	0.24995	0.23991	0.00006	0.00006
	ω=0.9				ω=0.95			
	d = 0.1							
0.0	0.22006	0.23777	0.24422	0.26434	0.23535	0.25437	0.24463	0.26474
0.1	0.22052	0.23785	0.23811	0.25754	0.23563	0.25415	0.23882	0.25819
1.0	0.22751	0.23896	0.16652	0.17810	0.23961	0.25408	0.16516	0.17597
4.0	0.24915	0.24224	0.00473	0.00489	0.24968	0.24369	0.00442	0.00454
	d = 1.0							
0.0	0.22648	0.23880	0.17588	0.18842	0.23904	0.25151	0.17503	0.18688
0.1	0.22751	0.23896	0.16652	0.17810	0.23961	0.25108	0.16516	0.17597
1.0	0.23804	0.24058	0.08120	0.08541	0.24492	0.24711	0.07720	0.08069
4.0	0.24977	0.24233	0.00119	0.00123	0.24991	0.24352	0.00114	0.00116
	d = 3.0							
0.0	0.24615	0.24180	0.02371	0.02462	0.24848	0.24454	0.02194	0.02265
0.1	0.24662	0.24187	0.02062	0.02140	0.24868	0.24440	0.01907	0.01967
1.0	0.24902	0.24222	0.00550	0.00569	0.24963	0.24373	0.00513	0.00527
4.0	0.24999	0.24236	0.00005	0.00005	0.25000	0.24347	0.00005	0.00005

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