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Diffusion From a Point Source in an Urban Atmosphere

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ABSTRACT

In the present paper, a model for the diffusion of material from a point source in an urban atmosphere is incorporated. The plume is assumed to have a well-defined edge at which the concentration falls to zero. The vertical wind shear is estimated using logarithmic law, by employing most of the available techniques of stability categories. The concentrations estimated from the model were compared favorably with the field observations of other investigators.

Key Words: Urban Atmosphere/ Concentration/ Wind Shear/ Stability Categories

INTRODUCTION

Most models for urban air pollution are based on Gaussian plume diffusion or Sutton's equations or the K-theory and require a digital computer.

There are some simpler models such as (1,2,3) In these models the assumptions used , for example uniform wind, uniform mixing in the mixing layer, ground terrain, etc. are not very realistic and , therefore, these assumptions restrict the use of the model for some special cases which don't normally occur in real life.

The purpose of this study is to suggest a simple physical realistic model which depends on the surface roughness of the ground area over which diffusion of pollution takes place; also, wind speed is treated as a function of height and stability of the atmosphere.

This definitely more closely represents real life situations than does treating wind as a constant quantity. The problem of diffusion and advection of conservative material as it travels downwind is investigated

PROPOSED MODEL AND ITS COMPONENTS

A point source normal to mean wind direction with height "h" situated at the ground level and emission rate, Q in studying diffusion and advection of air pollutant. The problem is two –dimensional in nature because homogeneity in the lateral direction is assumed.

Fig. 1. describes the coordinate system direction of the mean wind. The effective height denoted by $H = hs + \Delta h$, where hs is the stack height and Δh is the plume height which increases as the plume travel downwind. The mean wind is $\bar{u}(z)$.

The analysis that follows assumes steady-state conditions; that is, the variables of interest; for example the mean wind, stability of the atmosphere, source strength, etc . don't change in the time interval of interest. The ground surface is treated as a complete reflector of matter; that is, no removal occurs.

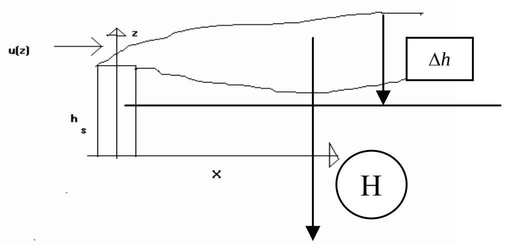

Fig. (1): Diagram of atmospheric diffusion until the edge of the plume.

Approach Used:

A very simple approach, namely the principle of conservation of mass with a steady state can be written as:

$$Q = \int_0^H \bar{u}(z)C(z)dz \qquad \textbf{(1)}$$

where
$\bar{u}(z)$ is the mean wind speed,
$C(z)$ is the concentration of material, and
H is the effective height of the plume.

In subsequent sections of this paper the different variables in equations (1), namely, the wind profile, concentration profile and effective height of the plume will be discussed in detail. The integration of equation (1) will lead to the mathematical model.

WIND PROFILE

a-Logarithmic Wind Law:

According to(4), the adiabatic wind profile is given by:

$$\frac{d\bar{u}}{dz} = \frac{\upsilon_*}{kz}\phi_m \qquad \textbf{(2)}$$

where Φ_m is a dimensionless parameter which depends on (z/L).

$$\phi_m = 1 + \beta_1(z/L) + \beta_2(z/L)^2 + \ldots$$

where β_1, β_2 etc. are constants and L is the stratification parameter, v_* is the friction velocity and k is the von-Karman's constant.

Equation (2) is valid for approximately the lower tenth of the planetary boundary layer. Several authors(4) shows that the following form for the function Φ_m.

$$\mathbf{\Phi_m = 1} \qquad \textbf{Neutral stratification} \qquad \textbf{(3a)}$$

$$\mathbf{\Phi_m = 1 + 5.2\ z/L} \qquad \textbf{stable stratification} \quad \mathbf{L>0} \qquad \textbf{(3b)}$$

$$\mathbf{\Phi_m = (1 - 16z/L)^{-1/4}} \qquad \textbf{unstable stratification} \ \mathbf{L<0} \qquad \textbf{(3c)}$$

Integrate equation (2) from roughness height z_o to height z using conditions (3) which yields the following schemes.

b- For Neutral Case:

$$u(z)=\frac{\upsilon_*}{k}\ln\left(\frac{z+z_0}{z_0}\right) \tag{4}$$

c- For Stable Case :

$$u(z)=\frac{\upsilon_*}{k}\left[\ln\left(\frac{z+z_0}{z_0}\right)+\frac{5.2z}{L}\right] \tag{5}$$

d- For Unstable Case:

$$u(z)=\frac{\upsilon_*}{k}\left[\ln\left(\frac{\left(1+f(z)\right)^{\frac{1}{4}}-1}{\left(1+f(z)\right)^{\frac{1}{4}}+1}\right)+2\tan^{-1}\left(1+f(z)\right)^{\frac{1}{4}}+\ln\left(\frac{\left(1+\frac{16z_0}{L}\right)^{\frac{1}{4}}+1}{\left(1+\frac{16z_0}{L}\right)^{\frac{1}{4}}-1}\right)+2\tan^{-1}\left(1+\frac{16z_0)}{L}\right)^{\frac{1}{4}}\right] \tag{6}$$

where $f(z)=16(z+z_o)$

HEIGHT OF PLUME

The problem of turbulent diffusion was first treated by$^{(5)}$. He solved the partial differential equation in the form:

$$u\frac{\partial C}{\partial x}=\frac{\partial}{\partial z}\left(K(z)\frac{\partial C}{\partial z}\right) \tag{7}$$

subject to the boundary conditions

$$\mathbf{C\rightarrow 0 \quad and \quad X\rightarrow\infty}$$
$$K(z)\frac{\partial C}{\partial z}\rightarrow \mathbf{0\ as\ \ z\rightarrow 0}$$

the ground boundary condition of zero vertical flux and the constant rate of emission of source Q is equal to $\int_0^x uCdz$ and $C\rightarrow\infty$ at (x=0=z).

Much work has been done on obtaining analytical solutions for K(z) and u. These solutions should be applied to ground-level sources only, for which eddy sizes are generally less than plume size. Reborts (1923) obtained a solution to equation (7) with wind varying as power of the height and eddy diffusivity varying as a power of height.

a- The Effective Height

Defining the plume height Δh of diffusing matter as the distance from the stack height h_s to the point at which concentration has fallen to one-tenth of the surface value. The plume height has been calculated adopting the following equation$^{(6)}$.

$$\Delta h = 3(w/u)D_1 \tag{8}$$

where w is the exit velocity of the pollutants, and D_1 is the internal stack diameter. The effective stack height H equals:

$$H = h_s + \Delta h = h_s + 3(w/u)D_1 \tag{9}$$

CONCENTRATION PROFILE

The existing theories of diffusion assume or specify the concentration profile in the vertical. It is very difficult to verify a concentration profile because of the practical difficulties in measuring concentration with sufficient accuracy in the atmospheric layer of interest. The atmospheric layer being 300-500m thick, there is just not enough reliable data obtained in controlled experiments on diffusion studies.

The concentration profile suggested here is of the form

$$C/C_0 = 1 + \alpha_1(z/H) + \alpha_2(z/H)^2 + ... \tag{10}$$

where C_o is concentration at the edge of the plume..
C is the concentration at distance z away from the axis of the plume.
H is the height of the plume and α_1 and α_2 , etc are constants.

The number of terms chosen in the above series will depend upon desired goodness of fit to the observed data as shown in figure (2). It was found that the series in equation (10) gives a fairly good fit to the observed data even if only the first two terms are retained; that is,

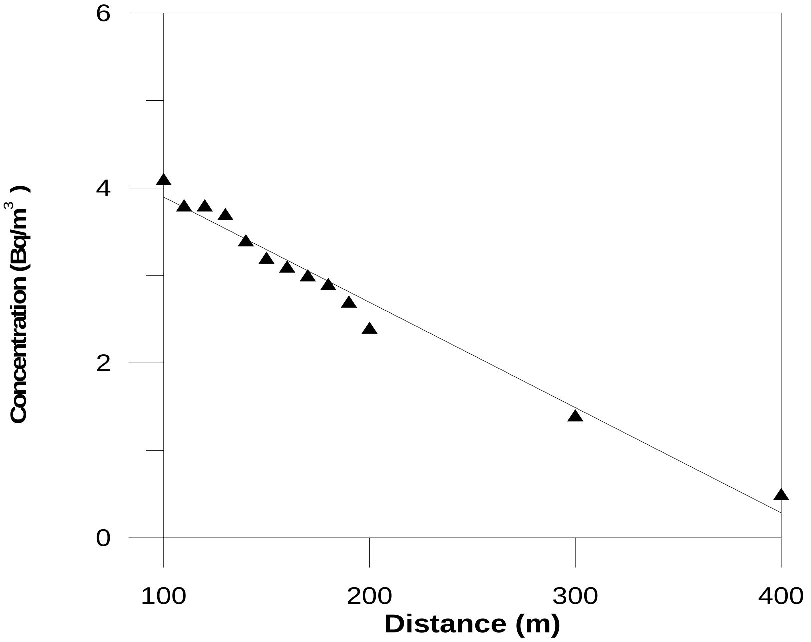

Fig. (2): The variation of the concentration of Iodine (I131) with distance from the reactor.Solid line is a straight line fit.

$$C/C_0 = 1 + \alpha_1 (z/H) \qquad \textbf{(11)}$$

The above equation is for a straight line. The value α_1 will depend upon the concentration desired at the edge of the plume. If the edge of the plume is defined as having r per cent of the concentration, then

$$\boldsymbol{\alpha_1 = -1 + 0.01\, r} \qquad \textbf{(11a)}$$

and if r = 0

$$\boldsymbol{C/C_o = 1-(z/H)} \qquad \textbf{(11b)}$$

PROPOSED MODEL

The model proposed here is meant for use in connection with conservative material. The components of the conservation of mass, equation (1), namely wind profile u(z), effective height H and concentration profile C(z) have been discussed in detail in sections 3, 4 5 respectively. Substituting for u(z) from equations (4), (5) and (6), H from equation (9) and C(z) from equation (11). We get on different formulas during different stabilities as follows:

a-Neutral Case:

$$Q = \int_0^H \left[\frac{\upsilon_*}{k} \ln\left(\frac{z + z_0}{z_0} \right) \right] C_0 \left(1 + \alpha(z/H)\right) dz \qquad \textbf{(12)}$$

which after integrating; yields:

$$C_0 = \frac{4kHQ}{\upsilon_*} \left[2\left(H + z_0\right)\left(2H + \alpha H - \alpha z_0\right) \ln \frac{H + z_0}{z_0} - H\left(4H + \alpha H - 2\alpha z_0\right) \right]^{-1} \qquad \textbf{(13)}$$

where C_o is the concentration at the edge of the plume at a place where the plume effective height is H. Q is the strength of the point source normal to the mean wind. α_1 is the constant in concentration profile, equation (11).

b-Stable Case:

$$Q = \int_0^H \left[\frac{\upsilon_*}{k} \ln\left(\frac{z + z_0}{z_0} \right) + \frac{5.2z}{L} \right] C_0 \left(1 + \alpha(z/H)\right) dz \qquad \textbf{(14)}$$

which after integrating; yields:

$$C_0 = \frac{kHQ}{\upsilon_*} \left\{ \frac{1}{4H} \left[2\left(H + z_0\right)\left(2H + \alpha H - \alpha z_0\right) \ln \frac{H + z_0}{z_0} - H\left(4H + \alpha H - 2\alpha z_0\right) \right] + \frac{5.2H^2}{L} \left(\frac{1}{2} + \frac{\alpha}{3} \right) \right\}^{-1}$$

------------------(15)

d-Unstable Case:

$$Q = \int_0^H \frac{\upsilon_*}{k} \left[\ln\left(\frac{\left(1+f(z)\right)^{\frac{1}{4}} - 1}{\left(1+f(z)\right)^{\frac{1}{4}} + 1} \right) + 2\tan^{-1}\left(1+f(z)\right)^{\frac{1}{4}} + \ln\left(\frac{\left(1 + \frac{16z_0}{L}\right)^{\frac{1}{4}} + 1}{\left(1 + \frac{16z_0}{L}\right)^{\frac{1}{4}} - 1} \right) + 2\tan^{-1}\left(1 + \frac{16z_0)}{L}\right)^{\frac{1}{4}} \right] C_0 \left(1 + \alpha(z/H)\right) dz$$

--------------- (16)

in which f(z) stands for:

$$f(z) = \frac{16(z+z_0)}{L}$$

By inserting the above equation into equation (16) and after integrating , one get:

$$C_0 = \frac{kQ}{\upsilon_*}\Omega^{-1}$$

in which Ω stands for

$$\Omega =$$

$$\left\{ \begin{array}{l} (z+z_o)\left[1+\frac{16\alpha}{H}\left(1+z+z_o-\frac{LL_o}{16}\right)\right]\ln\frac{\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{1}{4}}-1}{\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{1}{4}}+1}+2(z+z_o)\left[1+\frac{\alpha}{2H}(z-z_o)\right]\tan^{-1}\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{1}{4}} \\ -\frac{\alpha}{H}\left[\begin{array}{l} \frac{1}{7}\left(1+\left(\frac{L}{16}\right)^2\right)\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{7}{4}}+\frac{1}{5}\left(1-\left(\frac{L}{16}\right)^2\right)\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{5}{4}} \\ -\left(\frac{2}{3}+\frac{32L_o}{3L}\right)\left(1+\frac{16(z+z_o)}{L}\right)+\left(\frac{2}{3}+\frac{LL_o}{24}\right)\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{3}{4}}+\left(1+\frac{LL_o}{4}+\frac{3L^2}{32}-\frac{HL}{8\alpha}\right)\left(1+\frac{16(z+z_o)}{L}\right)^{\frac{1}{4}} \end{array}\right] \end{array} \right\}_{z=0}^{z=H}$$

CASE STUDY

It is useful to apply the derived expression for C_o/Q on the first research reactor at Inshas. A continuous Ventilation system is provided with the reactor to the areas where radioactive gases**,** volatile materials and suspended particles can exist due to either leakage or airborne radioactivity. The total ventilation rate which could be emitted from the reactor stack of 43 m height**,** 1 m internal diameter**,** and exist velocity 4 m/s is 39965 $m^3/hr^{(7)}$. Also we take α=-1

The calculated values of v_*, Δh, H and C_o/Q of neutral, stable and unstable conditions according are presented in Table(1), (2) and (3) respectively**.** The last columns in the three tables are given in 48 hours that are the usual continuous operation time of the reactor**.**

Figure (3) shows that a straight line fits well to this data in the case neutral, stable and unstable conditions between the concentration at the plume axis over emission rate Co/Q and the effective height H.

VERIFICATION

The dosage "D" i.e. the time integral of the concentration during the passage of a point source puff.

$$D = \int_0^t C\,dt \qquad \textbf{(17)}$$

where C is the concentration e.g. g/m^3 or Bq/m^3 .

Table (1): Wind speed, friction velocity, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in neutral classes.

U (m/s)	V_* (m/s)	Δh(m)	H (m)	$C_o/Q*10^3$ sec/m3
5.27	0.33	2.28	45.28	33.62
5.31	0.33	2.26	45.26	33.64
5.34	0.34	2.25	45.25	32.67
6.37	0.4	1.88	44.88	28.10
5.17	0.32	2.32	45.32	34.62
4.45	0.28	2.70	45.70	39.09
5.1	0.32	2.35	45.35	34.59
4.81	0.3	2.49	45.49	36.73
5.3	0.33	2.26	45.26	33.64
4.86	0.31	2.47	45.47	35.57
5.36	0.34	2.24	45.24	32.67
5.19	0.33	2.31	45.31	33.58
5.41	0.34	2.22	45.22	32.70
5.54	0.35	2.17	45.17	31.82
5.2	0.33	2.31	45.31	33.59
5.61	0.35	2.14	45.14	31.84
5.79	0.36	2.07	45.07	31.03
6.27	0.39	1.91	44.91	28.79
5.93	0.37	2.02	45.02	30.24
6.01	0.38	2.00	45.00	29.47
5.41	0.34	2.22	45.22	32.70
5.75	0.36	2.09	45.09	31.01
5.26	0.33	2.28	45.28	33.62

Table (2): Wind speed, friction velocity, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in stable classes using L=55m.

u (m/s)	v_* (m/s)	Δh(m)	H (m)	$C_o/Q*10^3$ sec/m^3
4.43	0.32	2.71	45.71	17.99
3.81	0.27	3.15	46.15	20.97
4	0.29	3.00	46.00	19.64
4.92	0.35	2.44	45.44	16.62
3.7	0.27	3.24	46.24	20.90
3.57	0.26	3.36	46.36	21.61
3.64	0.26	3.30	46.30	21.66
3.45	0.25	3.48	46.48	22.38
3.6	0.26	3.33	46.33	21.63
3.8	0.27	3.16	46.16	20.97
3.99	0.29	3.01	46.01	19.63
3.89	0.28	3.08	46.08	20.27
3.75	0.27	3.20	46.20	20.94
3.98	0.29	3.02	46.02	19.63
3.47	0.25	3.46	46.46	22.39
4.06	0.29	2.96	45.96	19.67
4.3	0.31	2.79	45.79	18.51
4.31	0.31	2.78	45.78	18.52
4.02	0.29	2.99	45.99	19.65
4.11	0.3	2.92	45.92	19.04
3.94	0.28	3.05	46.05	20.30
3.86	0.28	3.11	46.11	20.26
3.67	0.26	3.27	46.27	21.68

Table (3): Wind speed, friction velocity, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in unstable classes using L=-2.5m.

U (m/s)	v_* (m/s)	Δh(m)	H (m)	$C_o/Q * 10^6$ sec/m^3
4.43	1.33	2.71	45.71	73.97
3.81	1.28	3.15	46.15	76.15
4	1.29	3.00	46.00	75.80
4.92	1.37	2.44	45.44	72.22
3.7	1.27	3.24	46.24	76.60
3.57	1.25	3.36	46.36	77.64
3.64	1.26	3.30	46.30	77.13
3.45	1.24	3.48	46.48	78.07
3.6	1.26	3.33	46.33	77.07
3.8	1.27	3.16	46.16	76.74
3.99	1.29	3.01	46.01	75.79
3.89	1.28	3.08	46.08	76.26
3.75	1.27	3.20	46.20	76.67
3.98	1.29	3.02	46.02	75.78
3.47	1.25	3.46	46.46	77.48
4.06	1.3	2.96	45.96	75.29
4.3	1.32	2.79	45.79	74.40
4.31	1.32	2.78	45.78	74.41
4.02	1.29	2.99	45.99	75.82
4.11	1.3	2.92	45.92	75.34
3.94	1.29	3.05	46.05	75.73
3.86	1.28	3.11	46.11	76.22
3.67	1.26	3.27	46.27	77.17

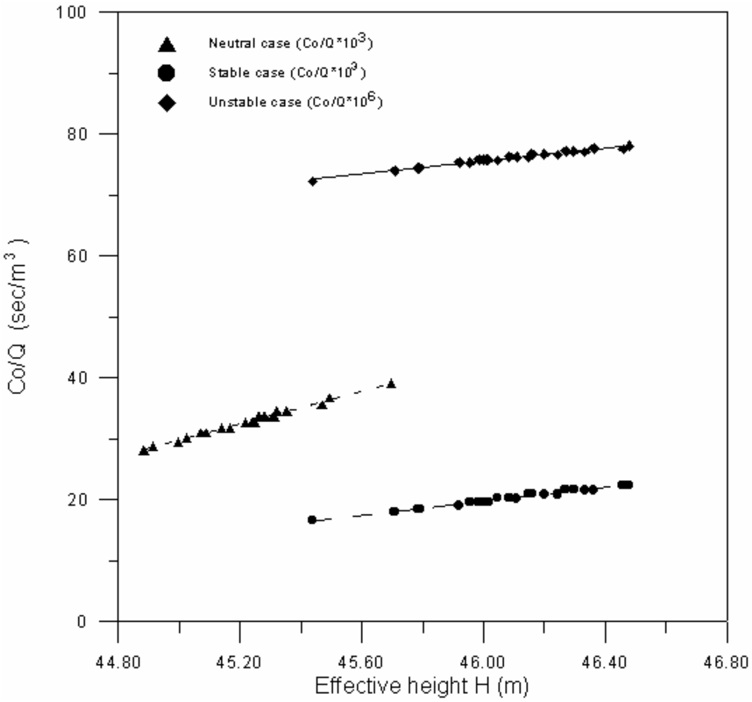

Fig. (3): Variation of the concentration at the plume axis over emission rate with the effective height H in neutral, stable and unstable conditions.

For an instantaneous point source and assuming non-absorbing boundary, the mass balance condition is:

$$Q = \int_0^z \int_0^t uCdzdt = \text{constant} \tag{18}$$

using equation (17)

$$Q = \int_0^z uDdz = \text{constant} \tag{19}$$

where

Q is the total material discharge per unit second of the source g/s or Bq.

U is the wind speed m/s.

D is the dosage of material in g. s. m^{-3} or Bq. s. m^{-3}.

For a point source locate at height H_1=27m (height of the source of the Second Research Reactor in A.E.A., Egypt (ETRR-2) from the ground. For Iodine (I131), the height of the plume (H)is 31.29m, the total material discharge per unit second (Q)is 35Bq, the wind speed (u)is 2.8m/s and the lapse rate ($\Delta T/\Delta Z$) *(^oc/100m)* is 0.36. This is the case of stable stratification. Using equation (15), we get the concentration at the axis of the plume (C_o) equals 1.208 Bq/m^3. Then the concentration at ground modifies to

$$C(ground) = 1.208\left(1 - \frac{H_1}{H}\right) = \mathbf{0.17\ Bq/m^3} \tag{20}$$

The observed concentration at a distance X=300m (with corresponding H=31.29m) was only 0.16Bq/m^3 . For the purpose of verification, they recommend that the source strength be adjusted to yield observed concentration at the first point of observation. For the test in question, the source strength is adjusted by

$$Q(corrected) = \frac{(0.16)(35)}{0.17} = 37.06\ Bq$$

By using Q(corrected), we get C(ground)=0.18 Bq/m^3.

SUMMARY AND CONCLUSIONS

The model presented described the pollutant concentrations downwind of a point source emitting pollutants into the atmosphere, the point source being located normal to the mean wind direction. The results of this study demonstrate:

(1) Wind shear, i.e. variation of wind with the height , is an important factor in turbulent diffusion problems in the lower atmosphere; therefore, the mean wind ū should be so treated instead of treating it as a constant quantity with height. The logarithmic law for wind profile is a good approximation for the planetary boundary layer. In this study The logarithmic law suggested by Businger et al. (1971) has been used to get three different formulas for logarithmic law (neutral, stable and unstable classes)., the model is being extended of the area and line source configurations.

(2) The average surface roughness of the area of interest is a factor of considerable importance. The surface roughness factor, z_o, may be up to 2-3 or more meters in an urban area in comparison with a z_o value of about 0.1m for flat even rural areas. Also the concentrations in a rural area may be to 4-5 times greater than concentration in an urban area which has considerable roughness, other factors remaining the same.

(3) The stratification factor L and the surface roughness z_o are important factors in determining pollution concentrations. The model proposed in this study accounts for these parameters.

We calculate the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate through different stability classes. Also we calculate the concentration at the ground of the Iodine (I131) which agree with the observed concentration value and adjusted its source strength.

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