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Comparison of Total Experimental and Theoretical Absolute γ-ray Detection Efficiencies of a Cylindrical NaI(Tl) Crystal.

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ABSTRACT

A new fit function has been developed to calculate theoretically the absolute γ-ray detection efficiencies (η_{Th}) of a cylindrical NaI(Tl) crystal, for calculating the absolute efficiency at any interesting γ–energy in the energy range between 10 and 1300 keV and distance between 0 and 8 cm. The total absolute γ -ray detection efficiencies have been calculated for five detectors, four are 2″x2″ and one is 3″x 3″ inches NaI(Tl) crystal at different distances. The absolute efficiency of the different detectors was calculated at the specific energy of the standard sources for each measuring distances. In this calculation, experimental (η_{Exp}) and theoretical (η_{Th}) have been calculated. The uncertainties of efficiency calibration have been calculated also for quality control. Measurements were performed with calibrated point source. Gamma-ray energies under consideration were 0.356, 0.662, 1.17 and 1.33 MeV. The differences between (η_{Exp}) and (η_{Th}) at these energies are 1.30E-06, 7.99E-05, 2.29E-04 and 2.42E-04 respectively. The results obtained on the basis of (η_{Exp}) and (η_{Th}) seem to be in very good agreement.

Keywords: ***Gamma Ray, Detection Efficiency and Cylindrical NaI (TI) crystal***

INTRODUTION

High-resolution gamma-ray spectrometry is a convenient method to measure the activity of radioactive nuclei emitting gamma- or X-rays [1]. In the spectrum all the photons that interact with the material within the sensitive volume of the detector are registered. Since radioactive atoms emit a discrete spectrum of photons, peaks in the spectrum occur at these photon energies due to interactions leading to full absorption of the photon energy within the sensitive volume.

The efficiency of a detector is a measure of how many pulses occur for a given number of gamma rays. Various kinds of efficiency definitions are in common use for gamma ray detectors:
1. *Absolute efficiency* is the ratio of the number of counts produced by the detector to the number of γ-rays emitted by the source into all directions.
2.*Intrinsic efficiency* is the ratio of the number of pulses produced by the detector to the number of gamma rays striking the detector, and
3.*Relative efficiency* is efficiency of one detector relative to another; commonly that of a germanium detector relative to a 3 inches long cylindrical NaI crystal with a diameter of 3 inches, each at 25 cm distance from a point source at a specified energy of 1.33 MeV.

This efficiency depends on detector properties, sample properties and the relative sample-detector position. For any detector the efficiency is a function of the sample shape, size, composition, the distance of the sample from the detector and the photon energy. To calculate the activities of gamma- and X- ray emitters from a spectrum measured with a given sample-detector arrangement, the counting efficiency must be known. Usually its uncertainty is the main factor influencing the uncertainty of the calculated activity. Therefore the measurement and/or calculation of the counting efficiency must be done with the greatest care and outmost precision.

MATHEMATICAL EXPRESSION AND SOLUTION

When a point source of radiation is located on the extended axis of a cylindrical detector, a mathematical expression for calculating the total absolute detection efficiency was used [(1)],

$$\eta_{Exp} = \frac{N_p \cdot 100}{I_\gamma [\%]} \cdot \frac{1}{A_{ref} \cdot e^{-\lambda \cdot t_c}} \quad \text{.......................(1)}$$

with: N_p = net peak area at E_γ I_γ = intensity of emitted γ-ray,
A_{ref} = activity of the standard source, tc = counting time
λ = decay constant for γ-ray

The absolute efficiency of the different detectors was calculated at the specific energy of the standard sources for each measuring distances. But, we need some fitting function to calculate the absolute efficiency for any considered γ–energy. We used for this purpose a new fit function for calculating the absolute efficiency at any interesting γ–energy in the energy range between 10 and 1300 kev and detector-sample distance between 0 and 3 cm.

THE FIT FUNCTION

Efficiency function was obtained by applying Weibull Model equation to the experimental efficiency data, which is in the next form:

$$\eta_{Th} = a - b \times \exp(-c \times E_\gamma^d) \quad \text{................(2)}$$

Where E_γ represents energy in MeV, where *A,b,c and d are* coefficient data .By equ. (2), we can determine the absolute efficiency, η_{Th}, at any specific energy E_γ if we know the energies and coefficient data. The efficiencies were repeatedly checked for every detector, and from the experimental efficiency curves, the coefficient data were determined at each distance for every detector by using a curve fitting system so-called (Curve Expert 1.34)[(2)]

THE UNCERTAINTY OF EFFICINENCY CALIBRATION

If a quantity *Y* being measured, called the measured, often is not measured directly, but is determined from *N* other quantities $X_1, X_2, \ldots, X_N$ through a functional relation *f*, often called the measurement equation[(3)]:

$$Y = f(X_1, X_2, \ldots, X_N) \quad \text{.........................(3)}$$

Included among the quantities X_i are corrections or correction factors as well as quantities that take into account other sources of variability, such as different observers, instruments, samples, laboratories, and times at which observations are made, e.g. different days of production and measurements. Thus, the function *f* of equ. (3) should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute significantly to the uncertainty of the measurement result.

An estimate of the measured or *output quantity Y*, denoted by *y*, is obtained from equ. (3) Using *input estimates* $x_1, x_2, \ldots, x_N$ for the values of the *N input quantities* $X_1, X_2, \ldots, X_N$. Thus, the *output estimate y*, which is the result of the measurement, is given by

$$y = f(x_1, x_2, \ldots, x_N) \quad \text{...........................(4)}$$

The combined standard uncertainty of *y* is then given by

$$u^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial X_i} \right)^2 \cdot u^2(x_i) \quad \text{......................(5)}$$

The absolute efficiency was calculated from next equation

$$Eff = \frac{100 \cdot N_p}{I_\gamma \cdot TOC \cdot A_{BOC}} \quad \text{.........................(6)}$$

with: N_p= net peak area at E_γ, I_γ = intensity of emitted γ-ray, TOC = time of counting and A_{BOC} = activity of the standard source at began of counting ,which calculated by equ.(7)

$$A_{BOC} = A_{DOC} \cdot \exp(-\lambda \,.\, (BOC-DOC)) \text{............(7).}$$

Where A_{DOC} is the activity of the standard source at date of calibration DOC, and λ is the decay constant.

The combined standard uncertainty of absolute efficiency $u(EFF)$ is consists of $u(N_p)$, $u(I_\gamma)$, $u(TOC)$ and $u(A_{BOC})$ so,

$$\left[\frac{u(EFF)}{EFF}\right]^2 = \left[\frac{u(N_p)}{N_p}\right]^2 + \left[\frac{u(I_\gamma)}{I_\gamma}\right]^2 + \left[\frac{u(TOC)}{TOC}\right]^2 + \left[\frac{u(A_{BOC})}{A_{BOC}}\right]^2 \text{...... (8)}$$

Because of $u(TOC) << TOC$, we neglected $u(TOC)$,while $u(A_{BOC})$ was calculated by equ. (9)

$$\left[\frac{u(A_{BOC})}{A_{BOC}}\right]^2 = \left[\frac{u(A_{DOC})}{A_{DOC}}\right]^2 + (BOC - DOC)^2 \cdot u^2(\lambda) \text{...... (9)}$$

We got $u(N_p)$ from the code Genie 2000 from Canberra [4] , while $u(\lambda)$ and $u(I_\gamma)$ were taken from the compilation of Reuss and Westmeier [5].

SET- UP, CALIBRATION, AND DATA COLLECTION

The following equipment and instruments were used in this experiment:
Five NaI (Tl) cylindrical detectors, four are 2"x2" and one is 3"x 3" inches NaI(Tl) crystal .
Nucleus, Inc. Multichannel Analyzer (MCA),
Canberra Multichannel Analyzer (MCA) with Genie 2000 software and,
^{137}Cs, ^{133}Ba, and ^{60}Co calibration sources.

In order to determine the peak efficiencies of each detector, several gamma-spectra were collected for several counting periods. Each detector was connected to an MCA and was used to collect long count spectra three hours from each source at each distance. By using ^{133}Ba, ^{137}Cs, and ^{60}Co point sources, the MCA was energy calibrated using the 356 keV, 661.7keV, 1172 KeV and 1332.5 keV gamma-ray peaks of these sources. After separately calibrating each of the detectors, the face of each detector was positioned against the surface of point sources to the alignment marks, to collect the spectra as shown in fig (1).

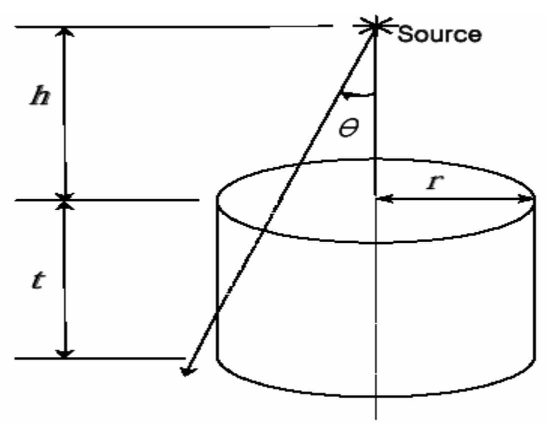


Fig. 1. Schematic Representation of the Axially Symmetric Arrangement of a Cylindrical Scintillation Detector.

The source whose plane is supposed to be parallel to and at distance h from the nearest face of the crystal cylinder should satisfy the following condition [6]: the distance from any edge-point to the point where the symmetry axis of the cylinder intersects the source plane must be smaller than detector radius *r*. In view of the actual source and crystal dimensions, which are generally adopted in experimental arrangements, this condition can hardly be called a restriction. Assuming a uniform source distribution and isotropic emission of the γ-rays of the energy *E* to be detected, one can define the quantity η_{Exp}, called the absolute detection efficiency.

When collecting data, the positions of the detectors placed in the center of the source at the particular distance. The distances between the detector face and each of the sources were measured from the outer surface of the detector to where the source is located. The counting time was three hours to establish a region of interest (ROIs) on the spectrum for each detector. All spectrums were analyzed by Genie 2000 software.

RESULTS AND DISCUSSIONS

Tables (1 and 2) summarized the calculated experimental total absolute detection efficiencies of 2"x2" and 3"x3" inches NaI (Tl) crystal respectively, for gamma rays from a point source at different distances.

Table (1): Experimental absolute detection efficiencies of 2"x2" detectors at different distances.

Detector (1)										
Gamma ray energy in keV	**Distance 1 cm**		**Distance 2 cm**		**Distance 3 cm**		**Distance 4 cm**		**Distance 5 cm**	
	Efficiency	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**
3.56E+02	1.82E-02	2.30E+00	1.10E-02	3.14E+00	6.70E-03	6.77E+00	4.56E-03	5.91E+00	3.54E-03	3.98E+00
6.62E+02	9.30E-03	4.76E+00	6.25E-03	1.96E+00	3.84E-03	6.49E+00	3.17E-03	2.94E+00	2.43E-03	4.76E+00
1.17E+03	3.93E-04	6.61E+00	1.85E-04	8.15E+00	1.19E-04	2.76E+00	5.35E-05	3.61E+00	4.29E-05	2.90E+00
1.33E+03	3.54E-04	4.92E+00	2.53E-04	2.83E+00	1.52E-04	2.76E+00	6.09E-05	3.77E+00	4.32E-05	2.83E+00
Detector (2)										
Gamma ray energy in keV	**Distance 1 cm**		**Distance 2 cm**		**Distance 3 cm**		**Distance 4 cm**		**Distance 5 cm**	
	Efficiency	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**
3.56E+02	1.34E-02	2.30E+00	7.52E-03	3.14E+00	5.13E-03	6.77E+00	3.70E-03	5.91E+00	3.54E-03	3.98E+00
6.62E+02	1.01E-02	4.76E+00	4.45E-03	1.96E+00	3.62E-03	6.49E+00	3.27E-03	2.94E+00	2.58E-03	4.76E+00
1.17E+03	5.83E-04	6.61E+00	2.65E-04	8.15E+00	1.55E-04	2.76E+00	9.71E-05	3.61E+00	5.90E-05	2.90E+00
1.33E+03	4.57E-04	4.92E+00	1.76E-04	2.83E+00	7.73E-05	2.76E+00	3.53E-05	3.77E+00	2.96E-05	2.83E+00
Detector (3)										
Gamma ray energy in keV	**Distance 1 cm**		**Distance 2 cm**		**Distance 3 cm**		**Distance 4 cm**		**Distance 5 cm**	
	Efficiency	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**
3.56E+02	2.43E-02	2.30E+00	1.38E-02	3.14E+00	8.46E-03	6.77E+00	6.41E-03	5.91E+00	4.91E-03	3.98E+00
6.62E+02	1.68E-02	4.76E+00	1.19E-02	1.96E+00	7.79E-03	6.49E+00	5.53E-03	2.94E+00	4.29E-03	4.76E+00
1.17E+03	2.03E-03	6.61E+00	1.22E-03	8.15E+00	6.57E-04	2.76E+00	5.39E-04	3.61E+00	4.09E-04	2.90E+00
1.33E+03	1.49E-03	4.92E+00	1.01E-03	2.83E+00	7.30E-04	2.76E+00	6.11E-04	3.77E+00	4.48E-04	2.83E+00
Detector (4)										
Gamma ray energy in keV	**Distance 1 cm**		**Distance 2 cm**		**Distance 3 cm**		**Distance 4 cm**		**Distance 5 cm**	
	Efficiency	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**	**Efficiency**	**U(eff)%**
3.56E+02	7.81E-03	2.30E+00	5.80E-03	3.14E+00	4.45E-03	6.77E+00	3.45E-03	5.91E+00	2.65E-03	3.98E+00
6.62E+02	4.62E-03	4.76E+00	3.15E-03	1.96E+00	2.17E-03	6.49E+00	1.66E-03	2.94E+00	1.41E-03	4.76E+00
1.17E+03	1.86E-04	6.61E+00	1.50E-04	8.15E+00	6.72E-05	2.76E+00	2.81E-05	3.61E+00	1.20E-05	2.90E+00
1.33E+03	1.21E-04	4.92E+00	7.25E-05	2.83E+00	5.59E-05	2.76E+00	3.97E-05	3.77E+00	1.98E-05	2.83E+00

Table (2): Experimental absolute detection efficiencies of 3"x3" detector at different distances.

Detector (5)								
Gamma-ray energy in keV	Distance 0 cm		Distance 2 cm		Distance 6 cm		Distance 8 cm	
	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)
356.01	3.21E-01	3.21E-01	1.27E-01	1.27E-01	3.10E-02	3.10E-02	3.04E-02	3.04E-02
661.6	6.19E-02	6.22E-02	6.11E-02	6.08E-02	1.76E-02	1.74E-02	1.44E-02	1.43E-02
1172.9	1.89E-02	1.97E-02	1.96E-02	2.13E-02	6.98E-03	7.61E-03	5.32E-03	5.75E-03
1332.5	1.84E-02	1.78E-02	1.85E-02	1.69E-02	6.87E-03	6.30E-03	5.22E-03	4.84E-03

In tables (1,2) one can observe that, all uncertainties are less than 10% for all detectors at different energies and distances. The smallest uncertainty value is 1.96% and the highest one is 8.15%. Now we ensure that we have reliable results and all calculations are correct.

For comparison, the total absolute detection efficiencies of the same γ-ray energies obtained by a new fit function for 2"x2" detectors at different distances are given in Table (3), while in table (4) the same comparison was listed for 3"x3" detector.

Table (3): Experimental and theoretical absolute detection efficiencies of 2"x2" detectors at different distances.

Detector (1)						
Gamma-ray energy in keV	Distance 1 cm		Distance 2 cm		Distance 3 cm	
	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)
3.56E+02	1.82E-02	1.82E-02	1.10E-02	1.10E-02	6.70E-03	6.70E-03
6.62E+02	9.30E-03	9.22E-03	6.25E-03	6.29E-03	3.84E-03	3.44E-03
1.17E+03	3.93E-04	6.23E-04	1.85E-04	4.23E-04	1.19E-04	1.86E-04
1.33E+03	3.54E-04	1.11E-04	2.53E-04	6.83E-05	1.52E-04	1.81E-06
Detector (2)						
Gamma -ray energy in keV	Distance 1 cm		Distance 2 cm		Distance 3 cm	
	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)
3.56E+02	1.34E-02	1.34E-02	7.52E-03	7.51E-03	5.13E-03	5.13E-03
6.62E+02	1.01E-02	9.54E-03	4.45E-03	3.53E-03	3.62E-03	3.37E-03
1.17E+03	5.83E-04	7.85E-04	2.65E-04	2.41E-04	1.55E-04	1.92E-05
1.33E+03	4.57E-04	9.93E-05	1.76E-04	1.40E-05	7.73E-05	4.99E-06
Detector (3)						
Gamma-ray energy in keV	Distance 1 cm		Distance 2 cm		Distance 3 cm	
	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)
3.56E+02	2.43E-02	7.81E-03	1.38E-02	5.80E-03	8.46E-03	8.59E-03
6.62E+02	1.68E-02	4.66E-03	1.19E-02	3.05E-03	7.79E-03	6.67E-03
1.17E+03	2.03E-03	3.61E-04	1.22E-03	2.50E-04	6.57E-04	4.27E-04

1.33E+03	**1.49E-03**	**7.88E-05**	**1.01E-03**	**8.07E-05**	**7.30E-04**	**4.38E-05**

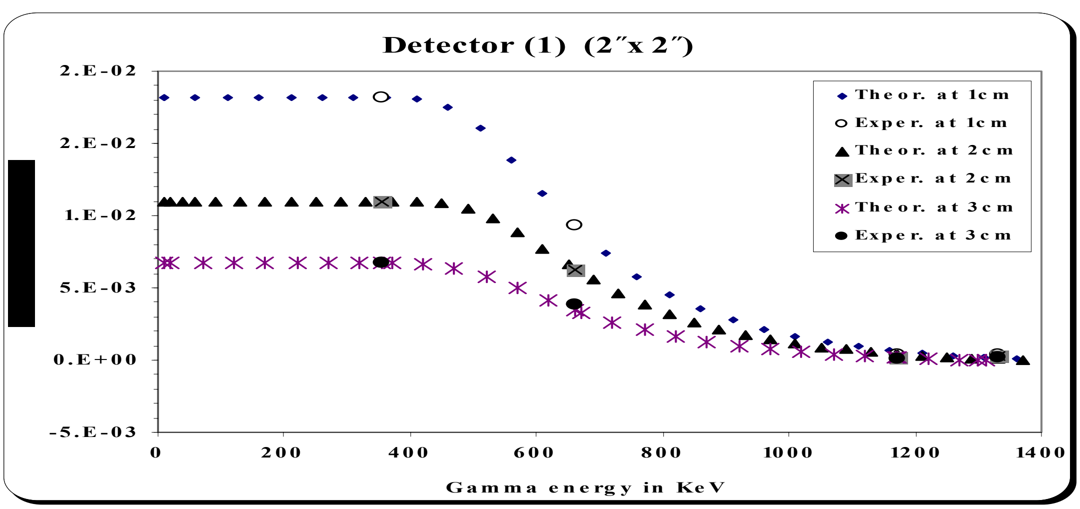
Table (3) continued

Detector (4)						
Gamma-ray energy in keV	**Distance 1 cm**		**Distance 2 cm**		**Distance 3 cm**	
	Efficiency	**Fit(eff)**	**Efficiency**	**Fit (eff)**	**Efficiency**	**Fit (eff)**
3.56E+02	**7.81E-03**	**7.81E-03**	**5.80E-03**	**5.80E-03**	**2.45E-03**	**2.54E-03**
6.62E+02	**4.62E-03**	**4.66E-03**	**3.15E-03**	**3.05E-03**	**2.17E-03**	**2.20E-03**
1.17E+03	**1.86E-04**	**3.61E-04**	**1.50E-04**	**2.50E-04**	**6.72E-05**	**2.49E-04**
1.33E+03	**1.21E-04**	**7.88E-05**	**7.25E-05**	**8.07E-05**	**5.59E-05**	**7.35E-05**

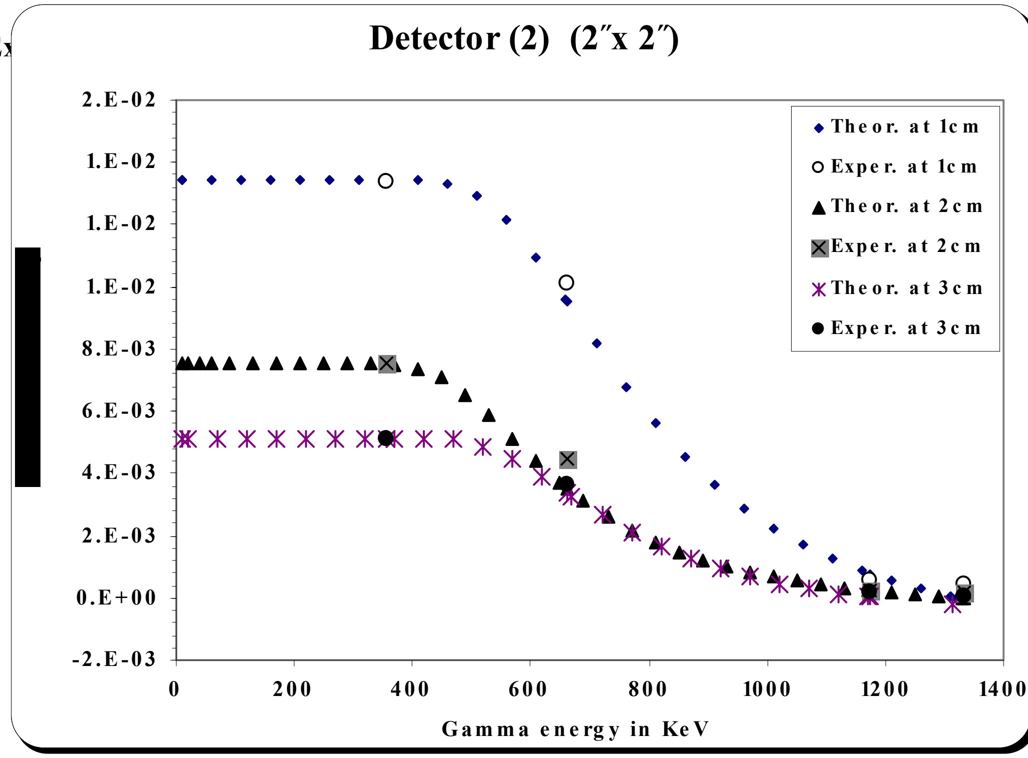
As can be seen in these tables (3), there is a very small difference between experimental and theoretical absolute detection efficiencies. For detectors (1, 2 and 4) at gamma energy 356 and 662KeV there is no difference, while at higher energies 1173and 1332 KeV, one can observe a small difference at different distances. In state of detector (3), there is a difference between experimental and theoretical absolute detection efficiencies. The smallest difference value is 0.00013 at energy 356KeV and the highest one is 0.0014 at energy 1332KeV. In table (4) one can see the best agreement between experimental and theoretical absolute detection efficiencies of 3″x3″ detector at different distances for all gamma energy.

Table (4): Experimental and theoretical absolute detection efficiencies of 3″x3″ detector at different distances.

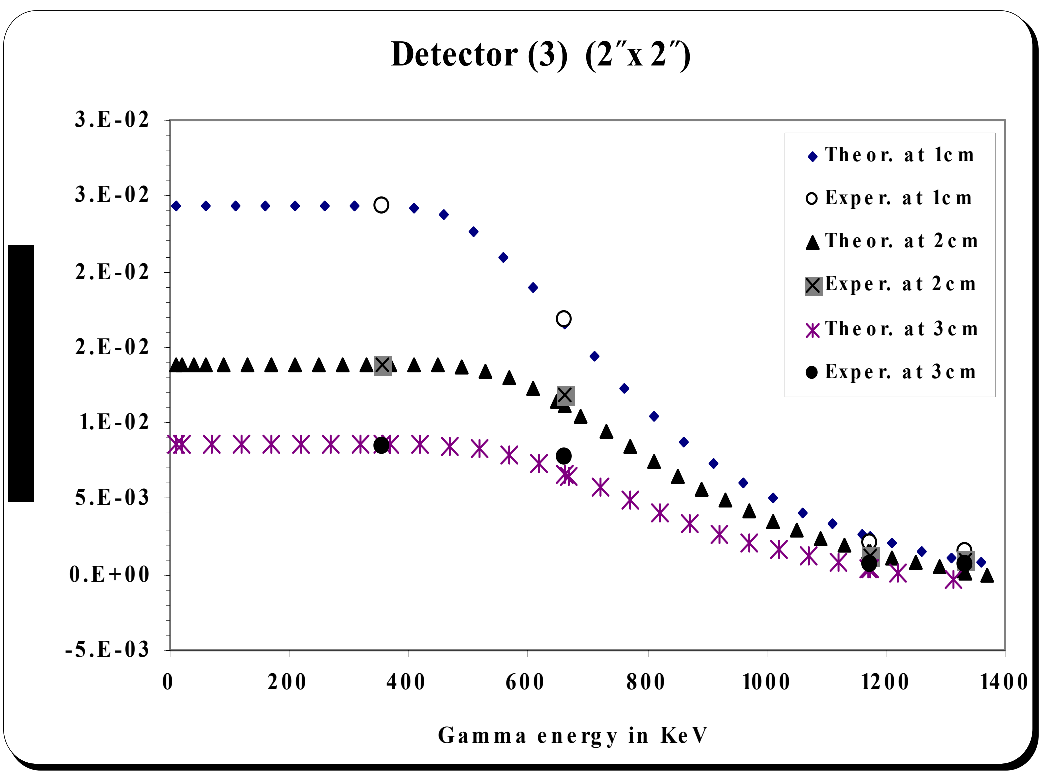
Detector (5)								
Gamma ray energy in keV	Distance 0 cm		Distance 2 cm		Distance 6 cm		Distance 8 cm	
	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)	Efficiency	Fit (eff)
356.01	3.21E-01	3.21E-01	1.27E-01	1.27E-01	3.10E-02	3.10E-02	3.04E-02	3.04E-02
661.6	6.19E-02	6.22E-02	6.11E-02	6.08E-02	1.76E-02	1.74E-02	1.44E-02	1.43E-02
1172.9	1.89E-02	1.97E-02	1.96E-02	2.13E-02	6.98E-03	7.61E-03	5.32E-03	5.75E-03
1332.5	1.84E-02	1.78E-02	1.85E-02	1.69E-02	6.87E-03	6.30E-03	5.22E-03	4.84E-03



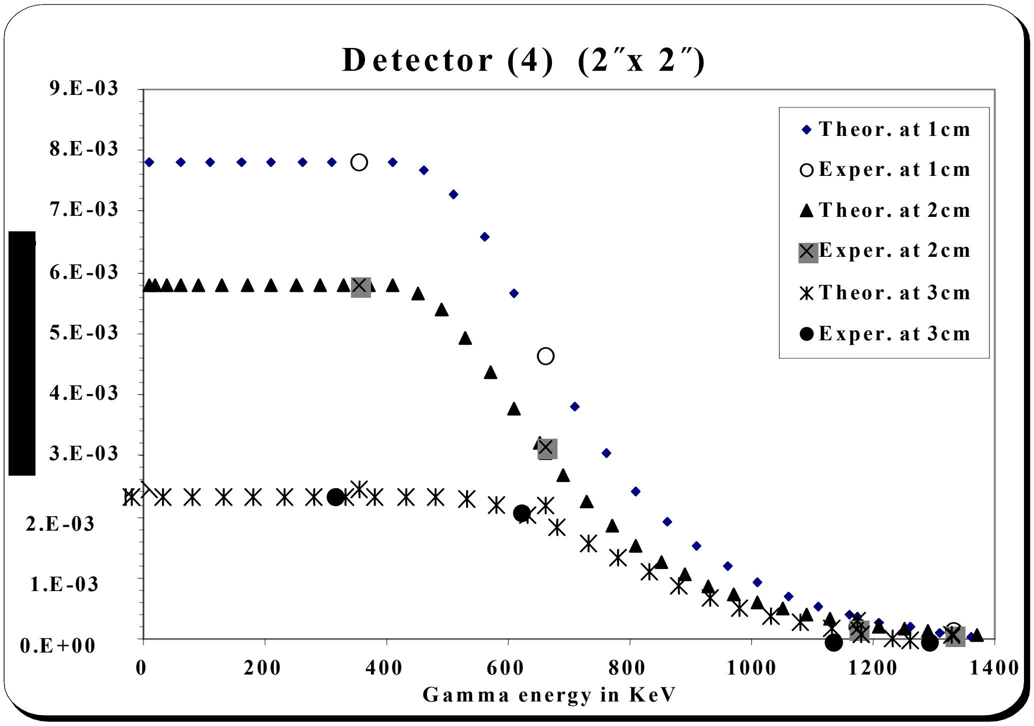
Fig(2):Ex … **ances.**



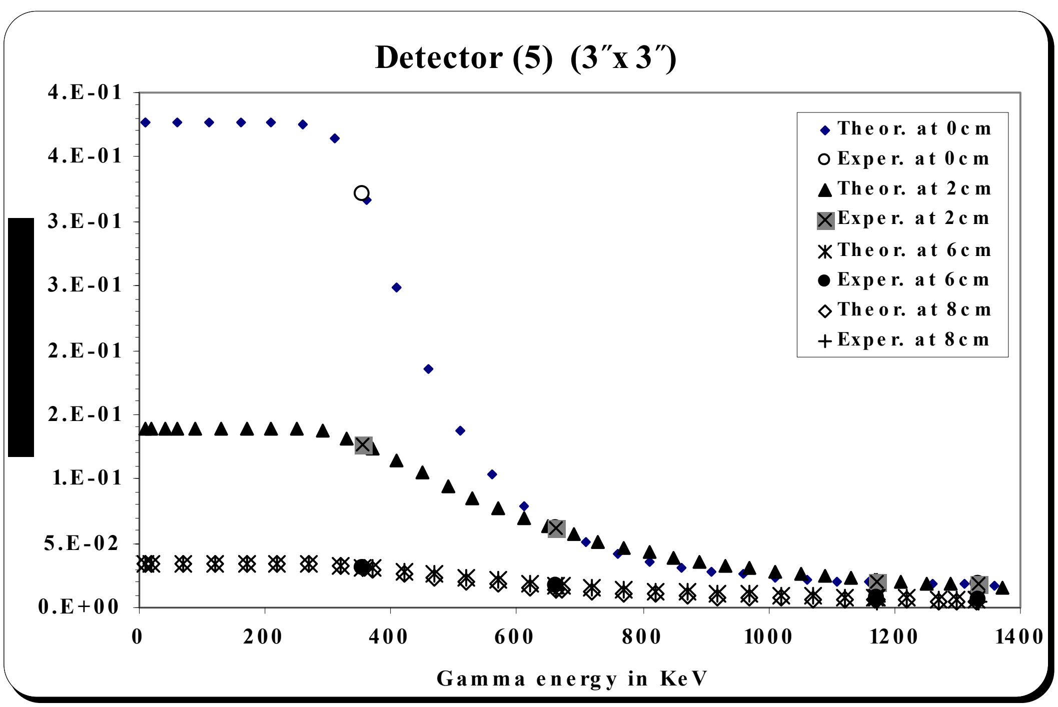
Fig(3):Experimental and theoretical efficiency curve for detector (2) 2″x2″ at different distances.



Fig(4):Experimental and theoretical efficiency curve for detector (4) 2″x2″ at different distances



Fig(5):Experimental and theoretical efficiency curve for detector (3) 2″x2″ at different distances



Fig(6):Experimental and theoretical efficiency curve for detector (5) 3"x3" at different distance

Figures (2 to 6) the efficincy curves have been ploted for all detectors at diffrent distances and gamma rays .From these figures, one can observe a very good agreement between both experimental and theoretical absolute detection efficiencies for all detectors at different distances

CONCLUSIONS

With available data for the total absolute detection efficiencies of point source γ -rays have theoretically (η_{Th}) been calculated for (2"x2" and 3"x3") cylindrical NaI(Tl) crystal and compared with measured efficiencies (η_{Exp}). From the result, conclusions are drawn up as follows:

1-The results obtained on the basis of (η_{Exp}) and (η_{Th}) seem to be in very good agreement.

2- We can use this new fit function to calculate the absolute efficiency for any considered γ–energy.

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