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Absolute Measurement of the Subcriticality Based on the Third Order Neutron Correlation in Consideration of the Finite Nature of Neutron Counts Data

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We have studied a measurement of subcriticality by using the neutron correlation method. Furuhashi proposed an absolute measurement of subcriticality by using the third order neutron correlation factor X in addition to the second order neutron correlation factor Y. In actual experiments, the number of neutron counts data is not infinity so that we take the effect of the finite nature of the neutron counts data into account. We derived new formulas in consideration of the number of data and verified them.

KEYWORDS: absolute measurement, subcriticality, neutron correlation method, third order moment, finite nature of neutron counts data, Monte Carlo simulation

1. Introduction

From the viewpoint of the criticality safety, measurement of the subcriticality is one of the most important subjects in reactor physics. Among various techniques developed so far, we have paid attention to the Furuhashi's technique[1] that can evaluate the absolute value of subcriticality, because it does not require any special experimental equipments, such as the pulsed neutron source[2,3] or the ^{252}Cf installed ionization chamber[4], except for a stationary extraneous neutron source and an ordinary neutron detector.

The Furuhashi's technique was developed on the basis of the variance-to-mean (V-to-M) technique[5]. In the Furuhashi's technique, in addition to the conventional V-to-M ratio Y, the third order neutron correlation factor X as a function of the counting gate width is also evaluated by measuring the neutron counts repeatedly. By using the simple theoretical formulas derived by Furuhashi and Izumi, one can obtain the absolute value of subcriticality from the saturation values of Y and X.

However, the Y value measured in an actual experimental condition is biased because of the finite nature of neutron counts data, so that simple conventional theoretical formula is not appropriate for analyzing the Y value measured in an actual experiment[6]. Furthermore, it was demonstrated by the authors that the X value was also biased due to the finite nature of data[7]. Therefore, we devised new definitions for Y and X to eliminate the effect of the finite nature of data as much as possible.

In the present paper, the advantages of the newly defined Y and X will be illustrated and then the investigation performed by the Monte Carlo simulations will be reported.

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2. Theory

2.1 Principle of subcriticality determination

In the V-to-M technique, the mean $m(T)$ and the variance v(T) are defined as,

$$m(T) \equiv \langle C(T) \rangle, \qquad (1)$$

$$\mathrm{v}(T) \equiv \left\langle (C(T) - m(T))^2 \right\rangle, \qquad (2)$$

where $C(T)$ is the neutron counts with respect to a counting gate width T and $\langle x \rangle$ means the expected value of x. By using $m(T)$ and v(T), the Y value, which means a neutron correlation factor of second order, is defined as follows[5]:

$$Y(T) \equiv \frac{\mathrm{v}(T)}{m(T)} - 1. \qquad (3)$$

Assuming the one energy group of neutrons and the one-point reactor approximation without delayed neutrons, the theoretical expression of Y for a subcritical system with an extraneous stationary neutron source can be derived as follows:

$$Y(T) = Y_\infty \left(1 - \frac{1 - e^{-\alpha T}}{\alpha T} \right). \qquad (4)$$

By fitting Eq. (4) to measured $Y(T)$, one can evaluate the two kinds of the parameters, i.e. the saturation value Y_∞ and the prompt neutron decay constant α. Theoretical expressions of Y_∞ and α can be derived as,

$$Y_\infty = \frac{\lambda_d \lambda_f \langle \nu(\nu - 1) \rangle}{\alpha^2} \left\{ 1 + \frac{\langle \nu \rangle \langle q(q-1) \rangle}{\langle q \rangle \langle \nu(\nu - 1) \rangle} (-\rho) \right\}, \qquad (5)$$

$$\alpha = \langle \nu \rangle \lambda_f (-\rho), \qquad (6)$$

where λ_d is the probability that one neutron is detected per unit time, λ_f the probability that one neutron causes the fission reaction per unit time, ν the number of neutrons emitted by one fission event, q the number of neutrons generated by one extraneous source event and $(-\rho)$ the subcriticality. One can find from Eqs. (5) and (6) that the absolute value of subcriticality cannot be determined directly from measured Y_∞ and α even if all the factorial moments of ν and q, i.e. $\langle\nu\rangle$, $\langle q\rangle$, $\langle\nu(\nu-1)\rangle$ and $\langle q(q-1)\rangle$, are available, because λ_d remains to be unknown.

To overcome this problem, Furuhashi and Izumi introduced the X value, which means a third order neutron correlation factor[1]. The X value is defined as follows:

$$X(T) \equiv \frac{s(T)}{m(T)} - 3\frac{v(T)}{m(T)} + 2 = \left(\frac{s(T)}{m(T)} - 1\right) - 3\left(\frac{v(T)}{m(T)} - 1\right), \quad (7)$$

where $s(T)$ is the third order moment of neutron counts with respect to T,

$$s(T) \equiv \left\langle (C(T) - m(T))^3 \right\rangle. \quad (8)$$

Introducing the same assumptions for the Y value, Furuhashi and Izumi derived the theoretical expression for expected value of X as follows:

$$X(T) = X_{2\infty}\left(1 + e^{-\alpha T} - 2\frac{1-e^{-\alpha T}}{\alpha T}\right) + X_{3\infty}\left(1 - \frac{3 - 4e^{-\alpha T} + e^{-2\alpha T}}{2\alpha T}\right), \quad (9)$$

$$X_{2\infty} = \frac{3\lambda_d^2\lambda_f^2\langle\nu(\nu-1)\rangle^2}{\alpha^4}\left\{1 + \frac{\langle\nu\rangle\langle q(q-1)\rangle}{\langle q\rangle\langle\nu(\nu-1)\rangle}(-\rho)\right\}, \quad (10)$$

$$X_{3\infty} = \frac{\lambda_d^2\lambda_f\langle\nu(\nu-1)(\nu-2)\rangle}{\alpha^3} \times \left\{1 + \frac{\langle\nu\rangle\langle q(q-1)(q-2)\rangle}{\langle q\rangle\langle\nu(\nu-1)(\nu-2)\rangle}(-\rho)\right\}, \quad (11)$$

where $X_{2\infty}$ and $X_{3\infty}$ are the saturation values of X.

Therefore, from the following relationships,

$$\frac{X_{2\infty}}{3Y_\infty^2} = \left\{1 + \frac{\langle\nu\rangle\langle q(q-1)\rangle}{\langle q\rangle\langle\nu(\nu-1)\rangle}(-\rho)\right\}^{-1}, \quad (12)$$

$$\frac{X_{3\infty}}{Y_\infty^2} = \frac{\langle\nu\rangle\langle\nu(\nu-1)(\nu-2)\rangle(-\rho)}{\langle\nu(\nu-1)\rangle^2} \times \frac{\left\{1 + \frac{\langle\nu\rangle\langle q(q-1)(q-2)\rangle}{\langle q\rangle\langle\nu(\nu-1)(\nu-2)\rangle}(-\rho)\right\}}{\left\{1 + \frac{\langle\nu\rangle\langle q(q-1)\rangle}{\langle q\rangle\langle\nu(\nu-1)\rangle}(-\rho)\right\}^2}, \quad (13)$$

one can obtain the absolute value of subcriticality by fitting Eqs. (4) and (9) to measured $Y(T)$ and $X(T)$, if statistics of q and ν are known.

It is possible to take the spatial effect into account. Considering the only the spatial fundamental mode, we can obtain the following relationships corresponding to Eqs. (12) and (13) in a bare homogeneous subcritical system[8].

$$\frac{X_{2\infty}}{3Y_\infty^2} = \left\{1 + \frac{\langle\nu\rangle\langle q(q-1)\rangle}{\langle q\rangle\langle\nu(\nu-1)\rangle}\frac{S_2}{S_1\int_V(\psi_0(r))^3\,dr}(-\rho)\right\}^{-1}, \quad (14)$$

$$\frac{X_{3\infty}}{Y_\infty^2} = \frac{\langle\nu\rangle\langle\nu(\nu-1)(\nu-2)\rangle(-\rho)}{\langle\nu(\nu-1)\rangle^2} \times \frac{\left\{\int_V(\psi_0(r))^4\,dr + \frac{\langle\nu\rangle\langle q(q-1)(q-2)\rangle}{\langle q\rangle\langle\nu(\nu-1)(\nu-2)\rangle}\frac{S_3}{S_1}(-\rho)\right\}}{\left\{\int_V(\psi_0(r))^3\,dr + \frac{\langle\nu\rangle\langle q(q-1)\rangle}{\langle q\rangle\langle\nu(\nu-1)\rangle}\frac{S_2}{S_1}(-\rho)\right\}^2}, \quad (15)$$

$$S_k \equiv \int_V S(r)(\psi_0(r))^k\,dr, \quad (16)$$

where $S(r)$ is a spatial distribution of extraneous neutron source and $\psi_0(r)$ is the spatial fundamental mode.

2.2 Estimate of Y and X values by sample variance and third order moment

In the previous section, we stated that the absolute value of subcriticality can be obtained by fitting Eqs. (4) and (9) to measured $Y(T)$ and $X(T)$. However, it was pointed out that the Y and X values were biased in an actual experimental condition because the mean $m(T)$, the variance $v(T)$ and the third order moment $s(T)$ were usually calculated as the sample mean $\overline{C}(T)$, the sample variance $v_S(T)$ and the sample third order moment $s_S(T)$,

$$\overline{C}(T) \equiv \frac{1}{N}\sum_{i=1}^{N} C_i(T), \quad (17)$$

$$\mathrm{v}_S(T) \equiv \frac{1}{N}\sum_{i=1}^{N}\left(C_i(T)-\bar{C}(T)\right)^2, \tag{18}$$

$$s_S(T) \equiv \frac{1}{N}\sum_{i=1}^{N}\left(C_i(T)-\bar{C}(T)\right)^3, \tag{19}$$

where $C_i(T)$ is the i-th neutron counts (i = 1, 2, ..., N) with respect to the counting gate width T and N the number of neutron counts data. Thus, the Y and X values are obtained as the Y_S and X_S values that are defined by using the sample variance $\mathrm{v}_S(T)$ and the sample third order moment $s_S(T)$ as,

$$Y_S(T) \equiv \frac{\mathrm{v}_S(T)}{\bar{C}(T)}-1, \tag{20}$$

$$X_S(T) \equiv \frac{s_S(T)}{\bar{C}(T)}-3\frac{\mathrm{v}_S(T)}{\bar{C}(T)}+2. \tag{21}$$

Recently, Wallerbos and Hoogenboom derived the theoretical expression for the expected value of Y_S as follows[6]:

$$\begin{aligned}\langle Y_S(T)\rangle &= \frac{\langle \mathrm{v}_S(T)\rangle}{\langle \bar{C}(T)\rangle}-1 \\ &= Y_\infty f_1(\alpha T,N)-\frac{1}{N}\end{aligned}, \tag{22}$$

$$f_1(x,y) \equiv 1-\frac{1-e^{-x}}{x}-\frac{1}{y}\left(1-\frac{1-e^{-xy}}{xy}\right). \tag{23}$$

On the other hand, the theoretical expression for the expected value of X_S was derived as follows[7]:

$$\begin{aligned}\langle X_S(T)\rangle &= \frac{\langle s_S(T)\rangle}{\langle \bar{C}(T)\rangle}-3\frac{\langle \mathrm{v}_S(T)\rangle}{\langle \bar{C}(T)\rangle}+2 \\ &= X_{2\infty}f_2(\alpha T,N)+X_{3\infty}f_3(\alpha T,N), \\ &\quad -\frac{6}{N}Y_\infty f_1(\alpha T,N)+\frac{2}{N^2}\end{aligned} \tag{24}$$

$$\begin{aligned}&f_2(x,y) \\ &\equiv 1+e^{-x}-2\frac{1-e^{-x}}{x} \\ &\quad -\frac{1}{y}\left\{3+e^{-x}-4\frac{1-e^{-x}}{x}-2\frac{1-e^{-xy}}{xy}\left(1-\frac{xe^{-x}}{1-e^{-x}}\right)\right\}, \\ &\quad +\frac{2}{y^2}\left(1+e^{-xy}-2\frac{1-e^{-xy}}{xy}\right)\end{aligned} \tag{25}$$

$$\begin{aligned}&f_3(x,y) \\ &\equiv 1-\frac{3-4e^{-x}+e^{-2x}}{2x} \\ &\quad -\frac{1}{y}\left[3-3\frac{1-e^{-x}}{x}-\frac{\left(1-e^{-x}\right)\left(1-e^{-xy}\right)}{xy}\left\{1+\frac{1+e^{-xy}}{2\left(1+e^{-x}\right)}\right\}\right]\cdot \\ &\quad +\frac{2}{y^2}\left(1-\frac{3-4e^{-xy}+e^{-2xy}}{2xy}\right)\end{aligned} \tag{26}$$

One can see that the theoretical expressions of Y_S and X_S values, namely Eqs. (22) and (24), include the additional terms which originate from the number of neutron counts data N. If N goes to infinity, Eqs. (22) and (24) become identical to conventional expression Eqs. (4) and (9).

When only T goes to infinity in Eqs. (22) and (24),

$$\lim_{T\to\infty}\langle Y_S(T)\rangle = Y_\infty\left(1-\frac{1}{N}\right)-\frac{1}{N}, \tag{27}$$

$$\begin{aligned}\lim_{T\to\infty}\langle X_S(T)\rangle &= \left(X_{2\infty}+X_{3\infty}\right)\left(1-\frac{1}{N}\right)\left(1-\frac{2}{N}\right) \\ &\quad -\frac{6Y_\infty}{N}\left(1-\frac{1}{N}\right)+\frac{2}{N^2}\end{aligned}. \tag{28}$$

From Eqs. (27) and (28), one can see that if N is not enough large, this effect is not negligible. Therefore, under such condition, the saturation values of Y and X are underestimated compared to the expected values by the conventional theoretical expressions, Eqs. (4) and (9).

2.3 Unbiased estimate of Y and X values by unbiased variance and third order moment

In the present study, we devised new definitions for Y and X to eliminate the effect of the finite nature of neutron counts data as much as possible. For this reason, we introduced the unbiased variance $\mathrm{v}_U(T)$ and unbiased third order moment $s_U(T)$ instead of $\mathrm{v}_S(T)$ and $s_S(T)$ as,

$$\mathrm{v}_U(T) \equiv \frac{1}{N-1}\sum_{i=1}^{N}\left(C_i(T)-\bar{C}(T)\right)^2, \tag{29}$$

$$s_U(T) \equiv \frac{N}{(N-1)(N-2)}\sum_{i=1}^{N}\left(C_i(T)-\bar{C}(T)\right)^3. \tag{30}$$

The Y_U and X_U value are defined by using unbiased values $\mathrm{v}_U(T)$ and $s_U(T)$ as,

$$Y_U(T) \equiv \frac{\mathrm{v}_U(T)}{\bar{C}(T)}-1, \tag{31}$$

$$X_U(T) \equiv \frac{s_U(T)}{\bar{C}(T)}-3\frac{\mathrm{v}_U(T)}{\bar{C}(T)}+2. \tag{32}$$

and the expected values of Y_U and X_U were derived as follows:

$$\langle Y_U(T)\rangle = \frac{\langle v_U(T)\rangle}{\langle \bar{C}(T)\rangle} - 1 , \qquad (33)$$
$$= Y_\infty f_1'(\alpha T, N)$$

$$\langle X_U(T)\rangle = \frac{\langle s_U(T)\rangle}{\langle \bar{C}(T)\rangle} - 3\frac{\langle v_U(T)\rangle}{\langle \bar{C}(T)\rangle} + 2 , \qquad (34)$$
$$= X_{2\infty} f_2'(\alpha T, N) + X_{3\infty} f_3'(\alpha T, N)$$

$$f_1'(x,y) \equiv \frac{y}{y-1} f_1(x,y), \qquad (35)$$

$$f_2'(x,y) \equiv \frac{y^2}{(y-1)(y-2)} f_2(x,y), \qquad (36)$$

$$f_3'(x,y) \equiv \frac{y^2}{(y-1)(y-2)} f_3(x,y). \qquad (37)$$

One can find that Eqs. (33) and (34) are simpler than Eqs. (22) and (24).

If T goes to infinity, Y_U and X_U approach to their saturation values that do not include N,

$$\lim_{T\to\infty}\langle Y_U(T)\rangle = Y_\infty , \qquad (38)$$

$$\lim_{T\to\infty}\langle X_U(T)\rangle = X_{2\infty} + X_{3\infty} . \qquad (39)$$

These results are obtained since the unbiased values $v_U(T)$ and $s_U(T)$ are used for definition of Y and X.

3. Monte Carlo Simulation

To investigate the behavior of Y_S, X_S, Y_U and X_U values due to the finite nature of neutron counts data, we carried out Monte Carlo simulations on a neutron correlation experiment that used a ^{252}Cf spontaneous neutron source in an infinite homogeneous system without delayed neutrons. The one-group constants for four subcritical states (k = 0.99, 0.95, 0.90, 0.80) used in the simulations are listed in Table 1.

Table 1 One-group constants for Monte Carlo simulation

v (cm/s)	Σ_s (1/cm)	Σ_f (1/cm)	Σ_c (1/cm)
1.395×10^{6}	9.394×10^{-1}	1.530×10^{-2}	0

Σ_d (1/cm)			
k = 0.99	k = 0.95	k = 0.90	k = 0.80
2.293×10^{-2}	2.454×10^{-2}	2.676×10^{-2}	3.202×10^{-2}

In Table 1, v is the neutron velocity, Σ_s the macroscopic scattering cross section, Σ_f the macroscopic fission cross section, Σ_c the macroscopic capture cross section and Σ_d the macroscopic detection cross section. It should be noted that all of the neutron capture were regarded as the neutron detection. The notations of λ_f and λ_d are defined as follows:

$$\lambda_f = v\Sigma_f , \qquad (40)$$

$$\lambda_d = v\Sigma_d . \qquad (41)$$

We assumed that the fissile material consisted of only ^{235}U. Then, the probability of emission of ν neutrons by one fission event, $P_f(\nu)$, could be approximated by the binomial distribution[9],

$$P_f(\nu) = \frac{\nu!}{5!(5-\nu)!}\left(\frac{2.474}{5}\right)^{\nu}\left(1-\frac{2.474}{5}\right)^{5-\nu} . \qquad (42)$$

Furthermore, the probability of emission of q neutrons by one source event of ^{252}Cf neutron source, $P_s(q)$, was also approximated by the binomial distribution[9],

$$P_s(q) = \frac{q!}{7!(7-q)!}\left(\frac{3.876}{7}\right)^{q}\left(1-\frac{3.876}{7}\right)^{7-q} . \qquad (43)$$

We assumed that a Multi Channel Scaler (MCS) was used for accumulating of successive neutron counts data. Its fundamental gate width and number of count gates with respect to T_0 are also assumed to be T_0 and N_0, as shown by Fig. 1. In order to avoid repeating calculation with different gate widths, we used bunching technique[10]. This technique requires the addition of successive i count-gates to obtain the time series data corresponding to a longer gate width $T = iT_0$ (i = 2,3,4...). Therefore the number of gates with respect to T, namely N, is in inverse proportion to i, as follows:

$$N = \left[\frac{N_0}{i}\right]_{\mathrm{Gauss}} . \qquad (44)$$

To get the high precision values of Y_S, X_S, Y_U and X_U, the simulations of MCS measurements are repeated M times. Table 2 shows N_0, T_0 and M used in the simulations.

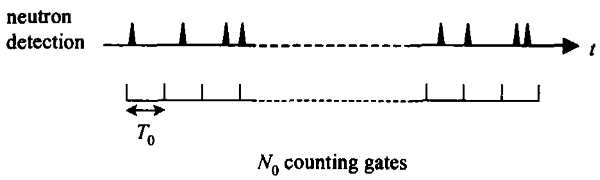

Fig. 1 Measurement of neutron counts by multi channel scaler (MCS)

Table 2 N_0, T_0 and M used in the simulations

	N_0	T_0 (μs)	M
k = 0.99	4000	250	4000
k = 0.95		50	20000
k = 0.90		25	40000
k = 0.80		10	100000

4. Results and discussions

4.1 The effect of finite nature of neutron counts data

Figures 2 and 3 show the results of Monte Carlo simulation and theoretical curves for Y_S and X_S when k = 0.95. From Figs. 2 and 3, it is found that the present theoretical curves, agree well with the results of Monte Carlo simulations, but the conventional ones differ from the results of Monte Carlo simulations as the gate width increases, i.e. N decreases. Therefore conventional formulas of Y and X are not applicable to fitting analysis, and the effect of the finite nature of data must be taken into account.

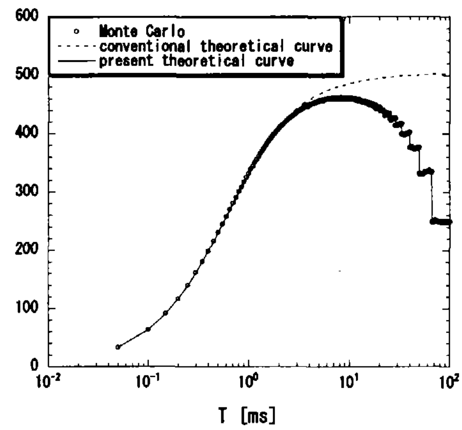

Fig. 2 Example results of Y_S (k = 0.95)

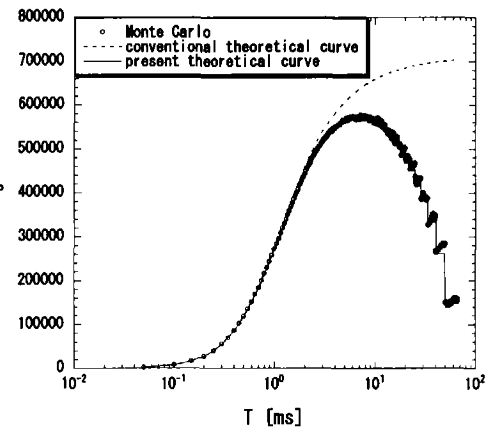

Fig. 3 Example results of X_S (k = 0.95)

Figures 4 and 5 show the results of those for Y_U and X_U. From Figs. 4 and 5, it is found that both present and conventional theoretical curves agree well with the results of Monte Carlo simulations. Therefore, it seems that we can almost eliminate the effect of the number of data and conventional formulas are available for fitting analysis.

Above discussions hold true for the results of Y_S, X_S, Y_U and X_U simulated at other subcritical states.

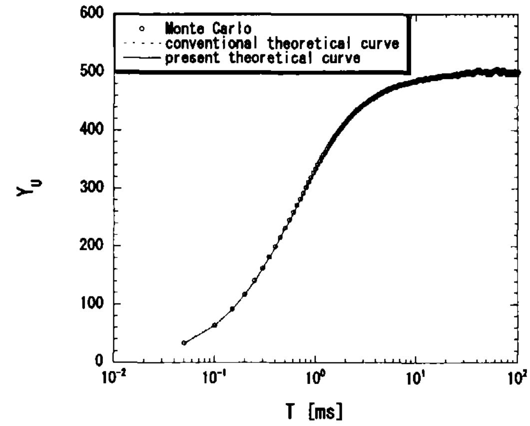

Fig. 4 Example results of Y_U (k = 0.95)

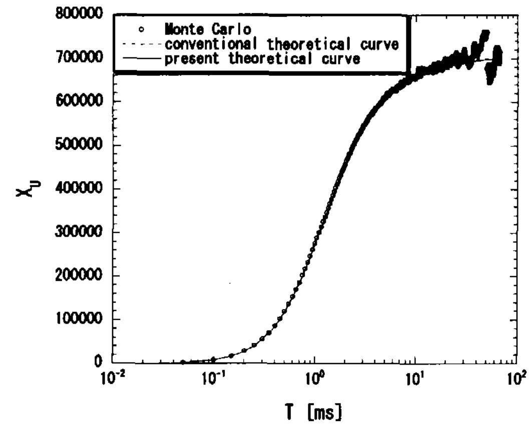

Fig. 5 Example results of X_U (k = 0.95)

4.2 Estimate of subcriticality by using Y_U and X_U

In the case of Y_S and X_S, the present theoretical formulas must be used for fitting analysis, and conventional ones are not available. On the other hand, in the case of Y_U and X_U, it seems that both formulas are available for fitting analysis. In this section, we discuss the results of subcriticality estimated by using Y_U and X_U. Table 3 shows the results of subcriticality, which were estimated by fitting the conventional formulas or present ones to Y_U and X_U simulated at each subcritical state. From Table 3, we can see that the results of subcriticality obtained by both formulas are nearly equal to theoretical value.

Figure 6 shows the ratios of present theoretical curve to conventional one when $k = 0.95$. From Fig. 6, we can see that the present theoretical curves are not much deference than conventional ones. These results hold true for other critical states. Therefore, we can use conventional formulas are available for fitting analysis, if Y_U and X_U are measured.

Table 3 Estimate of subcriticality for Y_U and X_U
(a): By fitting the conventional formulas.
(b): By fitting the present formulas.

	$(-\rho)$ (%Δk/k)		
	theoretical value	(a)	(b)
$k = 0.99$	1.010	1.517	1.435
$k = 0.95$	5.263	5.151	5.081
$k = 0.90$	11.11	11.20	11.12
$k = 0.80$	25.00	25.26	25.14

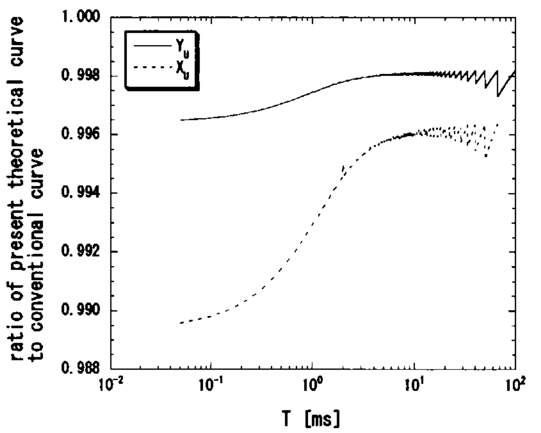

Fig. 6 The ratios of present theoretical curve to conventional one for Y_U and X_U ($k = 0.95$)

5. Conclusion

We derived new formulas for the neutron correlation factor Y and X, in consideration of the finite nature of neutron counts data. By using the Monte Carlo simulations, we verified the finite nature of data for Y and X. If Y and X are estimated by using the sample variance and the sample third order moment, the effect of the number of data is not negligible, so that conventional formulas are not appropriate and present ones must be used for fitting analysis. On the other hand, if Y and X are estimated by using the unbiased variance and the unbiased third order moment, we can eliminate the effect of the number of data as much as possible, so both conventional and present formulas are available for fitting analysis.

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