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A New Set of Parameters for 5 Gaussian Fission Yields Systematics

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A new set of parameters for 5 Gaussian-type fission yields systematics has been proposed for applying to high energy neutron or proton fission and to various kinds of fissioning systems including minor actinides. The mass yields calculated using the systematics were compared with various kinds of measured data including the fission with incident energy higher than 100 MeV and the fission of minor actinide nuclides. The comparisons showed rather good agreement between the calculated values and measured ones for various kinds of fissioning systems.

KEYWORDS: fission yields, mass distribution, systematics, minor actinide, high energy fission

I. Introduction

When a nuclear criticality accident occurs, many radioactive fission product (FP) nuclides are produced. These radioactive nuclides emit radiations which cause radiation exposure for personnel accessing the accident place for after-accident treatment. As the energy and the radiation type of the released radiations depend on the FP nuclides produced, it is important to know what kinds of FP nuclides are produced after the fission. The distribution of the FP nuclides produced depends on the kind of fissioning nuclide and the energy of incident particle impinging on the fissioning nuclide.

The evaluation of the yields of FP nuclides has been traditionary performed for reactor and fusion application where important fissioning nuclides are uranium and plutonium isotopes and the energy ranges of the incident neutrons are from thermal to 14 MeV. Recently several programs are proposed to transmute long-lived radioactive nuclides produced in nuclear reactors into short-lived ones in order to reduce the burden managing radioactivity in a long term. In such a program FP yields data of minor actinides by neutron and proton with energies from thermal to higher energy than 100 MeV, which have attracted less attention than major actinides like uranium and plutonium, are needed to access the feasibility of the system proposed and the safety related problems.

Fission yields data for fission of such minor actinides by high energy incident particles have not sufficiently measured. Then systematics to predict the fission yields is required. From the examination of available measured mass distribution, the systematics with 5 Gaussian functions has been developed.[1] The systematics is rather simple but has wide applicability to various kinds of fissioning systems with incident energy from thermal to higher than 100 MeV. In the present paper fission yields systematics of 5 Gaussian functions with new parameters set is described.

II. Description of 5 Gaussian mass yield systematics

Systematics has been developed based on the description by Moriyama-Ohnishi.[2] In the Moriyama-Ohnishi systematics, mass yields $\psi(A)$ are expressed as follows:

$$\begin{aligned}\psi(A) &= N_s\psi_s(A) + N_a\psi_a(A) \\ &= N_s\psi_s(A) \\ &\quad + N_a\left[\psi_{h1}(A) + \psi_{l1}(A)\right. \\ &\quad \left. + F\left\{\psi_{h2}(A) + \psi_{l2}(A)\right\}\right],\end{aligned}$$

where $\psi_s(A)$ and $\psi_a(A)$ are symmetric and asymmetric components respectively. The asymmetric component, $\psi_a(A)$, is then divided into heavy $\psi_h(A)$ and light $\psi_l(A)$ components with two further components (1 and 2). Each component is assumed to be Gaussian function.

$$\psi_x(A) = \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left\{-(A-A_x)^2/2\sigma_x^2\right\},$$

where subscript x denotes s, $h1$, $h2$, $l1$ and $l2$. Then there are 5 Gaussian functions in this systematics. The heavy components $\psi_{h1}(A)$ and $\psi_{h2}(A)$, however, relate to the light components $\psi_{l1}(A)$ and $\psi_{l2}(A)$ by reflecting about symmetric axis $A_s = (A_f - \bar{\nu})/2$. This relationship reduces the number of independent Gaussian functions to 3. Each function has two parameters, A_x and σ_x. Then there are 6 independent parameters. Other parameters needed in the systematics are normalization factors, N_s and N_a, which determine the contributions of symmetric and asymmetric components. They are given by:

$$\begin{aligned}N_s &= 200/(1+2R), \\ N_a &= 200R/\left\{(1+F)(1+2R)\right\},\end{aligned}$$

where R is the ratio of the asymmetric component to the symmetric component and F the ratio of the asymmetric component 1 to the asymmetric component 2. The normalizations given above assure the total yield, T, to be 200 as follows:

$$T = N_s + N_a \cdot [2 + 2F],$$

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\begin{aligned}
&= \frac{200}{1+2R} + \frac{200R \times 2(1+F)}{(1+F)(1+2R)}, \\
&= 200.
\end{aligned}
$$

In these expressions of the fission yields systematics, there are 8 parameters to be determined, that is, $\bar{\nu}$, R, F, σ_s, σ_{h1}, σ_{h2}, A_{h1} and A_{h2}. These values were determined by examining available measured data excepting the $\bar{\nu}$ value which is adopted from the proposal by Wahl.[3] The available experimental mass distributions were decomposed to 5 Gaussian functions by least squares fit. The parameters mentioned above were calculated from the parameters of the decomposed 5 Gaussian functions. The obtained parameters were examined by assuming various kinds of functional form. In the examination, it was assumed that the parameters depended on mass and charge of fissioning nuclide, incident energy and binding energy of incident particle and shell structure of the fissioning nuclide. In order to take the shell structure into consideration, the shell factor given by Meyers and Swiatecki[4] was used as in the Moriyama-Ohnishi systematics.

Examples of the examination are shown in Figs. 1 and 2. Figure 1 shows the incident energy dependence of the R parameter. The solid circles are derived from the measured mass distribution of ^{238}U fission by various kinds of incident neutron energy.[5] The measured mass distribution was decomposed to 5 Gaussian functions and the R value was calculated from the parameters of the decomposed 5 Gaussian functions. From the measured data the energy dependence seemed to be proportional to $1/(E^{C_1} + C_2)$ as seen in Fig. 1 and the parameters C_1 and C_2 were determined by least-squares fit. Figure 2 shows the shell factor dependence of the R parameter. The solid circles are derived from the measured data using the similar procedure to the above incident energy dependence. The parameter derived from the measured data seemed to have sin function of the shell factor. Then the coefficients of the sin function were determined by least squares fit.

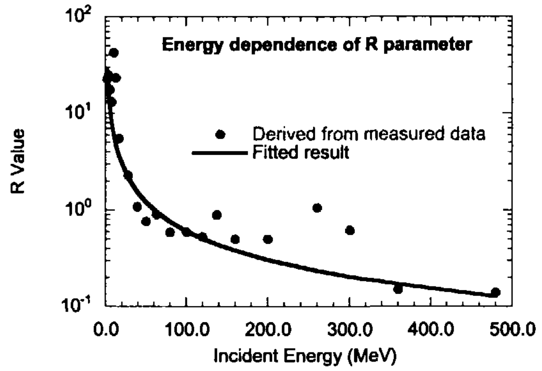

Fig. 1 Energy dependence of R parameter.

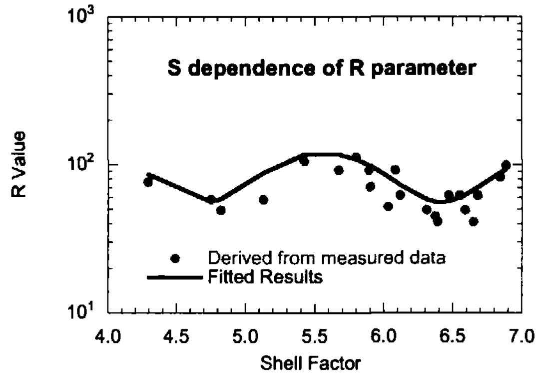

Fig. 2 Shell factor dependence of R parameter.

Other examples of the examination are shown in Figs. 3 and 4. Figure 3 shows the F parameter values derived from the mass distribution of ^{238}U neutron induced fission by various incident energy.[5] It is difficult from this figure to suppose the clear energy dependence of the F parameter. Then we assumed that there is no energy dependence for the F parameter. The shell

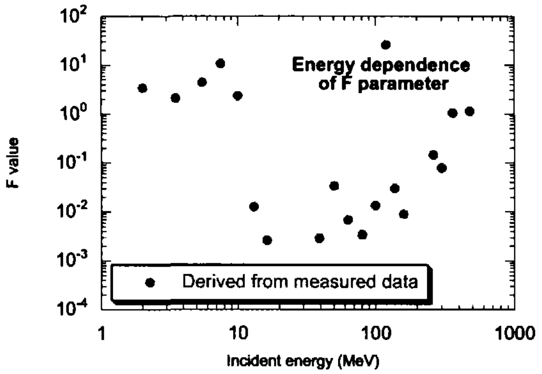

Fig. 3 Energy dependence of F parameter.

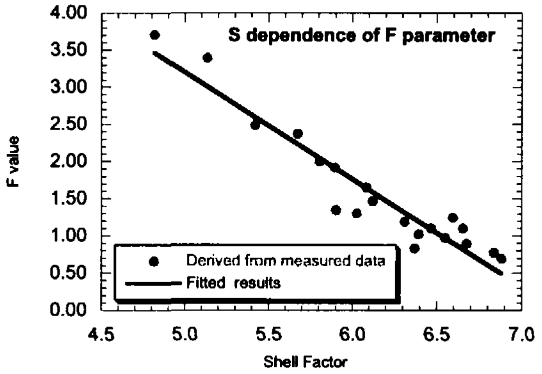

Fig. 4 Shell factor dependence of F parameter.

factor dependence, however, is clearly seen in Fig. 4. From this figure it was assumed that the F parameter linearly depends on the shell factor S. Similar examinations were performed for other parameters. By these examinations, the functional forms of the 8 parameters were determined.

The obtained parameters are listed below:

$$\begin{aligned}
\bar{\nu} &= 1.404 + 0.1067(A_f - 236) \\
&\quad + [14.986 - 0.1067(A_f - 236)] \\
&\quad \cdot [1.0 - \exp(-0.00858E^*)], \\
R &= [112.0 + 41.24\sin(3.675S)] \\
&\quad \cdot \frac{1.0}{BN^{0.331} + 0.2067} \cdot \frac{1.0}{E^{0.993} + 0.0951}, \\
F &= 10.4 - 1.44S, \\
\sigma_s &= 12.6, \\
\sigma_{h1} &= (-25.27 + 0.0345A_f + 0.216Z_f) \\
&\quad \cdot (0.438 + E + 0.333BN^{0.333})^{0.0864}, \\
\sigma_{h2} &= (-30.73 + 0.0394A_f + 0.285Z_f) \\
&\quad \cdot (0.438 + E + 0.333BN^{0.333})^{0.0864}, \\
A_{h1} &= 0.5393(A_f - \bar{\nu}) + 0.01542A_f \\
&\quad \cdot (40.2 - Z_f^2/A_f)^{1/2}, \\
A_{h2} &= 0.5612(A_f - \bar{\nu}) + 0.01910A_f \\
&\quad \cdot (40.2 - Z_f^2/A_f)^{1/2},
\end{aligned}$$

where E^* is the excitation energy of fissioning nuclide and is the sum of the incident energy E and the binding energy BN. Parameters A_f and Z_f are mass and charge of fissioning nuclides. S is the shell factor mentioned above. The $\bar{\nu}$ value is that given by Wahl as mentioned before. As for A_{h1} and A_{h2} values, we assumed that they have similar functional form used by Moriyama-Ohnishi but the coefficients were newly determined by using newer data than those they employed. As seen in the equations of A_{h1} and A_{h2}, they have the term proportional to $(40.2 - Z_f^2/A_f)^{1/2}$ which limits the applicability of the present systematics. As the inside of the parentheses has to be positive, the condition of $Z_f^2/A_f \leq 40.2$ has to be maintained when we use the systematics. Most of the minor actinides important for nuclear technology fields, however, are in the range of the limitation and the present systematics can be employed in practical use of nuclear technology fields.

Using the obtained systematics, comparisons with measured data were performed for various kinds of fissioning systems. The comparisons are shown in the next chapter.

III. Comparison with measured data

The systematics with new parameters set was used to compare with measured mass distributions of various kinds of fissioning systems. Some of the comparisons are shown in this chapter.

Figure 5 shows mass distribution of ^{245}Cm thermal neutron induced fission. Measured data are taken from the paper by Dickens.[6] The mass distribution of this case shows a typical asymmetric shape. The present result reproduces the asymmetric shape and shows good agreement with the measured data. Even in the wing part where yields are decreasing more than one order of magnitude, the decreasing trend of the yields seems to be reproduced. The scale of this figure, however, is logarithmic and the difference between the measured data and the calculated ones is not clearly seen. In order to make the difference clear, the difference is shown in Fig. 6. In this figure, percent differences, P_i, from the measured values are shown as defined follows:

$$P_i = \frac{C_i - E_i}{E_i} \times 100.0,$$

where C_i is calculated value and E_i measured one.

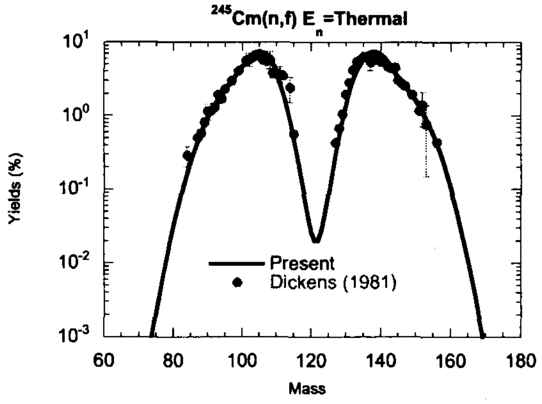

Fig. 5 ^{245}Cm thermal neutron induced fission.

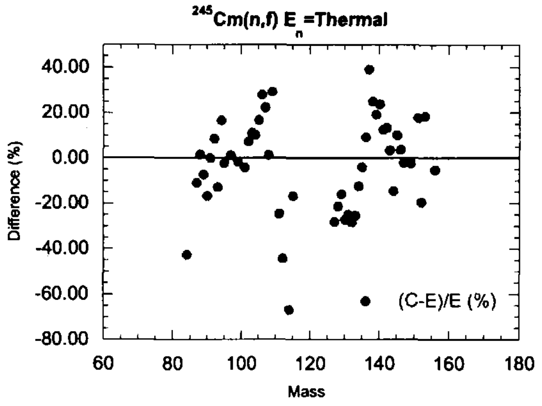

Fig. 6 Differences between measured values and calculated ones for ^{245}Cm thermal neutron induced fission.

Maximum difference in this case is about -70 % at A≈115. Excepting this point the difference is stayed within about 40 % in this case.

As another example, mass distribution of ^{238}U neutron induced fission by 160 MeV neutron is shown in

Fig. 7. Even for this kinds of high energy fission, mass distribution by the present systematics seems to reproduce the measured data. The measured mass distribution has no valley part and shows symmetric distribution. The calculated yields also shows symmetric shape although a slight dip at the peak area is seen in the calculation. Figure 8 shows percent difference between the measured data and the calculated ones. The differences seen in this figure scatters from -20 % to 40 %. The overall agreement between the measured data and the calculated ones of this case seems to be as good as that of the ^{245}Cm thermal neutron fission.

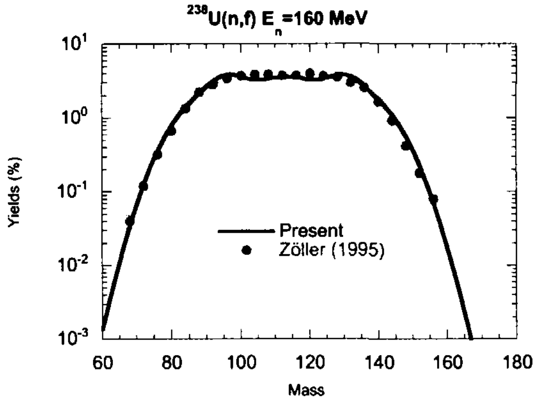

Fig. 7 ^{238}U fission by 160 MeV neutron.

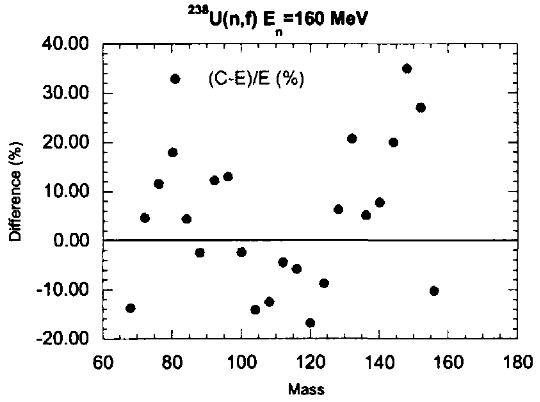

Fig. 8 Differences between measured values and calculated ones for ^{238}U fission by 160 MeV neutron.

From those comparisons the present systematics seems to be applicable to both low energy fission and high energy fission with similar precision. In the above examples, the mass distributions by neutron-induced fission are shown. The mass distributions by proton-induced fission were also calculated and compared with measured data. Even in the proton-induced fission similar agreement with the measured data was obtained. In Table 1 the average deviations from the measured values for various kinds of fissioning systems are shown. The average deviation is defined here as follows:

$$\overline{\left|\frac{C-E}{E}\right|} = \sqrt{\frac{1}{N}\sum_{i}^{N}\left(\frac{C_i - E_i}{E_i}\right)^2},$$

where C_i and E_i are calculated value and measured one respectively. As seen in Table 1 the average deviations do not exceed 1.0, that is, the deviation is less than 100 % on an average for various kinds of fissioning systems including proton-induced fission. Considering that the mass yields change from a few percents at the peak area to less than 1 % at the wing or the valley area, the systematics giving the deviation less than 100 % for various fissioning systems is regarded as a rather good one for applying to minor actinide fission by high energy incident particle.

Table 1 Average Deviations for Various Fission

Fissioning System	Incident Energy	Deviation
^{233}U + n	Thermal	0.2528
^{235}U + n	Thermal	0.6643
	8.1 MeV	0.2889
^{238}U + n	1.6 MeV	0.2917
	5.5 MeV	0.1900
	8.0 MeV	0.2715
	13 MeV	0.6197
	21 MeV	0.7204
	50 MeV	0.7069
	100 MeV	0.4358
	160 MeV	0.1447
^{238}U + p	55 MeV	0.3737
	340 MeV	0.3377
^{237}Np + n	5 MeV	0.6198
^{239}Pu + n	7.9 MeV	0.4843
^{242}Pu + n	15.1 MeV	0.2303
^{241}Am + p	16 MeV	0.7910
^{243}Am + p	15.6 MeV	0.1923
^{245}Cm + n	Thermal	0.2084
^{248}Cm + p	20 MeV	0.8355

IV. Summary

The new parameters set of FP yields systematics with 5 Gaussian functions has been developed. The systematics is able to reproduce the measured mass distributions of various kinds of fissioning systems including minor actinides with incident particle energy from thermal to higher than 100 MeV. The systematics, however, has the limitation of $Z_f^2/A_f \leq 40.2$. Then it is not used for the fissioning system having Z_f and A_f which fulfill the condition of $Z_f^2/A_f > 40.2$. The limitation, however, does not restrict practical use of the present systematics, because the range of the Z_f^2/A_f values of minor actinides interested in nuclear technology fields such as minor actinide incineration using ADS or reactor is stayed in the limitation. Then the present systematics will be used for the application fields like the feasibility study of the minor actinide incineration system.

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