11. Probing the "Nutcracker" by Two-Pion Correlation

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Abstract

We investigate the "nutcracker" phenomenon by studying two-pion correlation functions of non-central relativistic heavy-ion collisions. We apply the full (3+1)-dimensional hydrodynamic model with first order phase transition. Based on numerical results which shows the "nutcracked" freeze-out hypersurface, we calculate the two-pion correlation functions. In the case of snapshot hypersurfaces which clearly show the nut-shell structure, we find a interesting behavior at high relative momenta. We discuss the possibility of observing the phenomenon in heavy ion experiments.

Many kinds of candidates have been proposed for a signature of creation of the quark-gluon plasma (QGP) at high energy heavy ion experiments [1]. Teaney and Shuryak show that an unusual space-time evolution of hot and dense matter may occur in *non-central* heavy ion collisions if the phase transition takes place [2]. This phenomenon is named "nutcracker" scenario because the matter distribution looks like a "nut" and two "half-shells" and the characteristic distribution is a consequence of the existence of phase transition between the hadronic phase and the QGP phase. As the scenario exhibits a geometrical feature, pion interferometry is the most suitable tool to confirm the scenario. According to the (anti-)symmetry of identical quanta, particle interferometry gives us a geometrical information such as source sizes and lifetime [3]. In the present paper, we examine the possibility for the two-pion correlation function as a probe of the "nutcracker". In spite of the detailed analyses of the nutcracker scenario by Kolb *et al.* [4], the initial condition for the scenario is not clear. Here we select several sets of parameters which give typical structures in space-time evolution and focus our discussion on the behaviors of correlation functions.

Because the scenario happens in non-central collisions, we have to apply the full (3+1)-dimension calculations for the hydrodynamical equations [5]. Here we adopt an equation of state (EoS) with first order phase transition. The QGP phase consists of a free gas of u, d and s-quarks and gluons and the hadronic phase is a free resonance gas with excluded volume correction where heavy resonances are included up to 2 GeV [6]. Assuming the critical temperature T_c as 160 MeV at zero net baryon density, the bag constant becomes $B^{1/4} = 233$ MeV. As for the impact parameter, we put it as 7 fm. Maximum energy density E_{max} at the initial state is fixed at 7 GeV/fm^3. Though this value is much higher than the value for the 158 GeV/A Pb+Pb collisions at SPS [7], much higher energy density is expected at RHIC.

Whether the nut-shell structure appears or not depends on the shape of the initial transverse profile [4]. We assume that the transverse profile is proportional to the Woods-Saxon function. Here we vary a diffuseness parameter δ_r of the Woods-Saxon function so that four different scenarios occur, then we compare correlation functions which are obtained from those scenarios. Figure 1 shows the volume fraction distributions of freeze-out hypersurface projected onto the transverse plane. Here we show only the volume in $|z| \leq 0.5$ fm. Each column corresponds to one scenario. Each row corresponds to chronological snapshots before the freeze-out. The time interval of the snapshots is 0.4 fm, respectively. The leftmost series corresponds to $\delta_r = 0.75$, where two shells and a nut are created at the final stage ($t - t_0 = 6.8$ fm). We call this scenario "nut-shell". The next column corresponds to $\delta_r = 1.0$ ($t - t_0 = 6.0$ fm $\sim$ 7.6 fm). Similar structure also appears in this scenario but two shells survive longer than a central nut. So we call the scenario "shell". In the third series, though we can also see the structure in the second figure ($t - t_0 = 6.0$ fm), the shells vanish at the next step and only a central nut has been left. We call the scenario "nut" ($\delta_r = 0.5$). The rightmost series where $\delta_r = 0.25$ shows no unusual space-time evolution. We call the scenario "usual".

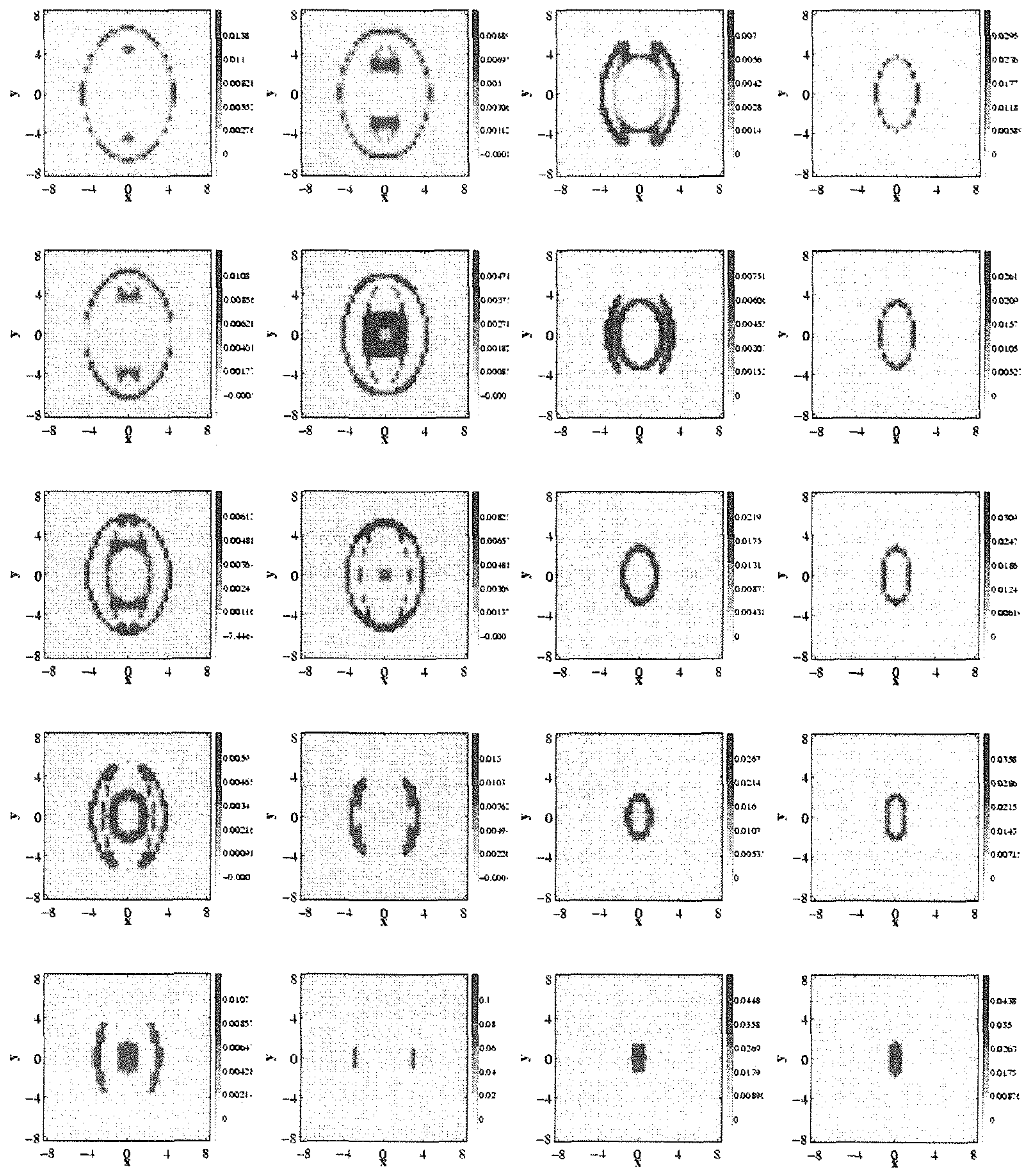

Figure 1: Volume fraction distributions of freeze-out hypersurfaces at each time step. See text for detail.

Assuming that the source is chaotic, the two-pion correlation function can be written as

$$C(q^\mu, K^\mu) = 1 + R(q^\mu, K^\mu),$$

$$R(q^\mu, K^\mu) = \frac{1}{\mathcal{N}} \int K_\mu d\sigma^\mu(x) \int K_\nu d\sigma^\nu(x') \sqrt{F(k_1,x)F(k_2,x')F(k_1,x')F(k_2,x)} \cos(q_\rho(x-x')^\rho) \quad (1)$$

where $K^\mu = 1/2(k_1^\mu + k_2^\mu)$ is the average momentum of two emitted pions, $q^\mu = k_1^\mu - k_2^\mu$ is the relative momentum and $\mathcal{N}$ is the normalization given by one-particle distribution. $F(k,x)$ denotes the Bose-Einstein distribution function which is given as

$$F(k,x) = \frac{1}{\exp(u_\mu(x)k^\mu/T_{\mathrm{f}}) - 1}. \quad (2)$$

The four velocity of the fluid $u^\mu(x)$ causes the space-momentum correlation which is observed as collective flow. As is well known, we can obtain the geometrical information on a space-time direction from the correlator as a function of a corresponding component of the relative momentum [8].

As seen in Fig.1, the nut-shell structure appears in the shape of the freeze-out hypersurface. But geometrical information obtained from the correlator corresponds to the source function (*i.e.*, emission probability of a particle with a momentum from a space-time point), which is deformed by the collective flow through the thermal distribution (2). First of all, we consider the particles at midrapidity because the nut-shell structure has been created in *central* region. We fix the transverse momentum as $K_T = 50$ MeV. Putting the longitudinal relative momentum as $q_z = 0$, we have three variables; azimuthal angle ϕ and the transverse relative momentum q_x and q_y. (Note that the x axis is chosen to be parallel to impact parameter.) If we consider the case $\phi = 0$ ($\phi = \pi/2$), q_x is the outward (sideward) component and q_y is the sideward (outward) component. In the nut-shell structure (see the lowest left figure of Fig.1), shape of the source along x axis differs from the one along y axis because the outer shell is cracked on y axis. Hence, if we probe the source along x axis and y axis respectively, the result should show some differences. Therefore, we calculate the *sideward* correlation functions of both $\phi = 0$ and $\phi = \pi/2$.

Figure 3 shows the sideward correlation functions. Left figure is the case $\phi = 0$ and right one is the case $\phi = \pi/2$. In each figure, thick line denotes the correlation function calculated from the "nut-shell" *snapshot* hypersurface (see the lower left figure of Fig.1). As clearly seen, the correlation function has the second peak at $q_{\mathrm{side}} \simeq 250$ MeV in the case $\phi = \pi/2$. This is the clear signature of the nut-shell structure of the source. Hence, we see the signature of the nutcracker by comparing the sideward correlation function of $\phi = \pi/2$ with the one of $\phi = 0$ which has no secondary peak. Other four kinds of lines include the whole hypersurface. However, even the "shell" scenario shows no peak at high relative momentum in the case $\phi = \pi/2$. In order to discuss more quantitavely, let us use a simple two-dimensional source model with nut-shell structure,

$$S(x,y) = S_{\mathrm{nut}}(x,y) + S_{\mathrm{shell}}(x,y), \quad (3)$$

$$S_{\mathrm{nut}}(x,y) = \frac{1-\alpha}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right), \quad (4)$$

$$S_{\mathrm{shell}}(x,y) = \frac{\alpha}{8L\delta_x}\theta(L - |y|)\{\theta(\delta_x - |x - R_x|) + \theta(\delta_x - |x + R_x|)\}. \quad (5)$$

A nut is modeled into a gaussian source which has the width σ_x and σ_y (eq.(4)). Two half shells are modeled into walls parallel to y axis with length L and thickness δ_x. The source function

$S(x,y)$ describes the emission probability of particles at transverse coordinate x and y. Here we neglect the effect of the collective flow (*i.e.*, space-momentum correlation), for simplicity. A parameter α controls the ratio of the number of particles emitted from the shell to the whole source. We can easily obtain the correlation functions in analytical form,

$$C(q_{\rm side}, q_{\rm out}=0, \phi=\frac{\pi}{2}) - 1 = \left[(1-\alpha)\exp\left\{-\frac{1}{2}(q_{\rm side}^2\sigma_x^2)\right\} + \alpha\frac{\sin q_{\rm side}\delta_x}{q_{\rm side}\delta_x}\cos q_{\rm side}R_x\right]^2, \quad (6)$$

$$C(q_{\rm side}, q_{\rm out}=0, \phi=0) - 1 = \left[(1-\alpha)\exp\left\{-\frac{1}{2}(q_{\rm side}^2\sigma_y^2)\right\} + \alpha\frac{\sin q_{\rm side}L}{q_{\rm side}L}\right]^2. \quad (7)$$

The nut-shell structure appears in eq.(6) as $\cos(q_{\rm side}R_x)$. Figure 3 shows the above correlation functions for the cases $\alpha = 0.1$ and $\alpha = 0.3$. Parameters are set to $\delta_x = 0.5$, $\sigma_x = 2.5$, $\sigma_y = 4.0$, $R_x = 2.5$ and $L = 8.0$. In the case $\phi = \pi/2$, we see the peak which characterizes the nut-shell structure for $\alpha = 0.3$ but there is no peak for $\alpha = 0.1$. In our hydrodynamic analysis (case of Fig.2), α is about 0.12. This value is too few for the correlation function to behave as the case of the nut-shell snapshot.

In summary, we analyze the two-pion correlation functions for "nutcracker" scenario. We perform a hydrodynamical calculation to reproduce some unusual structure of freeze-out hypersurface. Based on the results, we calculate the correlation function and consider how the unusual structures affect the correlation function. We find that the second peak of the correlation function at high relative momentum results from the nut-shell structure. However, it is also shown that the number of pions emitted from the shell is not so large that we cannot see any second peak of the correlation function from our freeze-out hypersurface. More detailed discussion will be published later [9].

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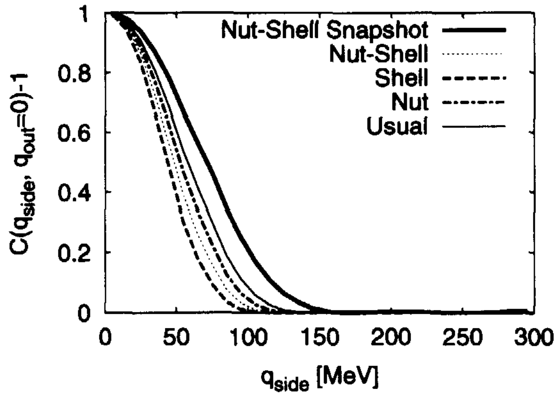

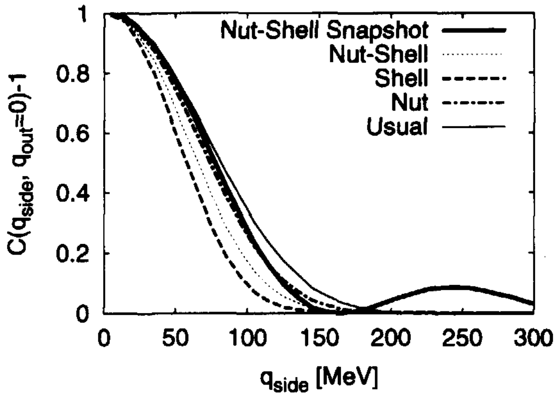

Figure 2: Left: the two-pion sideward correlation function at $\phi = 0$. Right: $\phi = \pi/2$. Thick lines show the function calculated from the nut-shell snapshot. Dotted lines, dashed lines, dashed-dotted lines and thin lines correspond to the "nut-shell" scenario, the "shell" scenario, the "nut" scenario and the "usual" scenario, respectively.

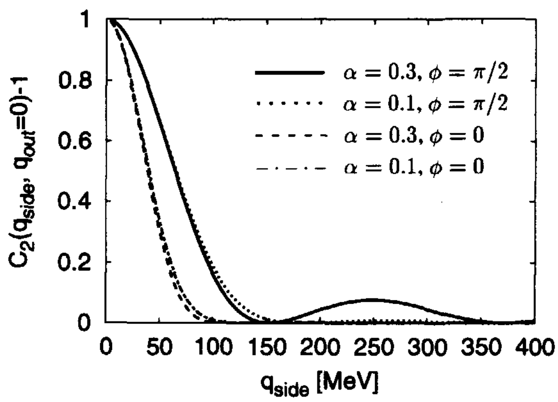

Figure 3: Correlation function calculated from a toy source model.