



25. クォーク物質での二重カイラル密度波 Dual chiral density wave in quark matter

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abstract

We prove that quark matter is unstable for forming a dual chiral density wave above a critical density, within the Nambu-Jona-Lasinio model. Presence of a dual chiral density wave leads to a uniform ferromagnetism in quark matter. A similarity with the spin density wave theory in electron gas and the pion condensation theory is also pointed out.

I. INTRODUCTION

Recently condensed matter physics of QCD has been an exciting area in nuclear physics. Superconducting model of the vacuum is a classic one, firstly considered by Nambu and Jona-Lasinio to understand the nucleon mass [1]. Recently their model has been considered as an effective model embodying spontaneous breaking of chiral symmetry in terms of quark degree of freedom [2]. Very recently color superconductivity in quark matter has been extensively studied by many authors [3]. There are also some works to inquire the coexistence of these two phases on the phase diagram of quark matter [3,4]. On the other hand, the possibility of ferromagnetism (FM) in quark matter has been also discussed [5].

It has been suggested that ferromagnetic phase exists at rather low densities in analogy with an electron gas discussed by Bloch [6] and demonstrated that relativistic effects play an important role in quark matter. However, the framework there is not self-consistent and simply based on the perturbation theory with one-gluon-exchange interaction. So, it might be interesting to inquire into the possibility of FM in quark matter more closely from elementary many-body theories; relativistic Hartree-Fock (RHF) calculations might be desirable as a first step. However, RHF calculations have two difficulties: one is due to the retardation effect in the interactions and the other is due to the vacuum effects. In particular, it is well known that the latter effects are indispensable in considering the phase diagram of quark matter at low densities, since they should give rise to confinement and spontaneously symmetry breaking of chiral symmetry.

We consider here the coexistence of FM and vacuum superconductivity. We prove that quark matter becomes unstable for a formation of a dual chiral density wave above a critical density, using the Nambu-Jona-Lasinio (NJL) model [2]. We shall also see a dual chiral density wave state is a ferromagnetic state. Thus there appears some coexistence region on the phase diagram of quark matter.

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II. DUAL CHIRAL DENSITY WAVE

We start with the NJL Lagrangian with $N_f = 2$ flavors and $N_c = 3$ colors,

$$\mathcal{L}_{njl} = \bar{\psi}(i\cancel{\partial} - m_c)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2], \quad (1)$$

where m_c is the current mass, $m_c \simeq 5\text{MeV}$. Under the Hartree approximation, we linearize Eq. (1) by partially replacing the bilinear quark fields by their expectation values with respect to the ground state. Note that we keep pseudo-scalar as well as scalar mean-field. In the usual treatment to generate a dynamical quark mass and study the restoration of chiral symmetry within the NJL model, authors implicitly assumed the pseudo-scalar mean-field should be vanished. However, this is justified only for the vacuum since it is a definite eigenstate of parity, and there is no compelling reason at finite baryon density. In the following we shall see two degrees of freedom play an important role at finite baryon density. We assume the following form for the mean-fields,

$$\langle\bar{\psi}\psi\rangle = \Delta \cos\theta(\mathbf{r}) \quad (2)$$

$$\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle = \Delta \sin\theta(\mathbf{r}), \quad (3)$$

and others are vanished¹. Both scalar and pseudo-scalar densities always reside on the chiral circle of the radius Δ , $|\langle\bar{\psi}\psi\rangle|^2 + |\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle|^2 = \Delta^2$, while each density is spatially variant. Taking a nontrivial form of the chiral angle θ such that $\theta(\mathbf{r}) = \mathbf{q} \cdot \mathbf{r}$, we call this set a dual chiral density wave (DCDW). Some authors considered similar configuration [7] and called a chiral density wave in analogy with spin density wave (SDW) by Overhauser in condensed-matter physics [8]. In their ansatz only the scalar mean-field oscillates with finite wave number and the pseudo-scalar one is discarded. It is also interesting to recall that DCDW configuration is similar to the pion condensation in high-density nuclear matter within the σ model, considered by Dautry and Nyman (DN) [9], where σ and π^0 meson condensates take the same form as Eq. (3).²

Accordingly, we define a new quark field ψ_W by the Weinberg transformation,

$$\psi_W = \exp[i\gamma_5\tau_3\mathbf{q} \cdot \mathbf{r}/2]\psi, \quad (4)$$

to get the transformed Lagrangian,

$$\mathcal{L}_{MF} = \bar{\psi}_W[i\cancel{\partial} - m - 1/2\gamma_5\tau_3\cancel{\partial}] \psi_W - G\Delta^2 + \delta\mathcal{L}, \quad (5)$$

where $\delta\mathcal{L}$ is a small residual term due to the current mass m_c ,

$\mathcal{L}_{SB} = -m_c\bar{\psi}_W[\exp(i\gamma_5\tau_3\mathbf{q} \cdot \mathbf{r}) - 1]\psi_W$, and we put $m \equiv m_c - 2G\Delta$ and $q^\mu = (0, \mathbf{q})$. In the following, we first take the chiral limit ($m_c = 0$) discarding $\delta\mathcal{L}$ and estimate the $m_c \neq 0$ case later. The form given in (5) appears to be the same as that in the superconductor model of the vacuum except an axial-vector field generated by the wave vector of DCDW;

¹We, hereafter, only consider the charge eigenstate for the ground state.

²The DN model has been recently revisited in ref. [10] in the context of saturation problem of nuclear matter within the σ model. In ref. [11] the similar configuration has been also considered in quark matter within a hybrid model with quark-meson couplings.

the *amplitude* of the DCDW produces the dynamical mass in this case. We shall see the axial-vector field gives rise to FM in quark matter and the wave vector \mathbf{q} is related to the magnetization: the *phase* of the DCDW induces the magnetization.

The Dirac equation for ψ_W then reads

$$(i\cancel{\partial} - m - 1/2\tau_3\gamma_5\cancel{q})\psi_W = 0. \quad (6)$$

We can find a spatially uniform solution for the quark wave function, $\psi_W = u_W(p) \exp(i\mathbf{p} \cdot \mathbf{r})$,³ and the eigenvalues are

$$E_p^\pm = \sqrt{E_p^2 + |\mathbf{q}|^2/4 \pm \sqrt{(\mathbf{p} \cdot \mathbf{q})^2 + m^2|\mathbf{q}|^2}}, \quad E_p = (m^2 + p^2)^{1/2} \quad (7)$$

for positive energy (valence) quarks with different polarizations. For negative energy quarks in the Dirac sea, they have a spectrum symmetric with respect to null line because of charge conjugation symmetry in the Lagrangian (5).

The mean-value of the spin operator is given by

$$\bar{s}_z = \frac{1}{2} u_W^\dagger \Sigma_z u_W = \frac{1}{2} \frac{q/2 \pm \beta}{E_p^\pm}, \quad (8)$$

with $\beta = \sqrt{p_z^2 + m^2}$. The integral of \bar{s}_z over the Fermi seas should be proportional to q . Thus we can see that q is proportional to the magnetization and the solution with $q \neq 0$ implies FM. The single-particle spectrum (7) clearly shows the *exchange splitting* between energies with two polarizations under the existence of magnetization; in the non relativistic limit, $|\mathbf{p}|, |\mathbf{q}| \ll m$, $E_p^\pm \rightarrow m + \frac{p^2}{2m} \pm |\mathbf{q}|/2$ [6]. Hereafter, we choose \mathbf{q}/\hat{z} , $\mathbf{q} = (0, 0, q)$, without loss of generality. We need not distinguish two flavors since all the results do not depend on the sign of q . The energy spectrum (7) shows some interesting features: first it breaks rotation symmetry to result in axially-deformed Fermi sea. Secondly, E_p^- first decreases and reaches zero at $\mathbf{p} = (0, 0, \sqrt{q^2/4 - m^2})$ as a function of $|\mathbf{p}|$ in case of $m < q/2$.

The thermodynamic potential is then given as

$$\begin{aligned} \Omega_{\text{total}} &= \gamma \int \frac{d^3p}{(2\pi)^3} [(E_p^- - \mu)\theta_- + (E_p^+ - \mu)\theta_+] - \gamma \int \frac{d^3p}{(2\pi)^3} [E_p^- + E_p^+] + (m - m_c)^2/4G \\ &\equiv \Omega_{\text{val}} + \Omega_{\text{vac}} + (m - m_c)^2/4G. \end{aligned} \quad (9)$$

where $\theta_\pm = \theta(\mu - E^\pm)$, μ is the chemical potential and γ is the degeneracy factor $\gamma = N_f N_c$. The first term Ω_{val} is the contribution by the valence quarks filled up to chemical potential, while the second term Ω_{vac} is the vacuum contribution which is formally divergent. Once Ω_{total} is properly evaluated, the equations to be solved to determine the optimal values of Δ and q are

$$\frac{\delta\Omega_{\text{total}}}{\delta\Delta} = \frac{\delta\Omega_{\text{total}}}{\delta q} = 0. \quad (10)$$

³This feature is very different from refs. [7], where the wave function is no more plane wave.

III. PROPER-TIME REGULARIZATION

Since NJL model is not renormalizable, we need some regularization procedure to get a meaningful finite value for the vacuum contribution. We cannot apply the usual momentum cut-off regularization (MCOR) scheme to regularize Ω_{vac} , since the energy spectrum is no more rotation symmetric. Instead, we adopt the proper-time regularization (PTR) scheme [12]. We think this is a most suitable one for our purpose, since Ω_{vac} counts the spectrum change under the “external” axial-vector field. It has been shown that the vacuum polarization effect under the external electromagnetic field can be treated in a gauge invariant way, where the energy spectrum is also deformed depending on the field strength. It is also known that the consequences from the NJL model are almost regularization-scheme independent [2], including the PTR scheme.

We eventually find

$$\Omega_{\text{vac}} = \frac{\gamma}{8\pi^{3/2}} \int_0^\infty \frac{d\tau}{\tau^{5/2}} \int_{-\infty}^\infty \frac{dp_z}{2\pi} \left[e^{-(\sqrt{p_z^2+m^2+q/2})^2\tau} + e^{-(\sqrt{p_z^2+m^2-q/2})^2\tau} \right] - \Omega_{\text{ref}}, \quad (11)$$

which is reduced to the standard formula [2] in the limit $q \rightarrow 0$.

We can easily see, from Eq. (11), that the q degree of freedom becomes redundant and theory must become *trivial* in the limit $m \rightarrow 0$, which is equivalent with $\Delta \rightarrow 0$ in the chiral limit: all the observables must be independent of q . This feature coincides with the form of DCDW. The integral with respect to the proper time τ is not well defined as it is, since it is still divergent due to the $\tau \sim 0$ contribution. Regularization proceeds by replacing the lower bound of the integration range by $1/\Lambda^2$, which corresponds to the momentum cut-off in the MCOR scheme,

$$\int_0^\infty d\tau \longrightarrow \int_{1/\Lambda^2}^\infty d\tau. \quad (12)$$

IV. INSTABILITY OF QUARK MATTER

Now we examine the instability of quark matter with respect to formation of the DCDW. In the following we consider sign change of the curvature of Ω_{total} at the origin (*stiffness*), which is proportional to the inverse of the magnetic susceptibility. Expanding Ω_{vac} with respect to q up to $O(q^2)$, we find

$$\begin{aligned} \Omega_{\text{vac}} &= \frac{\gamma\Lambda^2}{16\pi^2} J(m^2/\Lambda^2)q^2 + O(q^4) + \Omega_{\text{vac}}^0 \\ &\equiv \Omega_{\text{vac}}^{\text{mag}} + \Omega_{\text{vac}}^0 + O(q^4), \end{aligned} \quad (13)$$

where $J(x)$ is a universal function,

$$J(x) = -x\text{Ei}(-x), \quad (14)$$

with the exponential integral $\text{Ei}(-x)$. $\Omega_{\text{vac}}^{\text{mag}}$ is the pure magnetic contribution and provides a kinetic term for the DCDW. It originates from a vacuum polarization effect due to the presence of the DCDW and gives a ‘repulsive’ (positive) contribution irrespective of the value of m , so that the vacuum is stable against formation of the DCDW. Note that it gives a null contribution in case of $m = 0$, irrespective of q , as it should be.

The coefficient of q^2 term in $\Omega_{\text{vac}}^{\text{mag}}$, the vacuum stiffness parameter, has a definite physical meaning. Since the pion decay constant f_π is m dependent and is given in terms of $J(x)$ within the NJL model [2], $\Omega_{\text{vac}}^{\text{mag}}$ can be written as

$$\Omega_{\text{vac}}^{\text{mag}} = \frac{1}{2} f_\pi^2 q^2 + O(q^4), \quad (15)$$

except an irrelevant constant. There are two remarks in order. One is that there appears no divergence as $\Lambda \rightarrow \infty$ in the higher order terms with respect to q . Hence, strictly speaking, higher order terms should be discarded [13], since the divergent term should dominate over other finite terms. The other is that Eq. (15) suggests that the vacuum polarization effect precisely gives the corresponding kinetic term in the σ model, with a “running” pion decay constant.

For given μ and m , q we can evaluate the valence contribution Ω_{val} using Eq. (7) and write down the general formula analytically, but it is very complicated. However, it is sufficient to consider the case such that $q/2 < m$ and $\mu > m + q/2$ for our present purpose. Then the thermodynamic potential can be expressed as

$$\Omega_{\text{val}} = \epsilon_{\text{val}}(q) - \mu \rho_{\text{val}}(q), \quad (16)$$

where $\epsilon(q)$ and $\rho(q)$ are the energy density and the quark-number density, respectively. After some manipulation we find the valence contribution can be finally written as

$$\begin{aligned} \Omega_{\text{val}} &= \Omega_{\text{val}}^0 - \frac{\gamma}{8\pi^2} m^2 q^2 H(\mu/m) + O(q^4) \\ &\equiv \Omega_{\text{val}}^0 + \Omega_{\text{val}}^{\text{mag}} + O(q^4), \end{aligned} \quad (17)$$

where $H(x) = \ln(x + \sqrt{x^2 - 1})$ and $\Omega_{\text{val}}^0 = \epsilon_{\text{val}}^0 - \mu \rho_{\text{val}}^0$. Since the function $H(x)$ is always positive and a monotonously increasing function:

$$H(x) \longrightarrow 0 \quad \text{as } x \rightarrow 1 + 0, \quad \longrightarrow \ln x \quad \text{as } x \rightarrow +\infty, \quad (18)$$

the magnetic term $\Omega_{\text{val}}^{\text{mag}}$ contribute a negative energy and approaches to zero as $m \rightarrow 0$ (triviality). We may easily understand why the valence quarks always favor the formation of DCDW.

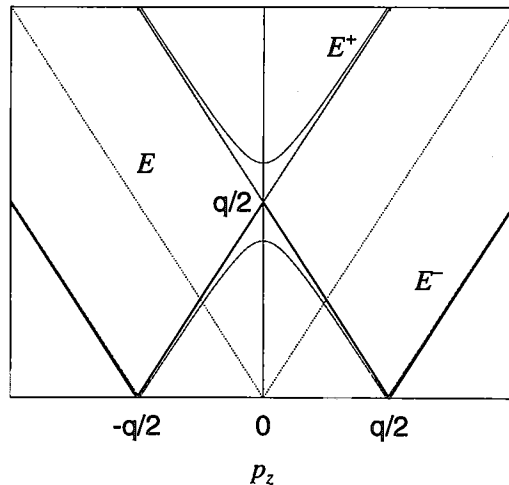


FIG. 1. Energy spectra for $\mathbf{p}_\perp = 0$. E^\pm with $m = 0$ are given by the thick lines, both of which overlap the one, E , with the definite chirality (dotted line) by the chirality dependent momentum-shift $\pm q/2$. We can see there is a level crossing at $p_z = 0$ with $m = 0$, while they are splitted by the mass (dash-dotted lines).

First, consider the energy spectra without the mass term (see Fig.1). As is already discussed, our theory becomes trivial in this case and we find two spectra

$$E_p^\pm = \sqrt{p_\perp^2 + (|p_z| \pm q/2)^2}, \quad \mathbf{p}_\perp = (p_x, p_y, 0), \quad (19)$$

which are essentially equivalent with $E_p^\pm = |\mathbf{p}|$; taking a linear combination of the eigenstates ψ_W^\pm , we can reconstruct the eigenstates of definite chirality γ_5 with the spectra, $\tilde{E}^\pm = \sqrt{p_\perp^2 + (p_z \pm q/2)^2}$. There is a level crossing at $\mathbf{p} = \mathbf{0}$. Once the mass term is taken into account this degeneracy is resolved and the exchange splitting appears at $\mathbf{p} = \mathbf{0}$. Hence it causes an energy gain, if $q = O(2\mu)$; we can also see this is the case by explicitly evaluating Ω_{val} . This mechanism is very similar to that of SDW by Overhauser [7,8].

Using Eqs. (9), (13), (17) we write the thermodynamic potential as

$$\Omega_{\text{total}} = \Omega_{NJJ} + \Omega_{\text{mag}} + O(q^4) \quad (20)$$

with

$$\begin{aligned} \Omega_{NJJ} &= \Omega_{\text{vac}}^0(m) + \Omega_{\text{val}}^0(m) + \frac{(m - m_c)^2}{4G} \\ \Omega_{\text{mag}} &= \frac{1}{2} f_\pi^2(m) q^2 - \frac{\gamma}{8\pi^2} m^2 q^2 H(\mu/m), \end{aligned} \quad (21)$$

where Ω_{NJJ} is the usual one without DCDW and $\Omega_{\text{mag}} = O(q^2)$. The dynamical mass m is given by the equation, $\partial\Omega_{\text{total}}/\partial m = 0$; At the order of q^0 the dynamical mass m^0 is determined by the equation,

$$\left. \frac{\partial\Omega_{NJJ}}{\partial m} \right|_{m^0} = 0. \quad (22)$$

Since $m - m^0 = O(q^2)$, the DCDW with wave vector \mathbf{q} onsets at a density where $\Omega_{\text{mag}}(m^0, q)$ becomes negative. Accordingly the critical chemical potential μ^{cr} is determined by the equation,

$$\frac{1}{2} f_\pi^2 - \frac{\gamma}{8\pi^2} m^{02} H(\mu^{cr}/m^0) = 0, \quad (23)$$

which means the stiffness parameter, which is proportional to the inverse of the magnetic susceptibility, vanishes at this point. This is *the threshold condition in a regularization-scheme independent way*, while the form of f_π depends on it. It is to be noted that this relation never depends on the values of the coupling constant G and the regularization parameter Λ included in the model.

V. RESULTS AND DISCUSSIONS

In Fig.2 we depict a critical line determined by Eq. (23) in the $\mu - m^0$ plane. In the limit $m^0 \rightarrow 0$, we find

$$\tilde{\mu}^{cr} = \frac{1}{2}e^{-\gamma_E/2} \simeq 0.375... \quad (24)$$

with γ_E being Euler's constant and $\tilde{\mu} = \mu/\Lambda$ in the PTR scheme. Thus, quark matter becomes unstable with respect to formation of DCDW and ferromagnetic phase appears beyond this chemical potential. Note that this is only a *sufficient* condition for the existence of ferromagnetic phase, and we can *never* excludes the possibility of the first order phase transition or metamagnetism.

There is left a delicate problem about the chiral symmetry restoration at finite density, especially about the order of the phase transition and the critical density, while general picture might be common that the dynamical mass and the pion decay constant decreases as chemical potential increases. There are devoted many studies about this issue but they are model dependent; even within the NJL model; some give a discontinuous jump of the dynamical mass as a function of chemical potential, while some give a smooth decrease, depending on the choice of values of the parameters [2]. We assume here the smooth decrease of the dynamical mass with respect to chemical potential.

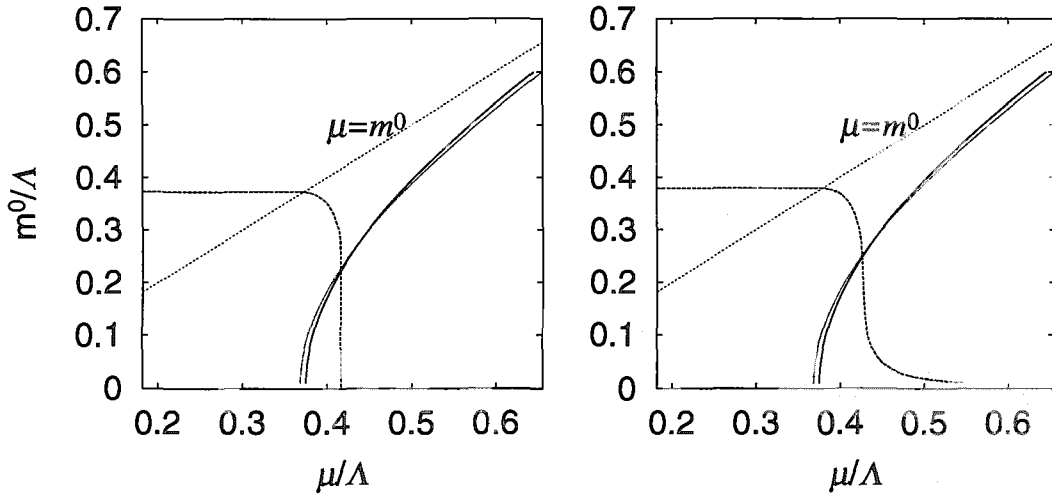


FIG. 2. Critical line and the dynamical mass in the $m^0 - \mu$ plane in the chiral limit (left panel) and in the case of nonvanishing current mass, $m_c = 5\text{MeV}$ (right panel). Critical line is given by the PTR method (solid curve) and by the MCOR method (crossed curve).

In the left panel of Fig.2 we depict an example of the dynamical mass as a solution of Eq. (22) in the chiral limit. We use a parameter set $g^2 \equiv G\Lambda^2 = 5$, $\Lambda = 1100\text{MeV}$ and $\gamma = 6$. We can see that the crossing point of two curves gives the critical chemical potential, above which quark matter is in the ferromagnetic phase until chiral symmetry is restored. Varying the coupling constant g^2 , chiral symmetry is restored at the chemical potential $\tilde{\mu}_{\text{res}}$,

$\tilde{\mu}_{\text{res}} = 1/2 - \pi^2/\gamma g^2$. Hence there is a critical coupling constant below which two curves have no crossing in the chiral limit,

$$g_{\text{cr}}^2 = \frac{4\pi^2}{N_c N_f (2 - \exp(-\gamma E))} \simeq \frac{27.46}{N_c N_f} \quad (25)$$

In the realistic case, where quarks have tiny but finite current mass, $\theta = \mathbf{q} \cdot \mathbf{r}$ is no more the self-consistent solution. We can estimate the explicit symmetry breaking (SB) effect \mathcal{L}_{SB} by relaxing the form of θ to satisfy the extremum condition such that $\delta[\Omega_{\text{total}}(\theta(\mathbf{r})) - \mathcal{L}_{\text{SB}}(\theta(\mathbf{r}))]/\delta\theta(\mathbf{r}) = 0$. If the form of θ is changed, the quark wave function should be also changed to satisfy the self-consistency. We only evaluate the SB effect by using the quark wave function in the chiral limit. Since the order of the SB effect is m_c , this procedure may be regarded as a perturbative expansion in terms of m_c . We can easily see this gives rise to no essential effect for validity of the critical line. In the right panel of Fig.2 we depict a critical line as well as the dynamical mass as functions of chemical potential for a small current mass $m_c = 5\text{MeV}$. We can easily see that two curves always have a crossing regardless of the coupling constant, since chiral symmetry is never restored at finite density in this case.

We also examine the regularization-scheme dependence of the critical line. Since Eq. (23) is regularization-scheme independent, we can use other expressions for the pion decay constant f_π . We examine here the MCOR scheme to find both schemes give similar critical lines (crossed curve in Fig. 2).

VI. SUMMARY AND CONCLUDING REMARKS

We have examined the stability of quark matter against the formation of DCDW, using the NJL model and found that Dirac sea is stiff, while Fermi sea is soft for it; the Overhauser mechanism works for Fermi sea. The critical condition is derived for the instability, which implies the ferromagnetic transition. We have seen that ferromagnetism can coexist with the vacuum superconductivity.

Up to this point we have only considered the direct channel (Hartree term) of the four Fermi interaction in the NJL model. When we consider the exchange contributions (Fock term), there appear quark-quark interactions in the vector and axial-vector channels. It is well known that the vector contribution is rather trivial and summed up as a renormalization of the chemical potential. We are interested in the axial-vector contribution in our context. We may see that it renormalizes the wave vector of the DCDW; this can be regarded as a kind of vertex renormalization of quark-DCDW coupling. This subject will be discussed elsewhere [15].

A DCDW instability in color superconducting phase might be another interesting subject to explore the phase diagram of quark matter, which may lead to a coexistence of superconductivity and ferromagnetism.

The author thanks K. Takahashi for stimulating discussions and useful comments. He also thanks T. Maruyama, E. Nakano and T. Tanaka for discussions. The present research is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute, and by the Japanese Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Culture, Sports, Science and Technology (11640272, 13640282).

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