Available at: http://www.ictp.trieste.it/~pub_off IC/2004/66

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

GLOBAL STABILIZATION OF LINEAR CONTINUOUS TIME-VARYING SYSTEMS WITH BOUNDED CONTROLS

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Abstract

This paper deals with the problem of global stabilization of a class of linear continuous time-varying systems with bounded controls. Based on the controllability of the nominal system, a sufficient condition for the global stabilizability is proposed without solving any Riccati differential equation. Moreover, we give sufficient conditions for the robust stabilizability of perturbation/uncertain linear time-varying systems with bounded controls.

MIRAMARE – TRIESTE
August 2004

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1 Introduction

The stabilizability problem of linear control systems has been studied by a number of authors for many years (see, e.g. [5, 8, 10, 16] and references therein). In all these works, it was supposed that the controls have not to satisfy any a priori bound. The situation is more interesting and realistic in the case when the control is restrained in some given subset. This constraint can arise for different reasons including physical considerations and technological limitations. There are several papers devoted to the bounded stabilization of linear time-invariant control systems; see, e.g. [2, 7, 13]. In the linear time-invariant control systems without bounded controls, whenever the system is global null-controllable, the problem of global stabilization via dynamic state feedback is standard.

For the infinite-dimensional systems, the bounded stabilization problem has been studied in [13] for time-invariant control systems using either the semigroup theory or an extension of the LaSalle's invariance principle to a Hilbert space. Among other generations of the bounded stabilization problem, the results obtained in [14, 15] are also worth mentioning. However, this problem is still not trivial for the time-varying case, especially for the uncertain time-varying systems.

In this paper, we consider the stabilizability problem via bounded control of linear time-varying systems

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad t \geq 0, \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control subjected to the constraint:

$$\|u(t)\| \leq r, \quad t \geq 0, \tag{2}$$

for some fixed positive real number r. For time-invariant systems, where the constant matrix A satisfies some appropriate spectral properties, the authors in [13] proposed the non-smooth stabilizing feedback controller of the form

$$u = \begin{cases} -r\frac{B^T x}{\|B^T x\|}, & \text{if } \|B^T x\| \geq r \\ -B^T x, & \text{if } \|B^T x\| \leq r. \end{cases}$$

The authors in [2, 3] extended to the smooth feedback control

$$u(t) = -r\frac{B^T x(t)}{1 + \|B^T x(t)\|},$$

and showed that this feedback controller globally stabilizes time-invariant system (1) provided some appropriate assumptions on the contraction semigroup. A state feedback control design is proposed in [1] for time-invariant systems in Hilbert spaces using a semigroup formulation, where A has compact resolvent. It is worth noting that the approach in these works cannot be readily applied to the time-varying systems. The main difficulty is that the investigation of the spectrum of the time-varying matrix/operator input $A(t)$ or of its evolution matrix/operator $U(t,s)$ is still complicated and there are no appropriate properties available as in the time-invariant case. Consequently, the problem of state feedback stabilization of linear time-varying systems with bounded controls is of interest in its own right. Further, in stability theory, an important investigation is to design a controller guaranteed the closed-loop system remaining asymptotically stable for all perturbations/uncertainties. The problem of stabilization for linear perturbation/uncertain systems has been the subject of research activity for many years; see, e.g. [3, 4, 11, 12, 14]. By perturbation/uncertain systems we mean systems which contain uncertain parameters. In this paper, we develop the result for the linear perturbed time-varying system

$$\dot{x}(t) = [A(t) + \Delta A(t)]x(t) + [B(t) + \Delta B(t)]u(t), \tag{3}$$

where the control $u(t)$ is subject to the constrained (2) and $\Delta A(t), \Delta B(t)$ are perturbation/uncertainties satisfying some norm-bounded constraint. New stabilizability conditions for the uncertain system (3) are derived based on the global null-controllability characterization of the nominal system.

This paper is organized as follows. Section 2 deals with the problem formulation and main notations. The relationship between the global controllability and the existence of a bounded solution of a Riccati differential equation (RDE) is also given in this section. Section 3 gives bounded stabilization conditions for the linear control system (1). Finally, in section 4, we study the problem of robust stabilization of the uncertain system (3). Illustrative examples and cited references are given.

2 Preliminaries

The following notation will be used throughout the paper.

R^+ denotes the set of all non-negative real numbers;

R^n denotes a n finite-dimensional space, with the scalar product $\langle .,. \rangle$;

$R^{n\times m}$ denotes the set of all $(n \times m)-$ matrices;

A^T denotes the transpose of the matrix A, matrix A is symmetric if $A = A^T$;

$L_2([t,s], R^m)$ denotes the set of all measurable L_2-integrable and R^m-valued functions on $[t,s]$;

Matrix $Q \in R^{n\times n}$ is semipositive definite ($Q \geq 0$) if $\langle Qx, x\rangle \geq 0$, for all $x \in R^n$. If $\langle Qx, x\rangle > 0$ for all $x \neq 0$, then Q is positive definite ($Q > 0$).

$M([0,\infty), R^n_+)$ denotes the set of all symmetric semipositive definite matrix functions, which are continuous on $[0,\infty)$.

Definition 2.1. The system (1) is globally stabilizable if there is a feedback control $u(t) = k(x(t))$ satisfying the constraint (2) such that the resulting closed-loop system of (1):

$$\dot{x}(t) = A(t)x(t) + B(t)k(x(t)), \quad t \in R^+,$$

is globally asymptotically stable in the Lyapunov sense.

Next, we consider the linear perturbed (uncertain) time-varying system (3), where the matrices $\Delta A(t), \Delta A_1(t)$ are real-valued functions representing time-varying parameter uncertainties. There are many different types of uncertain systems to be used depending on the type of uncertainty expected. In many cases, it is useful to consider a time-varying real parameter uncertainty in an uncertain system, which is typically a quantity unknown but bounded in magnitude in some way. That is, we do not know the value of the uncertainty but we know how big it can be (see, e.g. [2, 8, 12]). In this paper we assume that the perturbations/uncertainties $\Delta A(t), \Delta A_1(t)$ are real-valued functions and satisfy the following norm-bounded condition

$$\begin{cases} \Delta A(t) = H_1 F(t) E_1, \quad \Delta B(t) = H_2 F(t) E_2, \\ \|F(t)\| \leq 1, \quad \forall t \in R^+, \end{cases} \tag{4}$$

where $H_i, E_i, i = 1, 2$ are given constant matrices of appropriate dimensions. The uncertainties satisfying this condition will be called admissible.

Definition 2.2. Linear uncertain system (3) is robustly stabilizable if there is a feedback control $u(t) = k(x(t))$ satisfying the constraint (2) such that the resulting closed-loop system of (3) is globally asymptotically stable for all admissible uncertainties satisfying the condition (4).

The objective of this paper is to give stabilizability conditions for the systems (1) and (3).

It is well known that the main concepts of controllability was introduced by Kalman [6] and then developed by Ikeda et al. [5] in relation to the Riccati differential equations. Consider the linear unconstrained control system $[A(t), B(t)]$:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(0) = x_0, \quad t \in R^+,$$

where $x(t) \in R^n, u(t) \in R^m$. We recall that the system $[A(t), B(t)]$ is globally controllable (GC) in finite time if there is a number $N > 0$ such that for every $x_0 \in R^n$ there is a control $u(t) \in L_2([0, N], R^m)$ satisfying

$$U(N, 0)x_0 + \int_0^N U(N, s)B(s)u(s)ds = 0,$$

where $U(t, s)$ denotes the transition matrix of the linear time-varying system $\dot{x}(t) = A(t)x(t)$ defined by

$$\frac{\partial U(t, s)}{\partial t} = A(t)U(t, s), \quad t, s \geq 0, \quad U(t, t) = I.$$

Definition 2.3. [5] System $[A(t), B(t)]$ is uniformly globally controllable (UGC) in finite time if there are numbers $N > 0, c_1, c_2, c_3, c_4 > 0$ such that the following conditions hold for all $t \in R^+$:
(i) $c_1 I \le W(t, t+N) \le c_2 I$.
(ii) $c_3 I \le U(t+N, t) W(t, t+N) U^T(t+N, t) \le c_4 I$.
where

$$W(t, t+N) = \int_t^{t+N} U(N, s) B(s) B^T(s) U^T(N, s) ds.$$

It is obvious that if the system is UGC, then it is GC. Associated with the system $[A(t), B(t)]$ we consider the following RDE:

$$\dot{P}(t) + A^T(t)P(t) + P(t)A(t)P(t)B(t)B^T(t)P(t) + Q(t) = 0. \tag{5}$$

Proposition 2.1. [5] *If the system $[A(t), B(t)]$ is UGC in finite time, then the following assertions hold.*
(i) There is a number $c_5 > 0$ such that $\forall t_2 > t_1 \ge 0$:

$$\int_{t_1}^{t_2} U^T(s, t_1) U(s, t_1) ds \le c_5(t_2 - t_1) I.$$

(ii) The RDE (5), where $Q(t) = I$, has a solution $P(t) \in M([0, \infty), R^n_+)$, which is bounded from above and below. Moreover, we have

$$\|P(t)\| \le \Big[\frac{1}{c_1} + nc_5(1 + \frac{nc_2}{c_1})^2\Big], \quad \forall t \in R^+,$$

where the positive numbers c_1, c_2 are defined by Definition 2.3.

Proposition 2.2. *If the system $[A(t), B(t)]$ is UGC in finite time, then the RDE (5), where $Q = \eta I$, has a solution $P(t) \in M([0, \infty), R^n_+)$, satisfying the condition*

$$\|P(t)\| \le \Big[\frac{1}{\eta c_1} + nc_5(1 + \frac{nc_2}{c_1})^2\Big]\eta, \quad \forall t \in R^+.$$

Proof. Let the system $[A(t), B(t)]$ be UGC, then it is GC and, by Proposition 3.4 in [9], taking $Q(t) = \eta I$ the Riccati equation

$$\dot{P}(t) + A^T(t)P(t) + P(t)A(t) - P(t)B(t)B^T(t)P(t) + \eta I = 0,$$

has a solution $P(t) \in M([0, \infty), R^n_+)$. This implies that the Riccati equation

$$\dot{\bar{P}}(t) + A^T(t)\bar{P}(t) + \bar{P}(t)A(t) - \bar{P}(t)\bar{B}(t)\bar{B}^T(t)\bar{P}(t) + I = 0, \tag{6}$$

where

$$\bar{P}(t) = \frac{1}{\eta} P(t), \quad \bar{B}(t) = \sqrt{\eta} B(t),$$

has a solution $\bar{P}(t)$. On the other hand, it is obvious that the system $[A(t), \bar{B}(t)]$ is also UGC, and hence, by Proposition 2.1, any solution of the RDE (6) satisfies the condition

$$\|\bar{P}(t)\| \le \Big[\frac{1}{\eta c_1} + nc_5(1 + \frac{nc_2}{c_1})^2\Big], \quad \forall t \in R^+.$$

Therefore

$$\|P(t)\| \leq \Big[\frac{1}{\eta c_1} + nc_5(1+\frac{nc_2}{c_1})^2\Big]\eta, \quad \forall t \in R^+,$$

as desired.

We conclude this section with the following technical propositions needed for the proofs of the main results of this paper.

Proposition 2.3 [4] *Let X, Y, F be real matrices of appropriate dimensions and $\|F\| \leq 1$. Then*

$$2XFY \leq \alpha X^T X + \frac{1}{\alpha} Y^T Y,$$

for all $\alpha > 0$.

Proposition 2.4. *Let $B(t), P(t)$ be bounded matrix functions. Then the function $f : R^+ \times R^n \to R^n$ defined by*

$$f(t,x) = -r\frac{B(t)B^T(t)P(t)x}{1+\|B^T(t)P(t)x\|}, \quad t \in R^+,$$

is global Lipschitz, i.e.,

$$\exists L > 0: \quad |f(t,x_1) - f(t,x_2)| \leq L\|x_1 - x_2\|, \quad \forall x_1, x_2 \in R^n, t \in R^+.$$

Proof. Let $x_1, x_2 \in R^n$ and

$$y_1(t) = B^T(t)P(t)x_1, \quad y_2(t) = B^T(t)P(t)x_2.$$

The proof is similar along to the proof of Lemma 1 in [3] using the boundedness of $B(t)$ and $P(t)$ and the following obvious inequalities

$$\|\frac{y_1(t)}{1+\|y_1(t)\|} - \frac{y_2(t)}{1+\|y_2(t)\|}\| \leq 3\|y_1(t) - y_2(t)\|,$$

for all $y_1(t), y_2(t), t \in R^+$.

3 Global stabilization

Consider the linear time-varying system (1). Denote

$$\alpha = \frac{1}{c_1}, \quad \beta = nc_5(1+\frac{nc_2}{c_1})^2,$$

$$\gamma = \frac{1}{2b^2}, \quad b = \sup_{t\in R^+} \|B(t)\|,$$

where c_1, c_2, c_5 are defined by Proposition 2.1. In the sequel, we need the following assumptions.

A.1. The system $[A(t), B(t)]$ is UGC in finite time

A.2. $\gamma \geq 4\alpha\beta$.

Let $\eta > 0$ be any solution of the inequation

$$\beta^2\eta^2 + (2\alpha\beta - \gamma)\eta + \alpha^2 < 0, \tag{7}$$

and consider the following RDE:

$$\dot{P}(t) + A^T(t)P(t) + P(t)A(t) - P(t)B(t)B^T(t)P(t) + \eta I = 0. \tag{8}$$

Theorem 3.1. *Suppose that assumptions A.1, A.2 hold. Then the system (1) is globally stabilizable and the stabilizing control is*

$$u(t) = -\frac{rB^T(t)P(t)x(t)}{1 + \|B^T(t)P(t)x(t)\|}, \tag{9}$$

where $P(t)$ is the solution of the RDE (8).

Proof. Assume that system $[A(t), B(t]$ is UGC in some time $T > 0$. By Assumption A.2, inequation (7) has a solution $\eta > 0$. Consider RDE (8) and by Proposition 2.2, this Riccati equation has a solution $P(t) \in M([0,\infty), R^n_+)$ such that

$$p = \sup_{t\in R^+} \|P(t)\| \leq \Big[\frac{\alpha}{\eta} + \beta\Big]\eta. \tag{10}$$

Let us consider the bounded feedback control (9). By Proposition 2.4, the function

$$f(x) = -r\frac{B(t)B^T(t)P(t)x}{1 + \|B^T(t)P(t)x\|}$$

is global Lipschitz and hence the closed-loop system

$$\dot{x}(t) = A(t)x(t) + f(x(t)), \quad x(0) = x_0, \tag{11}$$

has a unique solution $x(t)$. Define the scalar function

$$V(t,x) = \langle P(t)x, x\rangle.$$

We shall show that the function $V(t,x)$ is a Lyapunov function for the system (11). By Proposition 2.2 the matrix function $P(t) \in M([0,\infty), R^n_+)$ is bounded from above and below, there are positive numbers λ_1, λ_2 such that

$$\lambda_1\|x\|^2 \leq V(t,x) \leq \lambda_2\|x\|^2.$$

Furthermore, taking the derivative of $V(.)$ along the solution $x(t)$ of the system (11), we have

$$\begin{aligned}
\dot{V}(t,x)) &= \langle\dot{P}(t)x, x\rangle + 2\langle P(t)\dot{x}, x\rangle \\
&= -\eta\|x(t)\|^2 + 2\langle P(t)B(t)B^T(t)P(t)x, x\rangle \\
&\quad -\frac{2r}{1 + \|B^T(t)P(t)x\|}\langle P(t)B(t)B^T(t)P(t)x, x\rangle \\
&\leq -\eta\|x\|^2 + 2\langle P(t)B(t)B^T(t)P(t)x, x\rangle,
\end{aligned} \tag{12}$$

because of

$$\frac{2r}{1+\|B^T(t)P(t)x(t)\|}\langle P(t)B(t)B^T(t)P(t)x,x\rangle \geq 0.$$

Therefore, from (12) it follows that

$$\dot{V}(t,x(t)) \leq -(\eta - 2p^2b^2)\|x(t)\|^2,$$

and the derivative of $V(.)$ is negative if

$$\eta > 2p^2b^2. \tag{13}$$

Using the condition (10), we have

$$p^2 \leq (\alpha+\beta\eta)^2,$$

and by the chosen number η, from the condition (7), we can verify that

$$(\alpha+\beta\eta)^2 < \frac{\eta}{2b^2},$$

such that the required condition (13) holds. This completes the proof of the theorem.

Remark 3.1. It should be noted that the global uniform controllability of the system $[A(t),B(t)]$ guarantees the existence of the bounded solution of RDE (8) and therefore, we can verify the global stabilizability without solving any RDE. However, to contruct the stabilizing feedback control, we need to solve RDE (8).

Example 3.1. Consider system (1) in R^2, where the control constraint is

$$\|u(t)\| \leq r = 1,$$

and

$$A(t) = \begin{pmatrix} sin2t & 0 \\ 0 & -1 \end{pmatrix}, \quad B(t) = \begin{pmatrix} \frac{1}{500}e^{-cos^2t} & 0 \\ 0 & \frac{1}{500}e^{-t} \end{pmatrix}.$$

We can verify that the transition matrix $U(t,s)$ is given by

$$U(t,s) = \begin{pmatrix} e^{cos^2s-cos^2t} & 0 \\ 0 & e^{-(t-s)} \end{pmatrix}.$$

Then for every $x=(x_1,x_2)\in R^2$ and $T>0$ we have

$$\begin{aligned}\int_t^{t+T}\|B^T(s)U^*(T,s)x\|^2ds &= \frac{1}{250.000}e^{-2cos^2T}x_1^2\int_t^{t+T}ds \\ &\quad +\frac{1}{250.000}e^{-2T}x_2^2\int_t^{t+T}ds \\ &= \frac{1}{250.000}[e^{-2cos^2T}Tx_1^2+Te^{-2T}x_2^2].\end{aligned}$$

Since

$$e^{-2cos^2T} \geq e^{-2T}, \quad e^{-2cos^2T} \leq 1, \quad \forall T \geq 1,$$

we obtain that for $T \geq 1$:

$$\frac{1}{250.000} T\|x\|^2 \geq \int_t^{t+T} \|B^T(s)U^T(T,s)x\|^2 d\tau \geq \frac{1}{250.000} Te^{-2T}\|x\|^2.$$

On the other hand, we have for all $s \geq t_1 \geq 0$:

$$\begin{aligned} \|U(s,t_1)\|^2 &= [e^{2cos^2 t_1} e^{-2cos^2 s} + e^{-2(s-t_1)}] \\ &\leq 250.000(e^2+1), \end{aligned}$$

hence, taking $T = 1$ the system is UGC with

$$c_1 = \frac{1}{250.000} e^{-2}, \quad c_2 = \frac{1}{250.000},$$

$$c_5 = 250.000(e^2+1).$$

Therefore, we can verify the assumption A.2, where

$$\gamma = \frac{1}{2b^2} = 125.000 \geq 4\alpha\beta = 8e^2(e^2+1)(1+2e^2)^2,$$

and the system is stabilizable. To find the stabilizing feedback controller, taking $\eta = \frac{1}{2500}$, we solve the solution $P(t) = (p_1(t), p_2(t))$ of the following system of two Riccati equations

$$\begin{cases} \dot{p}_1(t) + 2p_1(t)sin2t - \frac{1}{2500}p_1^2(t)e^{-2cos^2t} + \frac{1}{500} = 0, \\ \dot{p}_2(t) - 2p_2(t) - \frac{1}{500}p_2^2(t)e^{-2t} + \frac{1}{2500} = 0. \end{cases}$$

and the feedback control is given by (9).

4 Robust stabilization

In robust stability theory, an important consideration is to design a feedback controller which guarantees the asymptotical stability of the closed-loop system for all uncertainties. In this section, we consider the linear perturbed system (3), where the perturbations $\Delta A(t), \Delta B(t)$ satisfy the norm-bounded constraint (4). Denote

$$h_i = \|H_i\|, \quad e_i = \|E_i\|, \quad i = 1, 2,$$

$$\zeta = 2b^2 + h_1^2 + r(h_2^2 + e_2 b^2),$$

where b is defined in the previous section. Let η be any solution of the inequation

$$\beta\zeta\eta^2 - (2\alpha\beta\zeta - 1)\eta + \zeta\alpha^2 + e_1^2 < 0, \tag{14}$$

where α, β are defined in the previous section, and consider the RDE (8).

Remark 4.1. Note that inequation (14), in general, has a solution, e.g. if the uncertainties, $h_i, e_i, i = 1, 2$ satisfy the condition

$$4\beta\zeta(\alpha + e_1^2\beta) \leq 1. \tag{15}$$

We have the following result.

Theorem 4.1. *Assume that the condition (15) holds and system $[A(t), B(t)]$ is UGC in finite time. Then the uncertain system (3) is robustly stabilizable and the stabilizing control is*

$$u(t) = -\frac{rB^T(t)P(t)x(t)}{1 + \|B^T(t)P(t)x(t)\|},$$

where $P(t)$ is the solution of the RDE (8).

Proof. By the condition (15), the determinant of the equation

$$\alpha\eta^2 - \eta + e_1^2 = 0$$

is non-negative, and hence inequation (14) has a solution. Let $\eta > 0$ be any solution of (14). Due to the UGC of the system $[A(t), B(t)]$, the RDE (8) with the chosen $\eta > 0$ has a bounded solution $P(t) \in M([0, \infty), R_+^n)$ such that

$$p < (\frac{\alpha}{\eta} + \beta)\eta.$$

Taking the Lyapunov function

$$V(t, x) = \langle P(t)x, x\rangle,$$

in the same way as in the proof of Theorem 3.1, we have

$$\begin{aligned}\dot{V}(t, x) &= -\eta\|x\|^2 + 2\langle PBB^TPx, x\rangle \\ &\quad - \frac{2r}{1 + \|B^TPx\|}\langle PBB^TPx, x\rangle + \langle 2PH_1FE_1x, x\rangle \\ &\quad - \frac{r}{1 + \|B^TPx\|}\langle 2PH_2FE_2B^TPx, x\rangle.\end{aligned}$$

Using Proposition 2.3 we have

$$\begin{aligned}\dot{V}(t, x) &\leq -\eta\|x\|^2 + 2\langle PBB^TPx, x\rangle \\ &\quad + \langle (PH_1H_1^TP + E_1E_1^T)x, x\rangle + \\ &\quad \frac{r}{1 + \|B^TPx\|}\langle (PH_2H_2^TP + E_2B^TPPBE_2^T)x, x\rangle \\ &\leq -[\eta - 2p^2b^2 - (p^2h_1^2 + e_1^2) \\ &\quad - rp^2(h_2^2 + e_2^2b^2)]\|x\|^2.\end{aligned}$$

Therefore, the derivative of $V(t, x)$ is negative if η satisfies the inequality (14). The theorem is proved.

Example 4.1. Consider the uncertain system (3) in R^2 where the control constraint is

$$\|u(t)\| \leq r = 1,$$

$\Delta A(t) = H_1F(t)E_1, \quad \Delta B(t) = H_2F(t)E_2, A(t), B(t)$ are defined in Example 3.1 and

$$F(t) = \begin{pmatrix} f(t) & 0 \\ 0 & f(t) \end{pmatrix},$$

$$E_i = \begin{pmatrix} e_i & 0 \\ 0 & 0.01 \end{pmatrix}, \qquad H_i = \begin{pmatrix} h_i & 0 \\ 0 & 0.02 \end{pmatrix}, \quad i = 1, 2,$$

$$|f(t)| \leq \frac{1}{2},$$

and $e_1 = 0.01, e_2 = 0.1, h_1 = 0.001, h_2 = 0.0001$. As before, we have

$$U(t,s) = \begin{pmatrix} e^{cos^2 s - cos^2 t} & 0 \\ 0 & e^{-(t-s)} \end{pmatrix}.$$

and we can verify the condition (14) and the uncertain system (3) is robustly stabilizable.

5 Conclusions

The global stabilizability and robust stabilizability problem of a class of linear continuous time-varying systems with bounded controls were studied. Based on the controllability approach, simple global stabilizability conditions are proposed without solving any Riccati differential equation. We also established some sufficient conditions for the robust stabilization of uncertain linear time-varying systems with bounded controls. Illustrative examples of the results are given.

Acknowledgments. This work was done during the author's visit to the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy as a Senior Associate. The work was supported by the Swedish International Development Agency (SIDA) and partly by National Basic Program in Natural Sciences, Vietnam.

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