# Unconditional quantum cloning of coherent states with linear optics

Gerd Leuchs, **Vincent Josse, Ulrik L. Andersen**

Institut für Optik, Information und Photonik,
Max-Planck Forschungsgruppe,

Universität Erlangen-Nürnberg

# Quantum engineered optical pulses

- comparison discrete $\leftrightarrow$ continuous variables
- linear optics c.v. quantum information
- optimal quantum cloning of coherent states

# discrete vs. continuous

**discrete**

**dichotomic variable**

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = $$

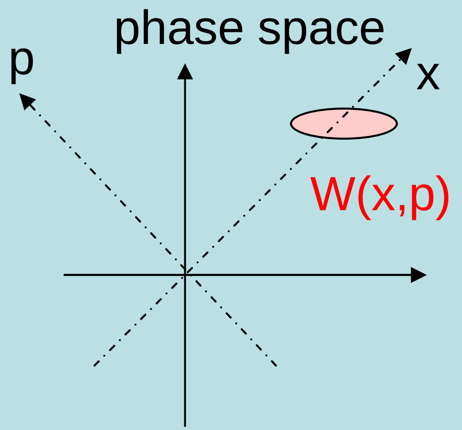
$$= \sum_{i=0}^{1} \alpha_i |i\rangle$$

**continuous variables**

$$|\psi\rangle = \sum_{i=0}^{\infty} \alpha_i |i\rangle$$ $\infty$ dim Hilbert space

alternatively : continuous variables x , p

$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} d\xi \, \exp(-\frac{ip\xi}{\hbar}) \, \Psi^*(x-\frac{1}{2}\xi) \, \Psi(x+\frac{1}{2}\xi)$$

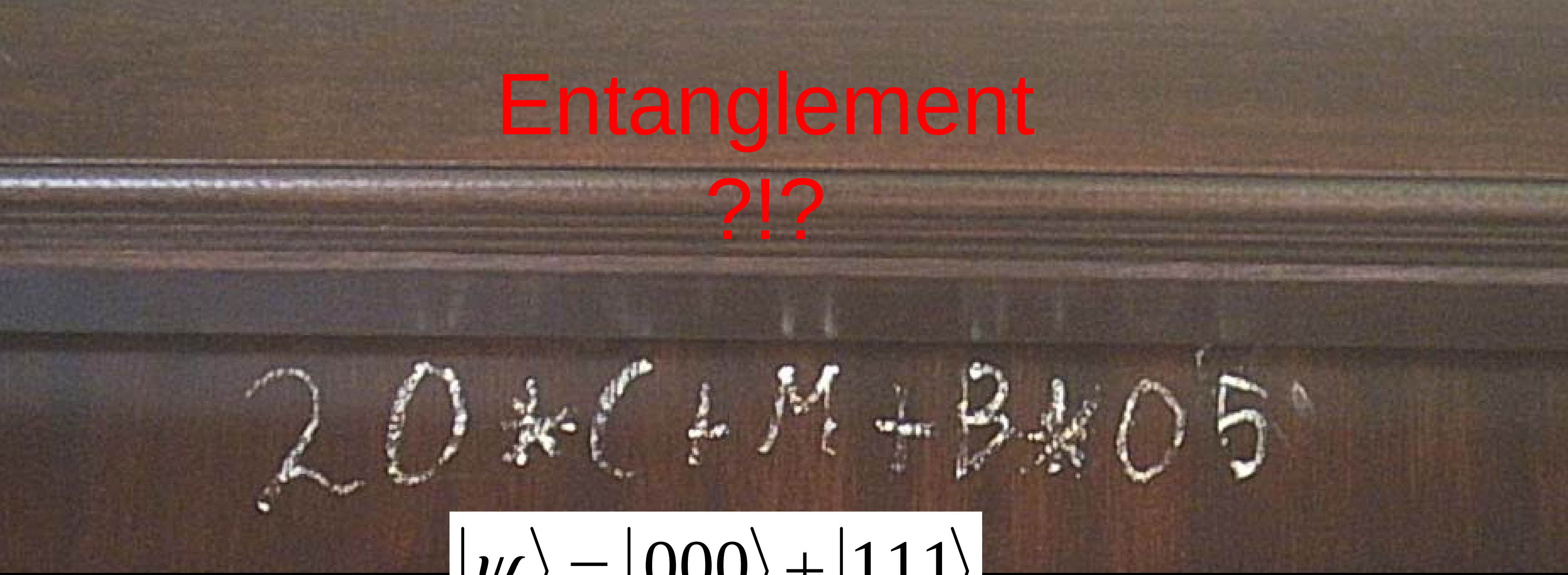



**types of continuous quantum variables**

- field quadratures
- Stokes variables (polarization)

# Entanglement generation

# Entanglement ?!?



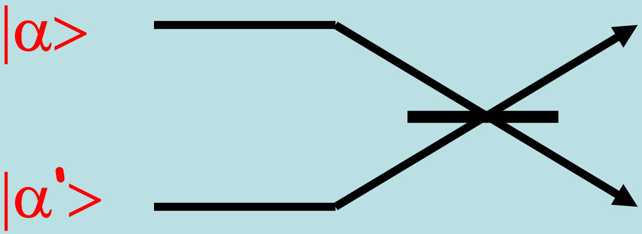

$$|\psi\rangle = |000\rangle + |111\rangle$$

$$|\psi\rangle = |123\rangle + |456\rangle$$

Christus
Mansionem
Benedicat

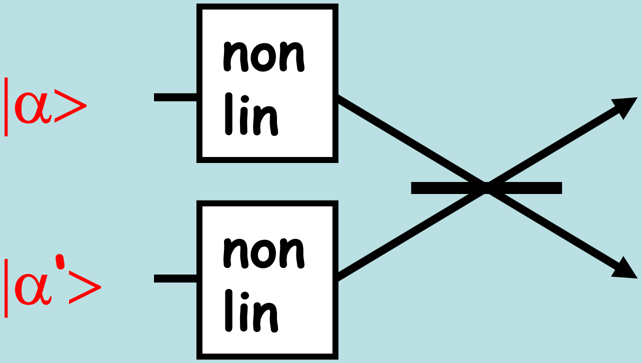
Caspar
Melchior
Balthasar

# Continuous Variables and Beam Splitters




not correlated

>>> coherent state cryptography (post selection)



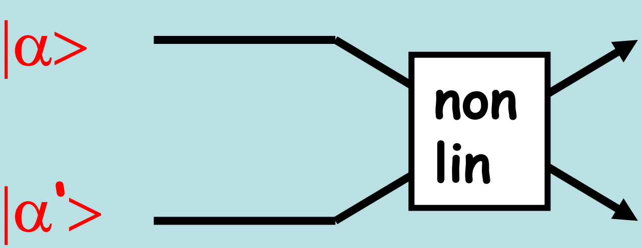

entangled

>>> -teleportation,

-secret sharing

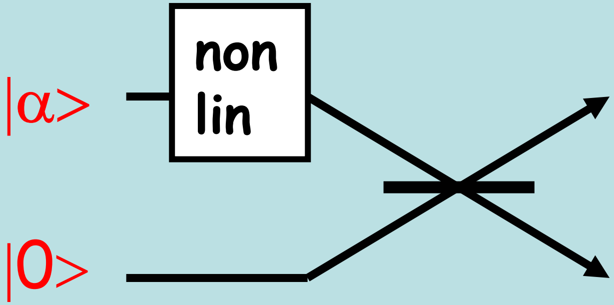
-quantum erasing

-etc.



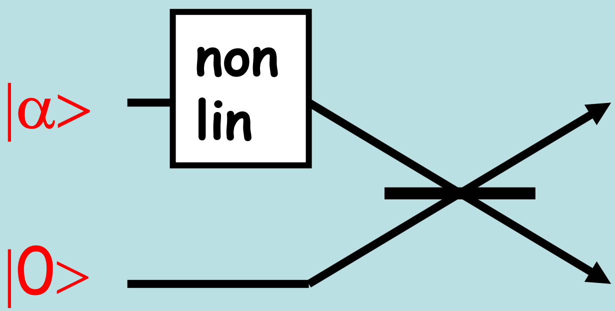

entangled

# Continuous Variables and Beam Splitters (2)




no detection (average loss 1 photon)

degradation of non-classical properties

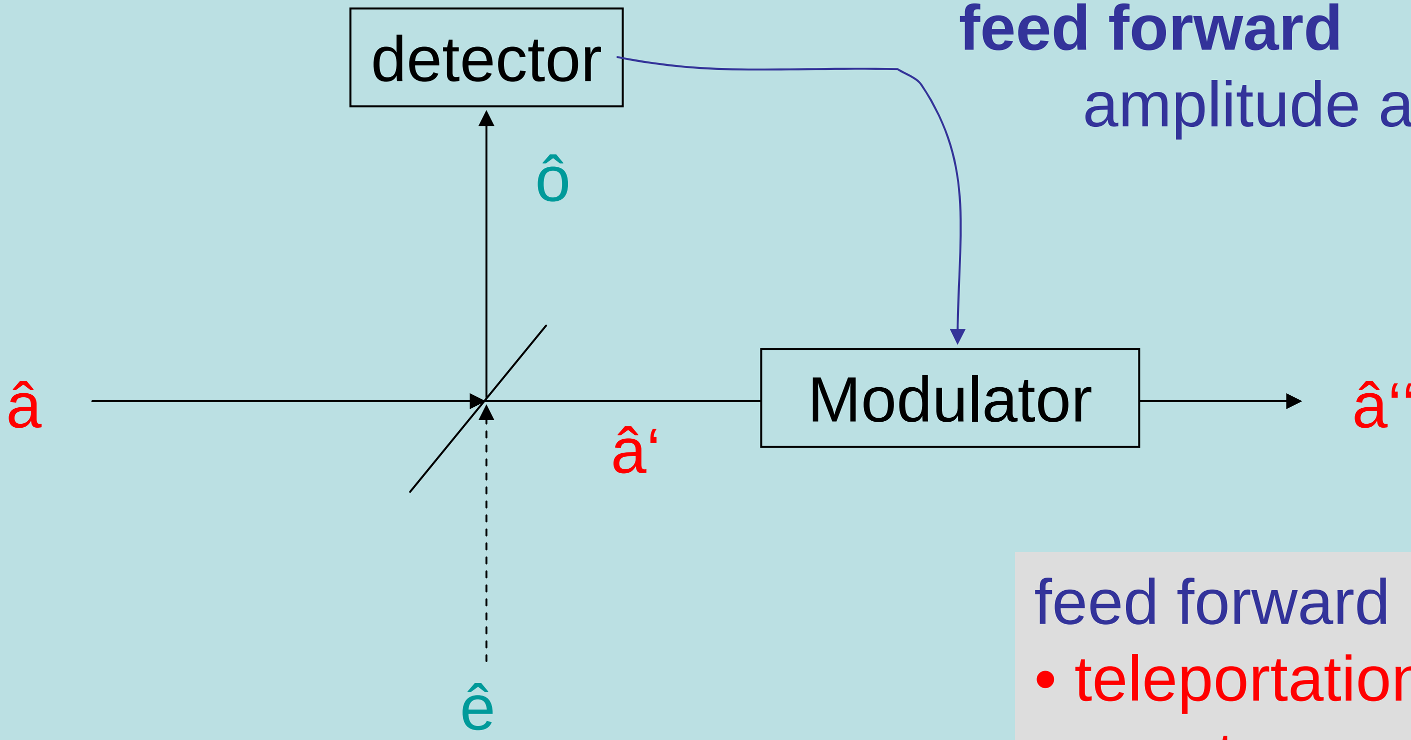



detection of 1 Photon

cat like state
(J. Wenger, R.T. Brouri, P. Grangier, PRL92_153601 (2004))

in discrete case: Knill, Laflamme, Milburn Nature **409**, 46 (2001)

# linear optics – detection – feed forward




**feed forward**
amplitude and/or phase

feed forward used in:
- teleportation
- quantum memory
- **here** ←
- …

# continuous variable protocols - experiments

| | |
|---|---|
| • quantum memory | (Copenhagen,Garching/ Paris) |
| • quantum eraser | (Erlangen, Olomouc) |
| • quantum cloner | (Erlangen) |
| • teleportation | (Pasadena / Canberra / Taiyuan / Tokyo / …) |
| • dense coding | (Taiyuan) |
| • key distribution | (Orsay / Erlangen / Canberra, Brisbane / North Western/Oregon/…) |
| • quantum interferometry | (Erlangen / Stockholm) |
| • secret sharing | (Canberra) |
| • purification | (Erlangen) |
| • … | |

# quantum cloning

no cloning theorem (Wootters and Zurek, 1982)

approximate cloning of single qubits (Buzek, Hillery, 1996)

approximate cloning of coherent states:

theory: Cerf et al, PRL 85, 1751 (2000),

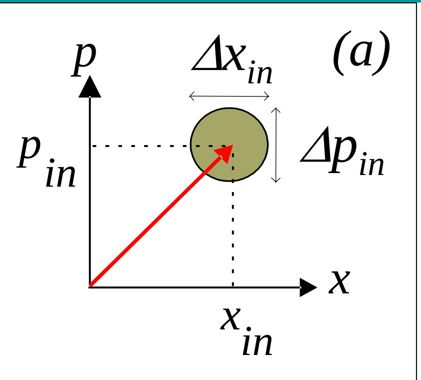
experiment: here

# quantum cloning

**quantum cloning of coherent states**

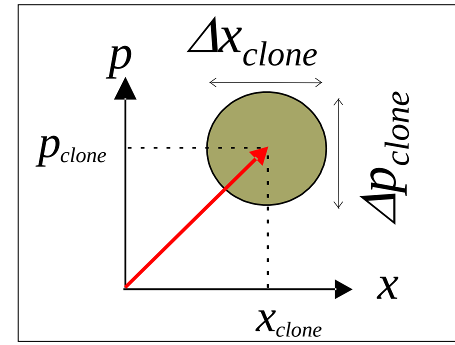
- fundamental aspects
- distribution of (partial) quantum information
- possible attack in quantum cryptography

# "classical" cloning of coherent state

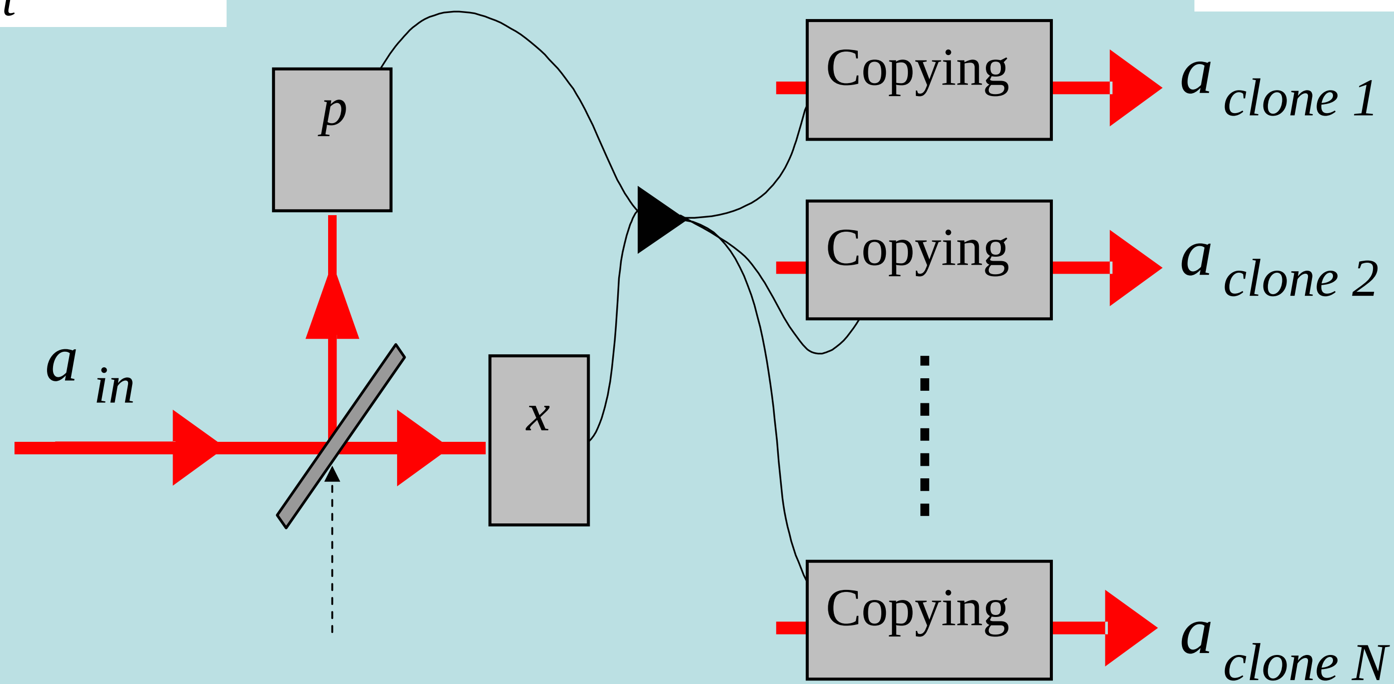



Input

1→2 cloner:
2 extra units of quantum uncertainty → F=1/2




Clones

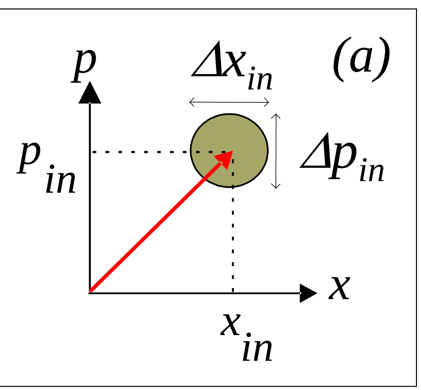


# quantum cloning of a coherent state

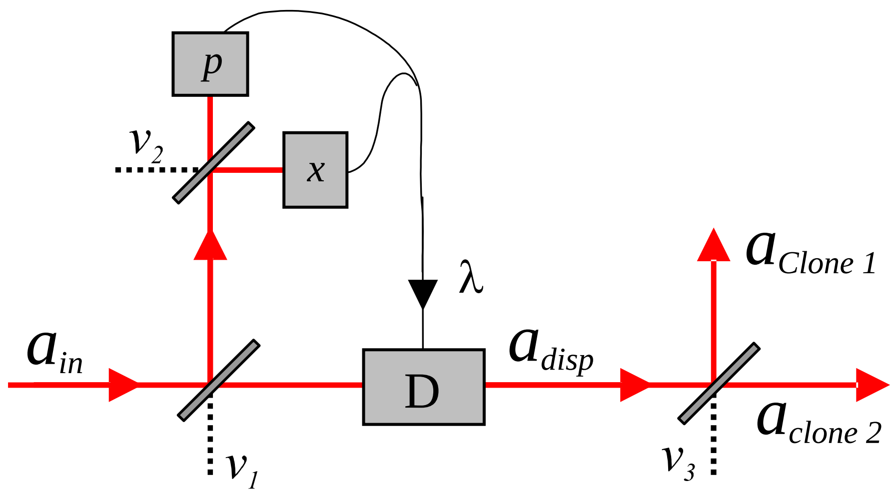
**other proposals**:
D'Ariano et al., PRL**86**, 914 (2001)
Braunstein et al., PRL**86**, 4938 (2001)
Fiurasek, PRL**86**, 4942 (2001)

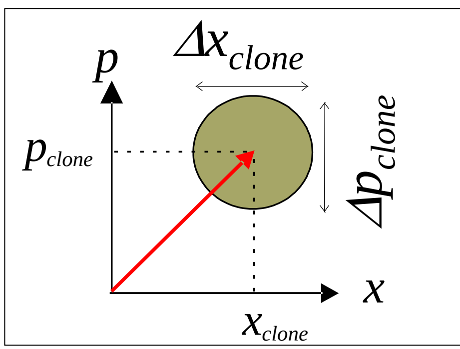
1→2 cloner:
1 extra units of quantum uncertainty

scheme using linear optics and feed forward:




*Input*




*Clones*

experiment:
U.L.Andersen, V.Josse, G.L., PRL 2005 to appear

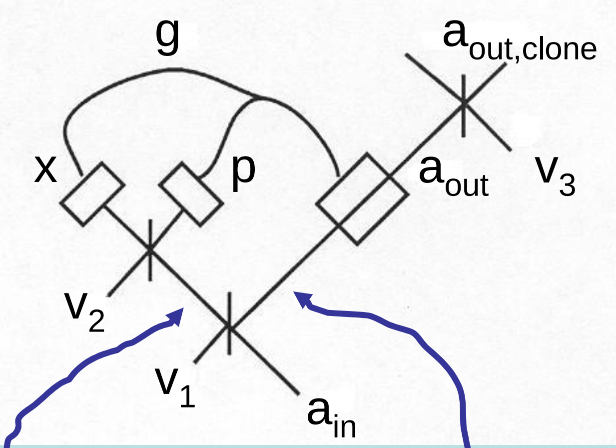
# Heisenberg description

$$
\begin{aligned}
x &= a^\dagger + a \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(a_{in}^\dagger + a_{in} - v_1^\dagger - v_1) - v_2^\dagger - v_2\right)
\end{aligned}
$$

$$
\begin{aligned}
p &= -i(a - a^\dagger) \\
&= -i\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(a_{in} - a_{in}^\dagger - v_1 + v_1^\dagger) + v_2 - v_2^\dagger\right)
\end{aligned}
$$

$$
\begin{aligned}
b &= \frac{1}{2}(x + ip) \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(a_{in} - v_1) - v_2^\dagger\right)
\end{aligned}
$$

$$
\begin{aligned}
a_{out} &= gb + \frac{1}{\sqrt{2}}(a_{in} + v_1) \\
&= \left(\frac{1}{\sqrt{2}} + \frac{g}{2}\right)a_{in} + \left(\frac{1}{\sqrt{2}} - \frac{g}{2}\right)v_1 - \frac{g}{\sqrt{2}}v_2^\dagger
\end{aligned}
$$



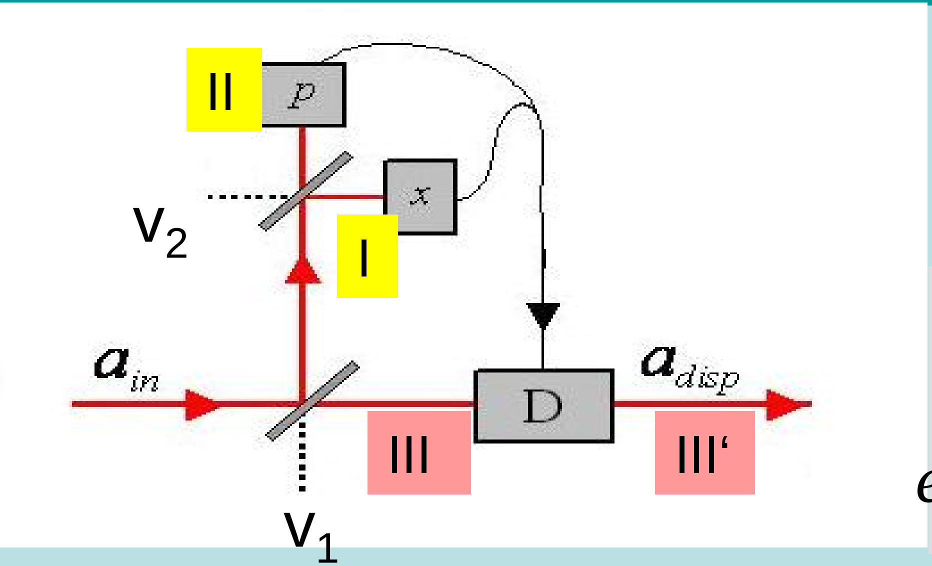

$$\frac{1}{\sqrt{2}}(\hat{a}_{in} - \hat{v}_1)$$

$$\frac{1}{\sqrt{2}}(\hat{a}_{in} + \hat{v}_1)$$

$$a_{out} = \sqrt{2}a_{in} - v_2^\dagger$$

$$a_{out}^{clone1,2} = a_{in} - \frac{1}{\sqrt{2}}(v_2^\dagger \pm v_3)$$

# Quantum description of feed forward action




after measurement of x and p $\langle x_I, p_{II} | \psi \rangle_{I,II,III}$

after displacement D:

$$e^{ig(x_I \hat{P}_{III} + p_{II} \hat{X}_{III})} \langle x_I, p_{II} | \psi \rangle_{I,II,III}$$

projection operator for sub-system III'

$$\langle x_I, p_{II} | e^{ig(\hat{X}_I \hat{P}_{III} + \hat{P}_{II} \hat{X}_{III})} | \psi \rangle_{I,II,III} \; {}_{I,II,III}\langle \psi | e^{-ig(i\hat{X}_I \hat{P}_{III} + i\hat{P}_{II} \hat{X}_{III})} | x_I, p_{II} \rangle$$

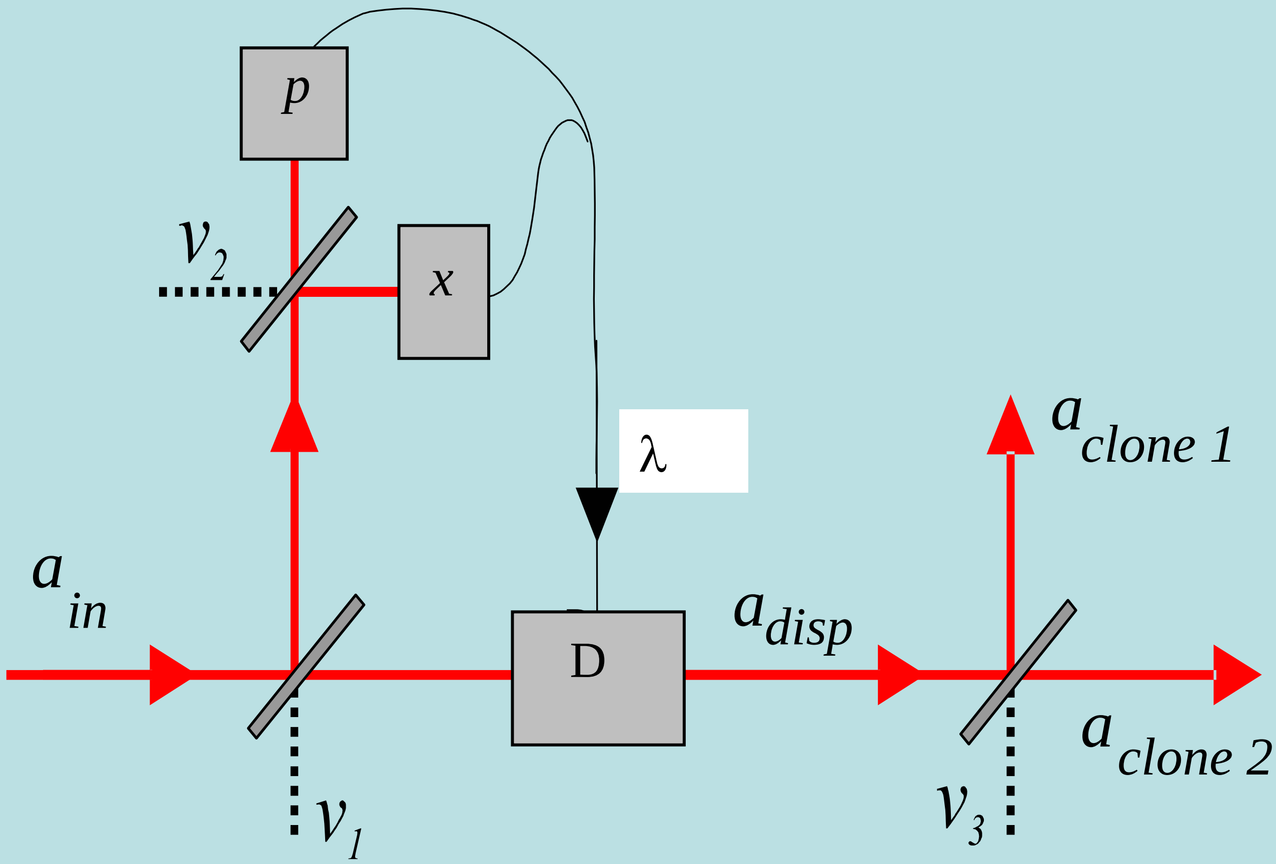
summing over all possible measurment outcomes → density matrix (III')

$$\hat{\rho}_{III'} = Tr_{I,II} \left\{ e^{ig(\hat{X}_I \hat{P}_{III} + \hat{P}_{II} \hat{X}_{III})} | \psi \rangle_{I,II,III} \; {}_{I,II,III}\langle \psi | e^{-g(i\hat{X}_I \hat{P}_{III} + i\hat{P}_{II} \hat{X}_{III})} \right\}$$

in Heisenberg representation: $\hat{X}_{III'} = \hat{X}_{III} + g\hat{X}_I, \;\; \hat{P}_{III'} = \hat{P}_{III} + g\hat{P}_{II}$

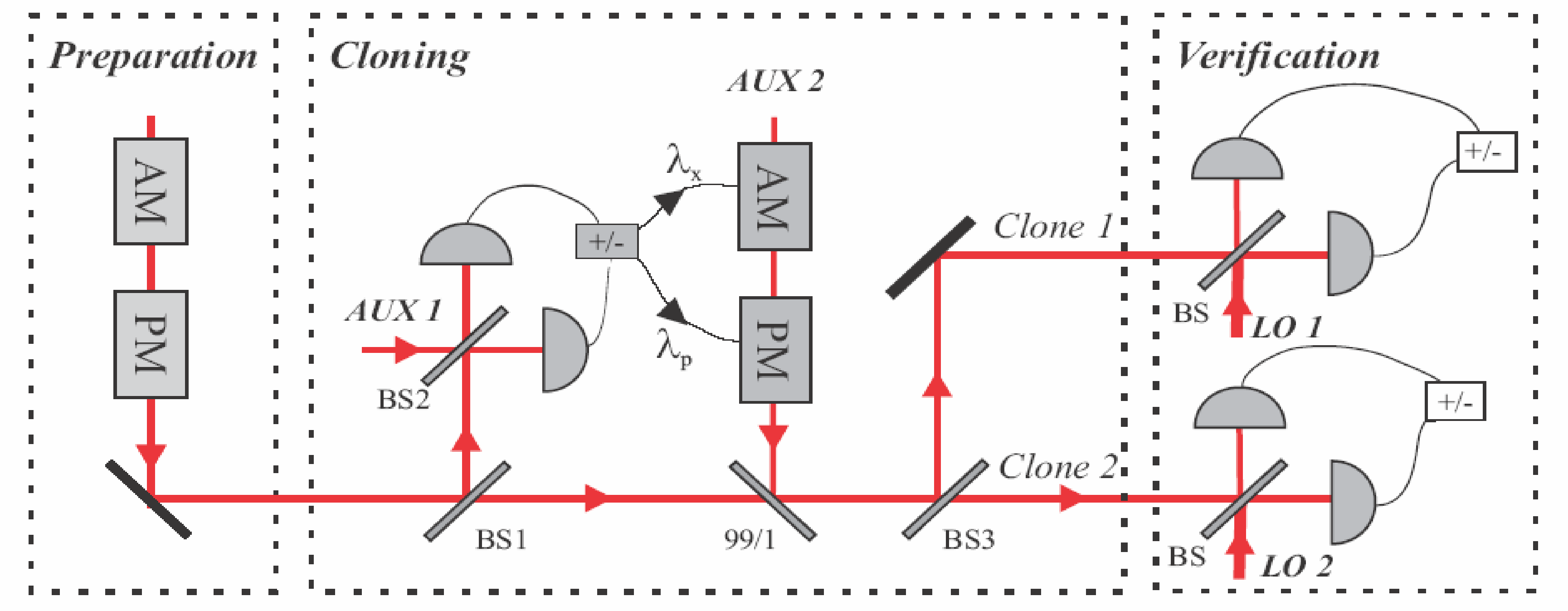
B. Julsgaard, J. Sherson, J. I. Cirac, J. Fiurasek, and E.S. Polzik, *Nature* **432**, 482 (2004)

# Quantum approach

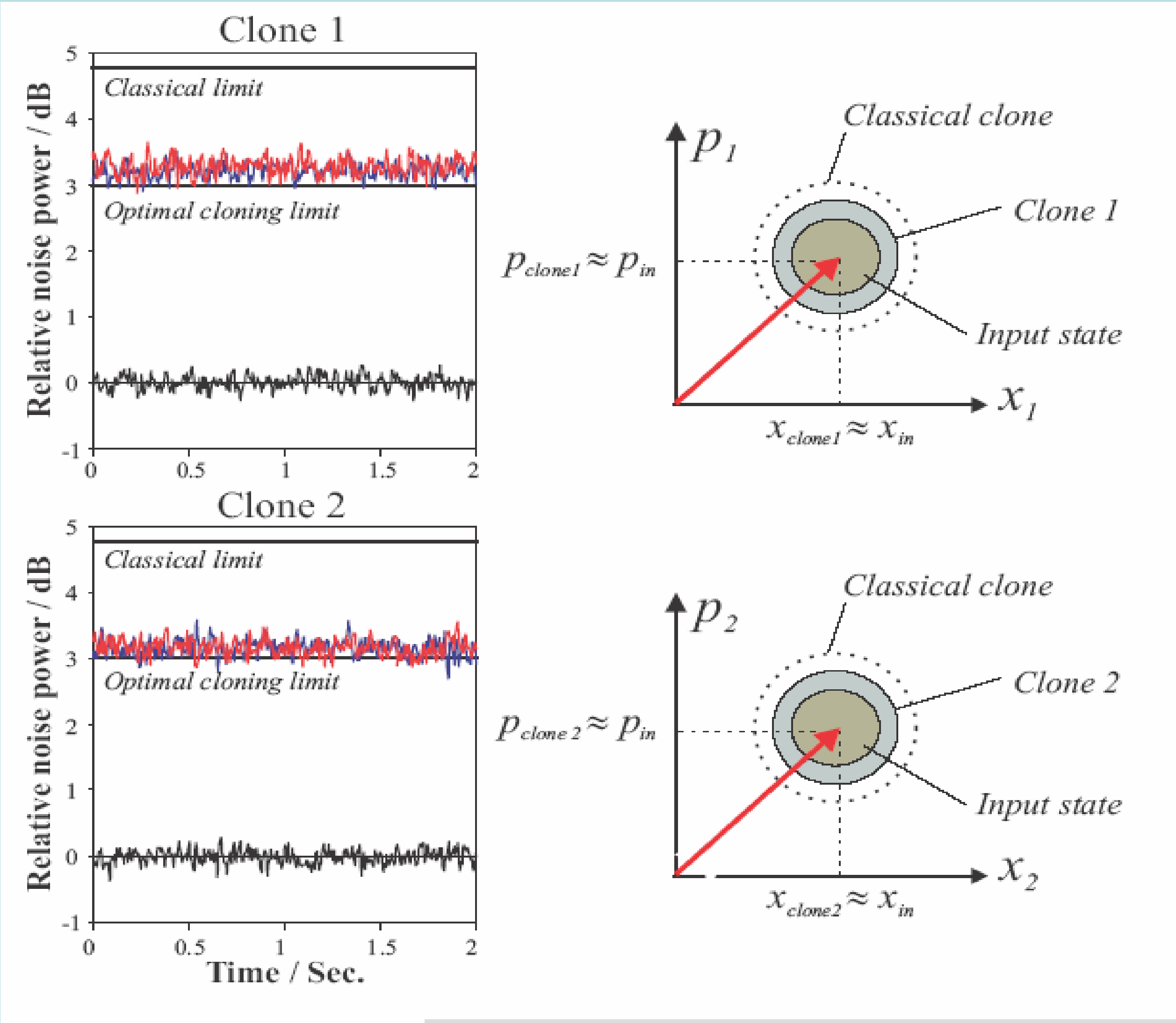


# quantum cloning of coherent side bands

Ulrik L. Andersen, Vincent Josse and G. L., quant-ph / 0501005,  PRL '05



# quantum cloning of coherent side band (2)



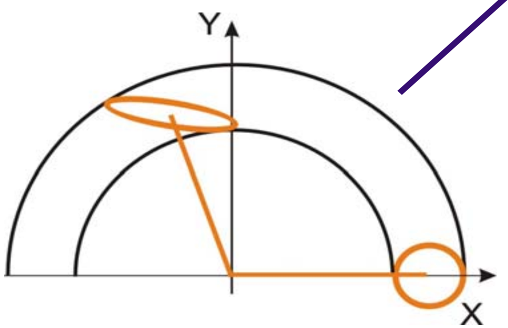

added noise:
3.15 →3.28 dB

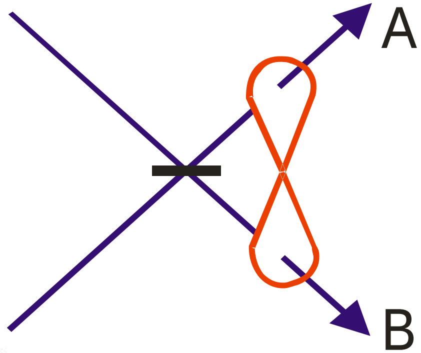
close to quantum limit of 3 dB

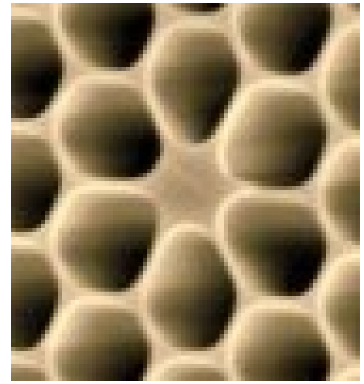
observed fidelity 64%

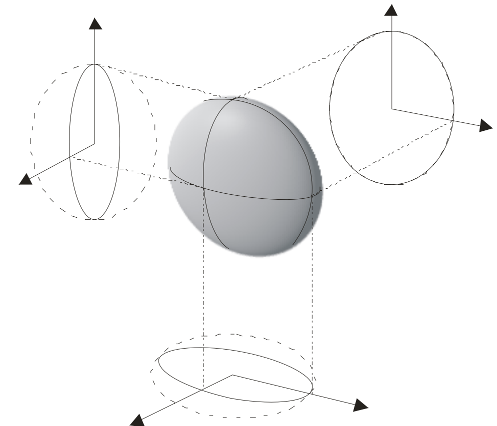
theoretical limit:
66.67% Gaussian
68.26% non Gaussian
N.Cerf et al quant-ph0410058

Ulrik L. Andersen, Vincent Josse and G. L., quant-ph / 0501005, PRL ‘05





theory group
N. Lütkenhaus

U. L. Andersen (N. Korolkova)

O. Glöckl (Ch. Silberhorn)
S. Lorenz
J. Heersink
Ch. Marquardt
J. Schneider
D. Elser
M. Sabuncu
J. Milanovic

R. Filip
T.C. Ralph
V. Josse
R. Loudon
M.D. Reid
P.D. Drummond
E.H. Huntington
H.A. Bachor
N. Cerf