ผลของอันตรกิริยาระหว่างแถบที่มีต่อสัมประสิทธิ์ไอโซโทปของตัวนำยวดยิ่งแมกนีเซียมไดโบไรด์

Effect of Interband Interaction on Isotope Effect Coefficient of MgB_2 Superconductors

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บทคัดย่อ : ในการวิจัยนี้ ได้คำนวณหาสมการแบบแม่นตรง ของอุณหภูมิและสัมประสิทธิ์ไอโซโทปของตัวนำยวดยิ่งคลื่น sชนิด สองแถบ ในขอบเขตการควบคู่แบบอ่อนโดยคำนึงถึงผลของอันตรกิริยาระหว่างแถบ และอันตรกิริยาการเกิดคู่ ในแต่แถบประกอบด้วย2 แบบคืออันตรกิริยาอิเล็กตรอน-โฟนอน และแบบที่ไม่ใช่อิเล็กตรอนโฟนอน จากการคำนวณเชิงตัวเลข คณะผู้วิจัยพบว่าสามารถหาค่าสัมประสิทธิ์ไอโซโทปของแมกนีเซียมไดโบไรด์ $\alpha = 0.3$ ที่มี $T_c \approx 40$ K ได้ในขอบเขตของค่าคงตัวอย่างอ่อนและอันตรกิริยาระหว่างแถบของโฟนอน-อิเล็กตรอนจะมีผลต่อสัมประสิทธิ์ไอโซโทปมากกว่าอันตรกิริยาระหว่างแถบของที่ไม่ใช่โฟนอน-อิเล็กตรอน

Abstract: In this research, the exact formula of T_c's equation and the isotope effect coefficient of two-band s-wave superconductors in weak-coupling limit are derived by considering the influence of interband interaction .In each band ,our model consist of two paring interactions : the electron-phonon interaction and non-electron-phonon interaction . According to the numerical calculation, we find that the isotope effect coefficient of MgB_2, $\alpha = 0.3$ with $T_c \approx 40$ K can be found in the weak coupling regime and interband interaction of electron-phonon show more effect on isotope effect coefficient than interband interaction of non-phonon-electron .

Introduction: The discovery of superconductivity in MgB_2 with a high critical temperature , $T_c \approx 39$ K, has attracted a lot of considerable attention. There is a large difference in the electron-phonon coupling on different Fermi surface sheet and this fact leads to multiband description of superconductivity in MgB_2. The experiments suggest that MgB_2 is the two-band s-wave superconductors(σ-band and π-band). It may have two energy gaps in each band. To recover this fact, we make the assumption that the paring interaction consists of 2 parts : a attractive electron-phonon interaction and a attractive non-electron-phonon interaction in σ-band and π-band , and the $\sigma-\pi$ scattering of interband pairs.

Methodology: The Hamiltonian of the corresponding system is taken in the form

$$H = H_\pi + H_p + H_{p\pi} \tag{1}$$

where H_π, H_p and $H_{p\pi}$ are the Hamiltonian of π band , σ band and interband respectively that

$$H_\pi = \sum_{k\sigma} \varepsilon_{k\sigma} \pi^+_{k\sigma} \pi_{k\sigma} - \sum_{kk'} V_{\pi\pi kk'} \pi^+_{k\uparrow} \pi^+_{-k\downarrow} \pi_{-k'\downarrow} \pi_{k'\uparrow} \tag{2.1}$$

$$H_p = \sum_{k\sigma} \varepsilon_{k\sigma} p^+_{k\sigma} p_{k\sigma} - \sum_{kk'} V_{ppkk'} p^+_{k\uparrow} p^+_{-k\downarrow} p_{-k'\downarrow} p_{k'\uparrow} \tag{2.2}$$

$$H_{p\pi} = -\sum_{kk'} V_{p\pi kk'} (p^+_{k\uparrow} p^+_{-k\downarrow} \pi_{-k'\downarrow} \pi_{k'\uparrow} + \pi^+_{k\uparrow} \pi^+_{-k\downarrow} p_{-k'\downarrow} p_{k'\uparrow}) \tag{2.3}$$

Here we use the standard meaning of parameters and $V_{\pi\pi kk'}$, $V_{ppkk'}$, $V_{p\pi kk'}$ are the attractive interaction potential in σ band and π band ,and interband respectively .

By performing a BCS mean field analysis of Eq.(1) and applying standard techniques, we obtain the gap equation as

$$\Delta_{pk} = -\sum_{k'} V_{ppkk'} \frac{\Delta_{pk'}}{2\sqrt{\varepsilon^2_{pk'} + \Delta^2_{pk'}}} \tanh\left(\frac{\sqrt{\varepsilon^2_{pk'} + \Delta^2_{pk'}}}{2T}\right) - \sum_{k'} V_{p\pi kk'} \frac{\Delta_{\pi k'}}{2\sqrt{\varepsilon^2_{\pi k'} + \Delta^2_{\pi k'}}} \tanh\left(\frac{\sqrt{\varepsilon^2_{\pi k'} + \Delta^2_{\pi k'}}}{2T}\right) \tag{3.1}$$

$$\Delta_{\pi k} = -\sum_{k'} V_{\pi\pi kk'} \frac{\Delta_{\pi k'}}{2\sqrt{\varepsilon^2_{\pi k'} + \Delta^2_{\pi k'}}} \tanh\left(\frac{\sqrt{\varepsilon^2_{\pi k'} + \Delta^2_{\pi k'}}}{2T}\right) - \sum_{k'} V_{\pi pkk'} \frac{\Delta_{pk'}}{2\sqrt{\varepsilon^2_{pk'} + \Delta^2_{pk'}}} \tanh\left(\frac{\sqrt{\varepsilon^2_{pk'} + \Delta^2_{pk'}}}{2T}\right) \tag{3.2}$$

After some calculation ,we can get the exact formula of critical temperature as

$$I_1 = \frac{A}{B + \sqrt{C^2 - D}} \tag{4}$$

where

$$A = 2(-1 + I_2\mu_p)(-1 + I_2\mu_\pi) - 2I_2^2\mu_{\pi p}^2$$

$$B = \mu_p + \mu_\pi + 2I_2(-\mu_p\mu_\pi + \mu_{\pi p}^2) + (\lambda_p + \lambda_\pi)((-1 + I_2\mu_p)(-1 + I_2\mu_\pi) - I_2^2\mu_{\pi p}^2))$$

$$C = \lambda_p + \lambda_\pi + \mu_p - I_2\mu_p(\lambda_p + \lambda_\pi) + \mu_\pi(1 + I_2^2\mu_p(\lambda_p + \lambda_\pi) - I_2(\lambda_p + \lambda_\pi + 2\mu_p))$$
$$- I_2\mu_{\pi p}^2(-2 + I_2(\lambda_p + \lambda_\pi))$$

$$D = 4((-1 + I_2\mu_p)(-1 + I_2\mu_\pi) - I_2^2\mu_{\pi p}^2)[\lambda_\pi\mu_p - 2\lambda_{\pi p}\mu_{\pi p} - (-1 + I_2\lambda_\pi)(\mu_p\mu_\pi - \mu_{\pi p}^2)$$
$$+ \lambda_p[(-1 + I_2\mu_p)(-\mu_\pi + \lambda_\pi(-1 + I_2\mu_\pi)) - I_2\mu_{\pi p}^2(-1 + I_2\lambda_\pi)]$$
$$+ \lambda_{\pi p}^2(-1 + I_2(\mu_p + \mu_\pi) + I_2^2(-\mu_p\mu_\pi + \mu_{\pi p}^2))]$$

and $$I_1 = \int_0^{\omega_D} d\varepsilon \frac{\tanh(\varepsilon / 2T_C)}{\varepsilon} \quad , I_2 = \int_{\omega_D}^{\omega_C} d\varepsilon \frac{\tanh(\varepsilon / 2T_C)}{\varepsilon}$$

and $$\lambda_\pi = N_\pi(0)V_{ph}^\pi , \lambda_p = N_p(0)V_{ph}^p , \lambda_{\pi p} = N_\pi(0)V_{ph}^{\pi p} = N_p(0)V_{ph}^{p\pi}$$
$$\mu_\pi = N_\pi(0)U_C^\pi , \mu_p = N_p(0)U_C^p , \mu_{\pi p} = N_\pi(0)U_C^{\pi p} = N_p(0)U_C^{p\pi}$$

Here λ and μ are the coupling constants of phonon and non-phonon respectively and $N(0)$ is the constant density of state.

The isotope effect coefficient is

$$\alpha = \frac{(1/2)}{\frac{\tanh(\omega_c / 2T_c)}{\tanh((\omega_D / 2T_c))} \frac{(\mu_\pi D' + \mu_p E' + \mu_{\pi p}^2 F')}{(\lambda_\pi A' + I_1 \lambda_{\pi p} B' + \lambda_p C')} - 1} \qquad (5)$$

Here

$$A' = -[-1 + \mu_p(I_1 + I_2)][-1 + \mu_\pi(I_1 - I_2)] + (I_1^2 - I_2^2)\mu_{\pi p}^2$$

$$B' = \lambda_{\pi p}[2(-1 + I_2\mu_p)(-1 + I_2\mu_p) + I_1(\mu_p + \mu_\pi - 2I_2\mu_p\mu_\pi)] + 4\mu_{\pi p} + 2(I_1 - I_2)I_2\lambda_{\pi p}\mu_{\pi p}^2$$

$$C' = (-1 + I_2\mu_p)(-1 + I_2\mu_\pi) + I_2^2\mu_{\pi p}^2 + I_1^2[-\mu_\pi + \mu_p(-1 + 2I_2\mu_\pi) - 2I_2\mu_{\pi p}^2]$$
$$+ I_1[\mu_p - \mu_\pi + 2\lambda_\pi(-(-1 + I_2\mu_p)(-1 + I_2\mu_\pi) + I_2^2\mu_{\pi p}^2)]$$

$$D' = I_1^2\lambda_{\pi p}^2 - (-1 + I_1\lambda_p)(-1 + I_1\lambda_\pi)$$

$$E' = -1 + I_1(\lambda_\pi + \lambda_p(1 - I_1\lambda_\pi) + I_1\lambda_{\pi p}^2) + \mu_\pi F$$

$$F' = 2I_2 + I_1(2 - 2I_2(\lambda_p + \lambda_\pi) + I_1(-\lambda_\pi + \lambda_p(-1 + 2I_2\lambda_\pi) - 2I_2\lambda_{\pi p}^2)$$

Eq.(5) can be easily reduced to be the isotope effect coefficient of BCS theory .

Results, Discussion and Conclusion : We calculate the exact formula of critical temperature and isotope effect coefficient numerically to find isotope effect coefficient of MgB_2, $\alpha = 0.3$ with $T_c \approx 40$ K .We can find many ranges of coupling constant agree with these conditions, example as $\mu_\pi = \mu_p = 0.05$, $\lambda_{\pi p} = 0.05$, $\mu_{\pi p} = 0.142$, $0.034 < \lambda_p < 0.114$, and $0.01 < \lambda_\pi < 0.1$. The interband interaction of electron-phonon interaction show more effect on isotope effect coefficient than of non-phonon interaction and both of them increase the isotope effect coefficient in the same way . Within our calculation, the strength of the coupling parameters indicate that the MgB_2 superconductor is in the weak coupling regime.

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