ผลของศักย์ของอันตรกิริยาแบบ3 หลุมต่อสัมประสิทธิ์ไอโซโทปของตัวนำยวดยิ่งอุณหภูมิสูง

Influence of Three-square-well Interaction Potential on Isotope Effect Coefficient of High-T_C Superconductors

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บทคัดย่อ : การวิจัยนี้ได้คำนวณสมการแบบแม่นตรงของสัมประสิทธิ์ไอโซโทปของตัวนำยวดยิ่งแบบ คลื่น s และ คลื่น d ภายใต้ขอบเขตการควบคู่อย่างอ่อนโดยใช้ศักย์ของอันตรกิริยาแบบ3 หลุม ทั้งนี้ในอันตรกิริยาการเกิดคู่ประกอบไปด้วย 3 ส่วน คือ อันตรกิริยาดึงดูดของอิเล็กตรอน-โฟนอน อันตรกิริยาดึงดูดของที่ไม่ใช่อิเล็กตรอน-โฟนอน และอันตรกิริยาผลักของแรงคูลอมบ์ ซึ่งมีพลังงานcutoff เป็น ω_{ac} , ω_{op} และ ω_c ตามลำดับ โดย$\omega_{ac} \alpha M^{1/2}$, และ ω_{op} , ω_c ไม่ขึ้นกับมวลไอโซโทป(M) คณะผู้วิจัยพบว่าผลการคำนวณของสัมประสิทธิ์ไอโซโทปในทุกกรณีจะได้สัมประสิทธิ์ของไอโซโทปมีค่าลู่เข้าหา 0.5 ที่อันตรกิริยาผลักของแรงคูลอมบ์มีค่าน้อยและที่อันตรกิริยาดึงดูดของอิเล็กตรอน-โฟนอนและอันตรกิริยาดึงดูดของที่ไม่ใช่อิเล็กตรอน-โฟนอนมีค่ามาก

Abstract : In this research, the exact formula of the isotope effect coefficient of s-wave and d-wave superconductor in weak-coupling limit are derived by using a three-square-well interaction potential that pairing interaction consists of 3 parts : an attractive electron-phonon interaction, an attractive non-electron-phonon interaction , and a repulsive Coulomb interaction . ω_{ac} , ω_{op} and ω_c is the characteristic energy cutoff of the Debye phonon , non-phonon ,and Coulomb respectively and $\omega_{ac} \alpha M^{1/2}$, and ω_{op} , ω_c do not depend on isotope mass(M). We find that, in all case of consideration, the isotope coefficient converges to 0.5 at lower value of Coulomb coupling constant and larger values of phonon and non-phonon coupling constant.

Introduction : The isotope effect coefficient, α, is one of the most interesting properties of superconductors. In the conventional BCS theory $\alpha = 0.5$ for all element. In high-T_c superconductors , experimenters found that α is smaller than 0.5. This unusual small value leads to suggestion that the pairing interaction might be predominantly of electronic origin with a possible small phononnic contribution.The purpose of this paper is to derive the exact formula of the isotope effect coefficient of s-wave and d-wave superconductors in weak-coupling limit by using a three-square-well interaction potential , two attractive potential and Coulomb potential.

Methodology: We recall the standard BCS gap equation at T_c .Within the three-square-well interaction potential model, we define interaction potential $V(\varepsilon_{kk'})$ which may be written as

$$V(\varepsilon_{kk'}) = \begin{cases} (-V_{ac} - V_{op} + V_c)\psi_\eta(k)\psi_\eta(k') & \text{for} \quad 0 \le |\varepsilon_{kk'}| \le \omega_{ac} \\ (-V_{op} + V_c)\psi_\eta(k)\psi_\eta(k') & \text{for} \quad \omega_{ac} \le |\varepsilon_{kk'}| \le \omega_{op} \\ V_c\psi_\eta(k)\psi_\eta(k') & \text{for} \quad \omega_{op} \le |\varepsilon_{kk'}| \le \omega_c \end{cases} \tag{1}$$

Here we assume that pairing interaction consists of 3 parts : an electron-phonon interaction V_{ac} , non-electron-phonon interaction V_{op} , and a Coulomb interaction V_c that V_{ac} and V_{op} is the attractive part and V_c is the repulsive part. ω_{ac} , ω_{op} and ω_c is the characteristic energy cutoff of the Debye phonon , non-phonon ,and Coulomb respectively. $\psi_\eta(k)$ is the basis function for the pairing symmetry considered and $\psi_\eta(k) = 1$ for s-wave pairing and $\psi_\eta(k) = \cos 2\theta_k$ for d-wave pairing where $\theta_k = \tan^{-1}(\frac{k_y}{k_x})$ is the angular direction of the momentum k in the ab-plane.

We can get T_c 's equation as

$$\frac{1}{Z_{ac}} = V_{eff} \tag{2}$$

Here

$$V_{eff} = V_{ac} - \cfrac{1}{Z_{op} - \cfrac{1}{V_{op} - \cfrac{V_c}{1 + V_c Z_c}}} \tag{3}$$

and

$$Z_{ac} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\psi_\eta^2(\theta)\int_0^{\omega_{ac}} d\varepsilon_k \frac{N(\varepsilon_k)}{\varepsilon_k}\tanh(\frac{\varepsilon_k}{2T_c}) \tag{4}$$

$$Z_{op} = \frac{1}{2\pi}\int_0^{2\pi} d\theta\psi_\eta^2(\theta)\int_{\omega_{ac}}^{\omega_{op}} d\varepsilon_k \frac{N(\varepsilon_k)}{\varepsilon_k}\tanh(\frac{\varepsilon_k}{2T_c}) \tag{5}$$

$$Z_c = \frac{1}{2\pi}\int_0^{2\pi} d\theta\psi_\eta^2(\theta)\int_{\omega_{op}}^{\omega_c} d\varepsilon_k \frac{N(\varepsilon_k)}{\varepsilon_k}\tanh(\frac{\varepsilon_k}{2T_c}) \tag{6}$$

In harmonic approximation , $\omega_{ac}\,\alpha\,M^{-1/2}$, and ω_{op} , ω_c do not depend on mass. The isotope effect coefficient of s-wave case , $\psi_\eta(\theta) = 1$, is

$$\alpha_s = \frac{1}{2}\left\{\frac{\frac{\lambda_{ac}}{\lambda_{eff}}(2-\frac{\lambda_{ac}}{\lambda_{eff}})}{1+(1-\frac{\lambda_{ac}}{\lambda_{eff}})^2(\frac{\tanh(\frac{\omega_{op}}{2T_c})}{\tanh(\frac{\omega_{ac}}{2T_c})}-1)}\right\} \qquad (7)$$

Here $\lambda_{eff} = N_0 V_{eff}$,$\lambda_{ac} = N_0 V_{ac}$,$\lambda_{op} = N_0 V_{op}$, and $\lambda_c = N_0 V_c$

with
$$\lambda_{eff} = \lambda_{ac} - \frac{1}{\frac{Z_{op}}{N_0} - \frac{1}{\lambda_{op} - \frac{\lambda_c}{1+\lambda_c Z_c / N_0}}} \qquad (8)$$

For d-wave case with constant density of state $N(\varepsilon_k) = N_0$, and $\psi_\eta(\theta) = \cos 2\theta$.We can get $\lambda^d_{eff} = 2\lambda^s_{eff}$, $\lambda^d_{ac} = 2\lambda^s_{ac}$,$\lambda^d_{op} = 2\lambda^s_{op}$,$\lambda^d_c = 2\lambda^s_c$ and

$$\alpha_d = \alpha_s \qquad (9)$$

Here α_d and λ^d are the isotope effect coefficient and coupling constant of d-wave superconductors, respectively.

Results, Discussion and Conclusion : We derive the exact formula of the isotope effect coefficient of s-wave and d-wave superconductor in weak-coupling limit by using a three-square-well interaction potential , two attractive potential and Coulomb potential. We get the same formula for both s-wave and d-wave case that agree with the physical meaning of isotope effect coefficient that depend only on type of mechanism. The isotope coefficient is tend to 0.5 at low value of Coulomb coupling constant λ_c and large values of phonon λ_{ac} and non-phonon λ_{op} coupling constant .

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