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THERMAL NUCLEATION OF KINK-ANTIKINK PAIRS IN A DEFORMABLE CHAIN: INFLUENCE OF THE NON-GAUSSIAN CORRECTION

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Abstract

Thermal nucleation of kink-antikink pairs in a nonlinear Klein-Gordon model with Remoissenet-Peyrard substrate potential coupled to an applied field is analyzed in the limits of moderate temperature and strong damping. We derive analytically the non-Gaussian correction to the nucleation rate formula of kink-antikink pairs previously calculated by Yemélé and Kofané [Phys. Rev. E **56**, 1037(1997)] and show that the correction factor depends on the intensity of the applied field, the temperature of the system and the shape of the substrate potential.

1 Introduction

Nucleation is generally defined as a phenomenon where a new phase appears locally in space. It is one of the most drastic phenomenon in various fields of physics, chemistry, biology, and also in engineering [1]. More precisely, the nucleation in condensed matter physics is most interesting in the sense that it can be controlled by parameters such as pressure, temperature, electric and magnetic fields and so on.

The studies of the nucleation connected to the formation of solitary structures in spatially one-dimensional ($1D$) and multistable systems are well developed theoretically, experimentally and numerically [2-11]. These studies offer the fundamental understanding of nucleation in homogeneous medium. More specifically, theoretical analysis of nucleation was introduced four decades ago by Seeger and Schiller [2] to describe kinetic process of dislocations and few years later by Langer [3] to investigate the problem of reversing of direction of magnetization in ferromagnetic systems. Also, Imawatsu [12] estimated the magnitude of maximum undercooling and superheating in the spherical crystallites in an undercooled liquid and the spherical liquid droplets in superheated crystals using the non classical nucleation theory. The same ideas, but where the approach is closely related to the concepts already developed in dislocation literature, were also developed by Büttiker and Landauer [4] to present detailed calculations of the nucleation rate of thermal kink-antikink pairs in the overdamped sine-Gordon (sG) chain. This theory was later reformulated by Marchesoni *et al.* [5] when analyzing the thermal nucleation of kink-antikink pairs in an elastic string. In fact, they showed that the assumption introduced to derive the nucleation rate of kink-antikink pairs previously calculated [4] is not consistent with the prescription of the nonlinear response theory and then, concluded that the Gaussian approximation implies by the nucleation rate formula is inadequate to describe the decay process of the critical nucleus and gives a simple estimate of the non-Gaussian corrections. In a hard-sphere system at high densities, where the study of the origin of anomalous diffusion and non-Gaussian effects were performed, Doliwa and Heuer [13] found that the anomalous diffusion is mainly due to homogenous contributions, whereas the non-Gaussian effect are mainly related to heterogeneous contributions. These authors concluded that the non-Gaussian effect, if present at all, only mildly related to jump contributions.

Many physical properties, such as structural phase transition and solitary waves, depend strongly on the description of the system-particle interaction [14]. It is difficult to include in any practical model all the forces that can enter rather complex system-particle interaction process in real physical system. Some attempts to understand system-particle interaction via nonlinear potentials where proposed. For example, polynomial functions such as ϕ^4, ϕ^6 and ϕ^8 or sine-Gordon, double sine-Gordon potentials were proposed as models describing system-particle interaction in nonlinear Klein-Gordon models [14]. Although interesting, theses potentials appear as a severe approximation because of the rigidity of their shapes, which is a drastic re-

striction in modeling a large amount of physical systems, as for example, the hydrogen-bonded ferroelectrics [15,16], or in the H/W (hydrogen atom adsorbed on a tungsten surface) [17].

Otherwise, it is well-known that under variation of some physical parameters, such as the temperature and the pressure, certain physical systems may undergo changes which are either shape distorsions, variation of crystalline structure or conformational changes. For example, many open problems concern DNA transcription through bubble opening, protein folding and biological machines which involve bond breaking/formation with a high degree of selectivity and specificity in conformational changes. Thus a few families of deformable potentials which are characterized by the variation of their shape in different manners, for example through the potential barrier, the bottom of the well as the positions of the degenerate minima, depending on the physical foundation of the model have appeared in the literature of nonlinear physics [18-27]. Let mention as an example among many, the well-studied nonsinusoidal Remoissenet and Peyrard [24] potential which takes into account deviations from sine-Gordon and double sine-Gordon potentials. However, some issues remain to be studied. It is the main purpose of this paper to present results concerning the improved formula of the nucleation rate of kink-antikink pairs in the driven and overdamped chain with the Remoissenet-Peyrard substrate potential [24].

The organization of this paper is as follows: In Sec.2, we present the generalized NKG model. In Sec.3, we reformulate the basic results on the nucleation rate of kink-antikink pairs in the homogeneous system [6] by taking into account the non-Gaussian correction in the spirit of Marchesoni *et al.* [5]. Finally, Sec.4 is devoted to concluding remarks.

2 Model interaction potential

We consider the model interaction potential here as a generalized nonlinear Klein-Gordon (NKG) model. It describes the dynamics of a chain of particles in a periodic nonsinusoidal substrate potential in the presence of external forces. The dynamical behavior of the system is governed by the following nonlinear Langevin equation (NLE)

$$Mu_{tt} - ku_{xx} + V_0 \frac{dV_{RP}\left(u,r\right)}{du} = -\gamma u_t + F + \zeta\left(x,t\right), \tag{1}$$

where u is the longitudinal dimensionless displacement of the particles from their equilibrium position along the x axis. The subscripts x and t denote the derivatives with respect to the space and time, respectively. The parameter V_0 is the amplitude of the substrate potential. The coupling of the scalar field $u\left(x,t\right)$ to the heat bath at absolute temperature T is described by a viscous term $-\gamma u_t$ and a zero-mean Gaussian noise source $\zeta\left(x,t\right)$. At Boltzmann equilibrium, the damping constant $\gamma = M\gamma_0$, where γ_0 corresponds to the rate of the energy exchange with the substrate, and the noise intensity are related through the fluctuation-dissipation relationship

$$< \zeta(x,t)\zeta(x',t') >= 2k_BT\gamma\delta(x-x')\delta(t-t'). \tag{2}$$

The constant force F in Eq.(1) is related to the applied field f through the relation $F = f/2\pi$. To model the on-site potential $V_{RP}(u,r)$, we shall use the nonsinusoidal substrate potential introduced by Remoissenet and Peyrard (RP) [24]

$$V_{RP}(u,r) = \frac{(1-r)^2(1-\cos u)}{1+r^2+2r\cos u}, \tag{3}$$

where r is the shape parameter, with $|r| < 1$. As this parameter varies, the amplitude of the potential remains constant with degenerate minima $2\pi n$ and maxima $(2n+1)\pi$, while its shape changes. When $r > 0$, it has flat bottoms separated by thin barriers while for $r < 0$, it has the shape of sharp wells separated by flat wide barriers (see Fig. 1). At $r = 0$, the RP potential reduces to the well-known sG potential.

The unperturbed NKG equation, obtained from the NLE(1) has been derived from the Hamiltonian

$$H = \int \frac{dx}{a}\left\{\frac{M}{2}u_t^2 + \frac{k}{2}u_x^2 + V_0 V_{RP}(u,r) - Fu\right\}, \tag{4}$$

where a is the lattice constant. In this expression, since u is the dimensionless displacement of particles, the parameters M, k and V_0 have the dimension of (mass)x(length), (energy)x(length), and (energy)x(length)$^{-1}$, respectively.

The dynamics of the system obtained from the NLE(1) by setting its right-hand side equal to zero is dominated by elementary excitations: phonons and solitons (kink and antikink). While phonons are extended modes of the system, solitons are localized modes and may be viewed as effective particles characterized by a mass and an energy. In a number of situations, kink dynamics may be described by equations of its collective coordinates namely the kink center of mass. If one assumes periodic boundary conditions on the chain of length L, $u(x,t) = u(x+L,t)$, kinks are only present as a result of thermal activation. These thermal kinks are created in pairs involving a kink and an antikink. On the other hand, if the system is not subjected to periodic boundary conditions or in other words, if the ends of the string are free, the so-called "geometric" solitons of the same sign appear in the system. In fact, the NLE(1) exhibits static kink solutions given implictly by (see [6,24,25])

$$\frac{x}{d^{(1)}} = sgn(\pi - u)\left\{(1-\alpha^2)^{1/2}\tanh^{-}1\left[\frac{(1-\alpha^2)}{1+\alpha^2\tan^2(u/2)}\right]^{1/2} - \tanh^{-}1\left[\frac{1}{1+\alpha^2\tan^2(u/2)}\right]^{1/2}\right\} \tag{5a}$$

for $0 \le r < 1$ and

$$\frac{x}{d^{(2)}} = sgn(u - \pi)\left\{[(1-\alpha^2)^{1/2}/\alpha]\tan^{-}1\left[\frac{(1-\alpha^2)}{\alpha^2+\tan^2(u/2)}\right]^{1/2} + \tanh^{-}1\left[\frac{\alpha^2}{\alpha^2+\tan^2(u/2)}\right]^{1/2}\right\} \tag{5b}$$

for $-1 < r \le 0$ with

$$d^{(1)} = \xi_0/\alpha, \qquad d^{(2)} = \xi_0\alpha, \qquad \xi_0 = (k/V_0)^{1/2}, \text{and} \qquad \alpha = \frac{1-\mid r \mid}{1-\mid r \mid}. \tag{6}$$

The terms $d^{(1)}$and $d^{(2)}$ have the dimensions of length and give the measure of the "pseudo-kink width" for $r \geq 0$ and $r \leq 0$, respectively. The parameter ξ_0 designates the characteristic length of the system. The antikink solutions are obtained by replacing u by $2\pi - u$ in Eqs.(5). It can be shown that the static kinks (and antikinks) energy E_s, and their rest mass M_s are given by

$$E_s^{(\ell)} = 8\sqrt{kV_0}G^{(\ell)}(r), \; M_s^{(\ell)} = \frac{8}{\zeta_0}G^{(\ell)}(r), \tag{7}$$

with $\ell = 1, 2$ and

$$G^{(1)}(r) = (\alpha/\alpha_*)\tanh^{-1}\alpha_*, \; G^{(2)}(r) = \alpha_* \tan^{-1}(\alpha_*/\alpha), \; \alpha_* = \sqrt{1-\alpha^2}, \tag{8}$$

where the superscripts (1) and (2) stand for $0 \leq r < 1$ and $-1 < r \leq 0$, respectively. For $r = 0$, the above kink soliton parameters reduce to those of the usual sG kink soliton.

In the presence of the applied field, the total on-site potential energy, that is, the sum of the substrate potential energy $V_0 V_{RP}(u, r)$ and the energy due to the applied field $-Fu$, is given by

$$V(u, F) = V_0 V_{RP}(u, r) - Fu. \tag{9}$$

The minima and the maxima of the above on-site potential energy (9) which are known as Peierls valleys and Peierls hills, respectively, are different from those of the substrate potential energy and may disappear if the applied field F is greater than the threshold value F_m[6]

$$\frac{F_m}{V_0} = \frac{2\sqrt{2}\alpha^2\left[\left(3\alpha^2 - 1\right) + \sqrt{\Delta}\right]\sqrt{\sqrt{\Delta} - 3\left(1-\alpha^2\right)}}{5\alpha^2 - 3 + \sqrt{\Delta}}, \tag{10}$$

with $\Delta = 9\alpha^4 - 14\alpha^2 + 9$ (see Fig.2). This means that kink solutions of the NLE(1) can only exist if $F < F_m$. Note that these Peierls valleys and Peierls hills obey the relation $dV(u, F)/(du) = 0$. More specifically, the NLE(1) describing the configuration of the nucleus in a pure system may also be viewed as the equation of motion of a classical "particle" with mass "k", time "x" in a potential $-V(u, F)$, where $V(u, F)$ is given by Eq.(9). The critical nucleus of amplitude Δu_m will be a configuration which deviated only in a localized region from the uniform state u_{sn} followed by the motion to the right until the turning-point $u_{sn} + \Delta u_m$ is reached. Then, the particle returns again asymptotically to the local maxima at u_{sn}. The corresponding stationary solution of the NLE(1) is the saddle-point configuration or the critical nucleus which departs from the stationary uniform state u_{sn} at $x = \pm\infty$. Its amplitude Δu_m strongly depends on the applied field. Furthermore, Δu_m decreases monotonically with respect to F: $\Delta u_m = 2\pi$ for $F = 0$, and $\Delta u_m = 0$ for $F = F_m$. The transition between two adjacent Peierls valleys due to thermal fluctuations called critical nucleus are the newly formed kink-antikink pairs, whose size depends on the applied field F. This transition is possible only if the fluctuations produce, within the system, a minimum of energy $\Delta E_N \gg k_B T$ necessary to create a critical nucleus $u_N(x, X)$, where X designates the newly formed kink position which in the continuum limit is linearly time dependent; that is, $X(t) = X_0 + vt$, where X_0 is the initial position of the

kink center of mass and v the kink velocity. For $F \ll F_m$, the nucleus $u_N(x,X)$ can be well approximated by the linear superposition of a kink and an antikink centered at $\pm X$, respectively, that is,

$$u_N(x,X) = u_+(x+X,0) + u_-(x-X,0), \tag{11}$$

where the solutions satisfie the NLE(1) without the right-hand side.

3 The Non-Gaussian correction

In this section, we consider the influence of the Gaussian approximation on the nucleation rate of kink-antikink pairs by calculating the non-Gaussian correction term following Marchesoni *et al.* [5]. In the overdamped limit $\left(\gamma \gg \sqrt{V_0}\right)$, where the inertial term (Mu_{tt}) is neglected, the substitution of Eq.(11) into the NLE(1) leads to the following equation for the nucleus

$$\frac{dR}{dt} = -\frac{dV_N^{(\ell)}}{dR} + \eta_R(t), \tag{12}$$

with the reduced coordinate $R = 2X$, where the potential of the critical nucleus is given by

$$V_N^{(\ell)}(R) = -\frac{2\pi F}{\gamma M_S^{(\ell)}} R - \frac{4E_s^{(l)}\vartheta^{(\ell)}}{\gamma M_s^{(\ell)}} \exp\left(-\frac{|R|}{d^{(\ell)}}\right), \tag{13}$$

with

$$\vartheta^{(1)} = \alpha_* \frac{\exp\left(-2\alpha_* \tanh^{-1}\alpha_*\right)}{\alpha^2 \tanh^{-1}\alpha_*},\ \vartheta^{(2)} = \alpha\alpha_* \frac{\exp\left(2(\alpha_*/\alpha)\tan^{-1}(\alpha_*/\alpha)\right)}{\tan^{-1}(\alpha_*/\alpha)}. \tag{14}$$

The noise $\eta_R(t)$, associated with Eq.(12) for the nucleus, verifies the following fluctuation-dissipation relationship

$$<\eta_R(t)\eta_R(t')> = 2\gamma^2 D_R^{(\ell)}\delta(x-x'),\ D_R^{(\ell)} = \frac{2k_BT}{\gamma M_s^{(\ell)}}. \tag{15}$$

To calculate the nucleation rate of kink-antikink pairs, it is necessary to determine the size of the critical nucleus $R_N^{(\ell)}$ and the negative eigenvalue $\lambda_0^{N(\ell)}$ of the non-uniform state. Thus, the nucleus size is set by the condition that

$$V_N^{(\ell)'}(R)|_{R_N} = 0, \tag{16a}$$

leading to

$$R_N^{(\ell)} = d^{(\ell)} ln\left[\frac{2E_s^{(\ell)}\vartheta^{(\ell)}}{\pi F d^{(\ell)}}\right], \tag{16b}$$

and the negative eigenvalue of the non-uniform state, which is the eigenvalue of the RP scattering potential in the presence of the applied field defined as $[dV(u,F)/du^2]_{u_N}/[d^2V(u,F)/du^2]_{u_{sn}}$, is given by

$$\lambda_0^{N(\ell)} = \left.\frac{d^2V_N^{\ell}(R)}{dR^2}\right|_{R_N} = -\frac{2\pi F}{\gamma M_s^{(\ell)} d^{(\ell)}}. \tag{17}$$

In the limit where the shape parameter $r \to 0$, the parameters $\vartheta^{(\ell)}$ defined by Eq.(14) reduce to 1 and then, the negative eigenvalue of the non-uniform state given by Eq.(17), reduces to that obtained for the sG systems.

With the above stated results, in the Gaussian approximation, the improved formula of the number of kink–antikink pairs per unit time and per unit length is then given by

$$J_0^{(\ell)} = K^{(\ell)}(r)\Omega^{(\ell)} \exp(-\beta \Delta E_N^{(\ell)}) \tag{18}$$

with $\beta = 1/k_B T$, where $K^{(\ell)}(r)$ is the non-Gaussian correction term, and where the prefactor $\Omega^{(\ell)}$ has been nicely work out by Yemélé and Kofané [6]. Its analytical expression is given by [6]

$$\Omega^{(\ell)} = \left(\frac{\Gamma}{2\pi}\right)^{3/2} \left(\frac{\gamma}{k}\right)^{1/2} \left(\frac{|\lambda_0^{N(\ell)}|}{\Gamma}\right)^{1/2} \prod_{n\,1}^{p-1} \left(\frac{\Gamma}{\lambda_n^{N(\ell)}}\right)^{1/2} \left(\frac{\Delta E_N^{(\ell)}}{k_B T}\right)^{1/2} (Q), \tag{19}$$

where $\lambda_n^{N(\ell)}$ are the eigenvalues of the non-uniform state, and $\Gamma = \big(V_0/\gamma\big)\big[d^2V(u,F)/du^2\big]_{u_{sn}}$ while Q is the product of the eigenvalues of the localized eigenmodes of the critical nucleus. In addition, p is the number of localized modes and strongly depends on the shape parameter r. In fact, when $r \geq 0$, the system possesses two bound states ($p = 2$), with $\lambda_n^{N(\ell)} = 0$ and $\lambda_0^{N(\ell)}$ given by Eq.(17). Moreover, internal modes appear when r decreases from 0 to -1. For example $p = 5$ for $r = -0.5$ and $p = 21$ for $r = -0.9$. The quantity $\Delta E_N^{(\ell)}$ which appears in Eq.(18) designates the energy of the critical nucleus whose accurate value at a given field $F \leq F_m$ is evaluated numerically through the relation

$$\Delta E_N^{(\ell)} = \int_{-\infty}^{\infty} dx \left[k \frac{du_N(x)}{dx}\right]^2, \tag{20}$$

where $u_N(x)$ satisfies the NLE (1) without the right-hand side. However, for some particular cases, the explicit analytical expression of $\Delta u_m^{(\ell)}$ may be obtained as follows:

(i) For small F values ($F \leq F_m$), the amplitude $\Delta u_m^{(\ell)}$ of the critical nucleus is large and very close to 2π:

$$\Delta u_m^{(1)} = 2\pi - \alpha\big(4\pi F/V_0\big)^{1/2} \qquad \text{and} \qquad \Delta u_m^{(2)} = 2\pi - \big(1/\alpha\big)\big(4\pi F/V_0\big)^{1/2}. \tag{21}$$

This nucleus is called large amplitude nucleus (LAN) with energy

$$\Delta E_N^{(\ell)} \approx 2E_s^{(\ell)} \left[1 - \frac{\pi d^{(\ell)} F}{E_s^{(\ell)}} - \frac{\pi d^{(\ell)} F}{E_s^{(\ell)}} ln \frac{2E_s^{(\ell)} \upsilon^{(\ell)}}{\pi F d^{(\ell)}}\right], \tag{22}$$

where $E_s^{(\ell)}$ designates the static kink energy defined in Eq.(7).

(ii) For large F values ($F \approx F_m$), the amplitude of the critical nucleus is close to zero. This critical nucleus, solution of the NLE (1), is called small amplitude nucleus (SAN) whose

analytical expression is given by

$$u_N(x) = \Delta u_m^{(\ell)} \sec^2(\frac{x}{2\xi}), \tag{23a}$$

with amplitude

$$\Delta u_m^{(\ell)} = 3\left(\frac{1+r^2}{1-r^2}\right)\tan u_{sn}\left[\frac{1-2\varepsilon\cos u_{sn}+\dfrac{4\varepsilon}{\cos u_{sn}}}{1-5\varepsilon\cos u_{sn}}\right] \tag{23b}$$

and size

$$\xi^2 = \xi_0^2\left(\frac{1+r^2}{1-r^2}\right)^2\left[\frac{(1+2\varepsilon cosu_{sn})^3}{cosu_{sn}+2\varepsilon(1+sin^2u_{sn})}\right], \tag{24}$$

where ξ_0 is given by Eq.(6) and $\varepsilon = r/(1+r^2)$. The energy of this SAN is given by

$$\Delta E_N^{(\ell)} = \frac{24}{5}\sqrt{kV_0}\Delta u_m^{(\ell)}\left(\sqrt{\frac{F}{V_0}}\right)\left(\frac{1+r^2}{1-r^2}\right)\Bigg]\frac{1}{\tan u_{sn}} + 4\varepsilon\left(1+2\varepsilon\cos u_{sn}\right)\frac{F}{V_0}\left(\frac{1+r^2}{1-r^2}\right)^2\Bigg]^2, \tag{25}$$

where $\Delta u_m^{(\ell)}$ is now given by Eq.(23b).

In the presence of random fields, the critical nucleus may always exist in the system even at $T \to 0$, resulting from the combined effects of the energy fluctuations and the applied field F. At high temperatures, thermal nucleus will play the main role. The non-Gaussian correction $K^{(\ell)}(r)$ to the nucleation rate formula of kink-antikink pairs obtained through the Gaussian approximation is given by

$$K^{(\ell)}(r) = \frac{\int_{-\infty}^{\infty}\exp\left(-\frac{\left|\lambda_0^{N(\ell)}\right|}{2D_R^{(\ell)}}R^2\right)dR}{\int_{-\infty}^{\infty}\exp\left(-\frac{V_N^{(\ell)}(R)}{D_R^{(\ell)}}\right)dR}. \tag{26}$$

In the absence of these correction terms, that is $K^{(\ell)}(r) = 1$, the expression of the nucleation rate of kink-antikink pairs given by Eq.(18), reduces to that obtained by Yemélé and Kofané [6]. For smaller F values, if we approximate $V_N^{(\ell)}(R)$ to $-2\pi F/\gamma M_s^{(\ell)}R$, the leading contribution from the denominator is analytically estimated and we obtain

$$K^{(\ell)}(r) = 2\pi\sqrt{\frac{Fd^{(\ell)}}{kT}}, \tag{27}$$

where $d^{(\ell)}$ ($\ell = 1, 2$) are the "pseudo-kink width" defined by Eq.(6). As it can be seen, in this limit the non-Gaussian correction, $K^{(\ell)}(r)$, to the nucleation rate formula of kink-antikink pairs is a linear function of the square root of the "pseudo-kink width" and of the applied field. Thus, this factor is an increasing function of the applied field while it decreases with the shape

parameter r. The results plotted in Figs.3 and 4 are based on the computational evaluation of Eq.(26) and depend on the reduced temperature $\tau = k_B T\sqrt{kV_0}$ as well as the applied field F and on the shape parameter. Although these plots confirm the qualitative behavior of the correction factor $K^{(\ell)}(r)$ with the applied field resulting from the approximated result (25), they show also that this correction factor exhibits a complex behavior with the shape parameter (see Fig. 5).

4 Conclusion

In summary, we have investigated in this paper the influence of the Gaussian approximation on the nucleation process of kink-antikink pairs in the NKG model with RP substrate potential, driven by an applied constant field. The results of our calculations, performed in the spirit of Marchesoni *et al.* [5], show that the correction factor resulting from the Gaussian approximation depends on the temperature, the applied field and also on the shape parameter of the system. More precisely, the correction factor increases with the applied field while it exhibits a complex behavior concerning the shape parameter of the system. In the moderate temperature which is under consideration in this paper, the non-Gaussian correction factor is always greater than 1. This means that the nucleation formula previously derived by Yemélé and Kofané [6], while investigating the nucleation rate of thermal kink-antikink pairs in a driven and overdamped deformable chain, has been underestimated. The improved formula of the nucleation rate of kink-antikink pairs calculated here will be considered to derive the nucleation rate formula in a model with impurity and, the relationship between the average velocity of particles [28] and the deformability parameter of the substrate potential [24] will be established.

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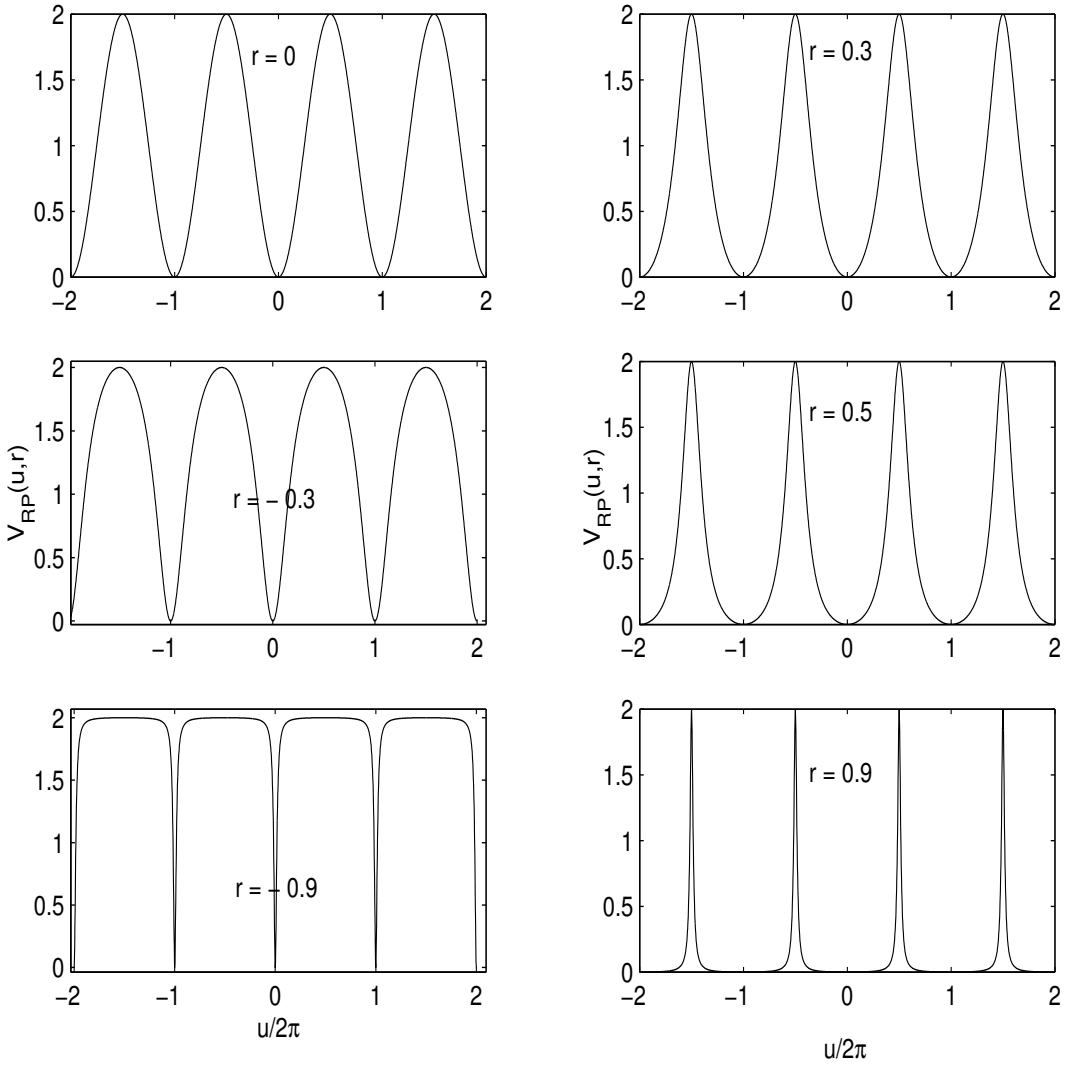

Figure 1: Substrate potential $V_{RP}(r, u)$ as a function of $u/2\pi$ for some values of the shape parameter.

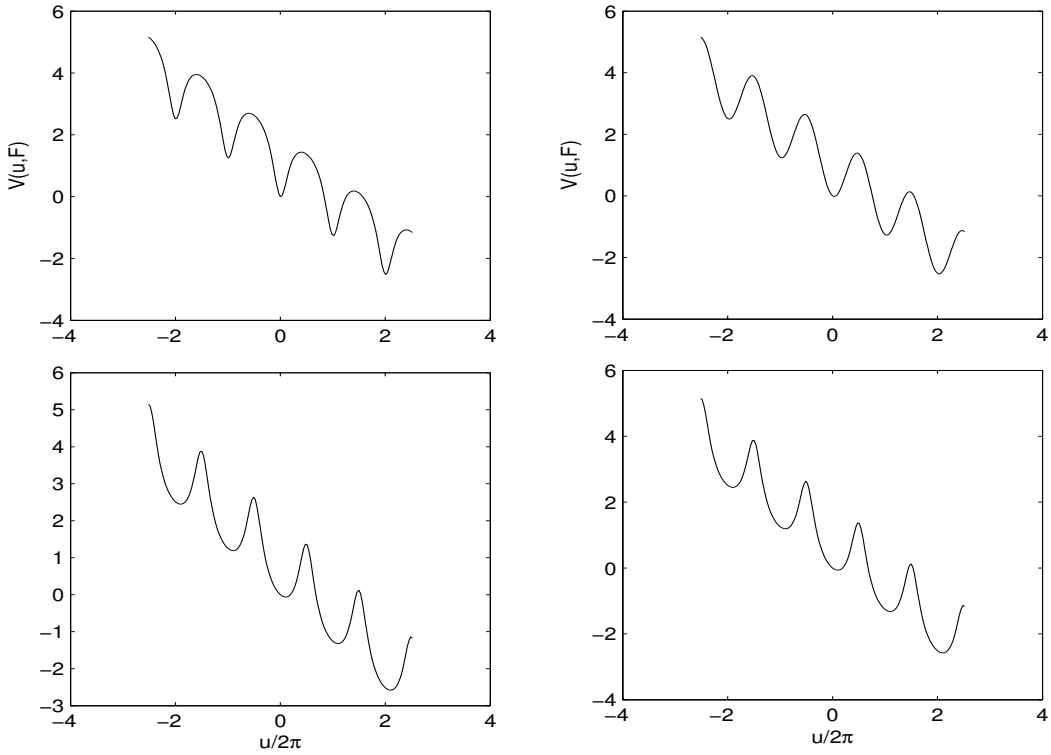

Figure 2: Total potential energy in the presence of the applied field, $V(u, F)$, as a function of $u/2\pi$ for some values of the shape parameter : (1) $r = -0.3$, (2) $r = 0.0$, (3) $r = 0.3$, (4) $r = 0.9$.

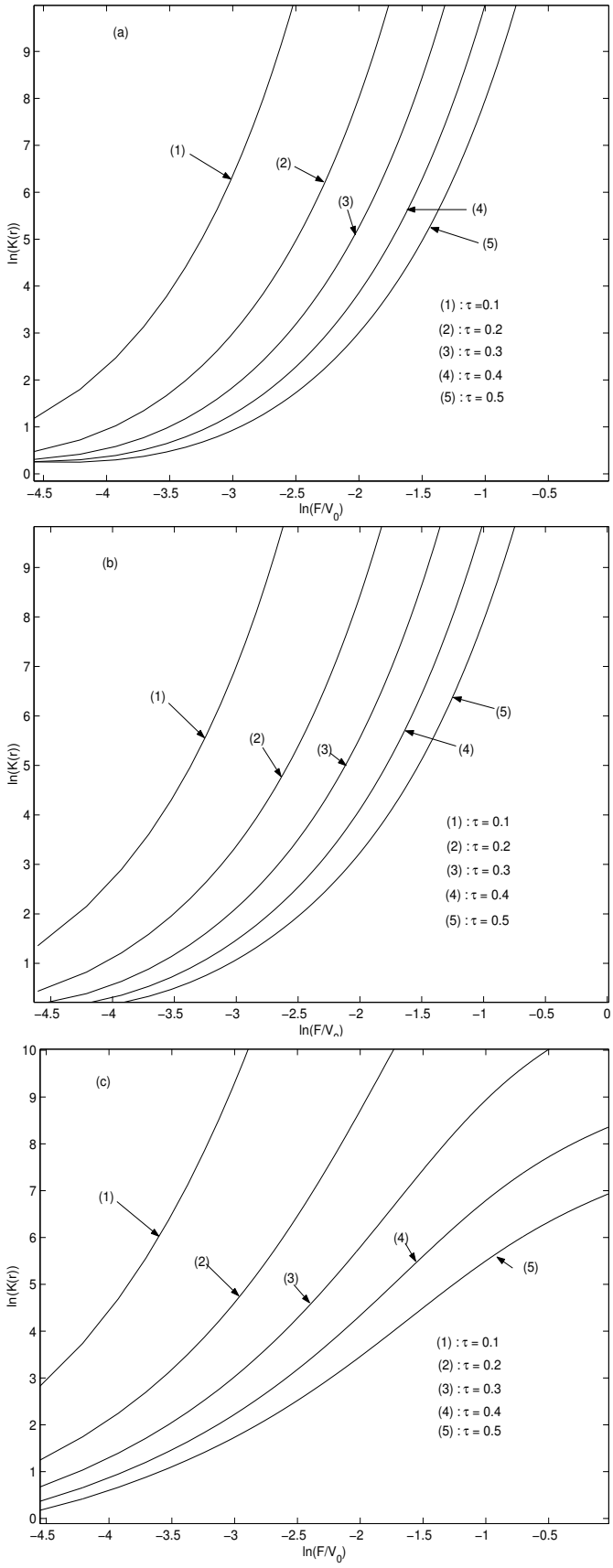

Figure 3: Non-Gaussian correction factor $\ln(K(r))$, given by Eq.(26), as a function of the applied field for different reduced temperature $\tau = k_B T\sqrt{kV_0}$ with : a) $r = -0.5$, b) $r = -0.3$ and c) $r = 0.3$.

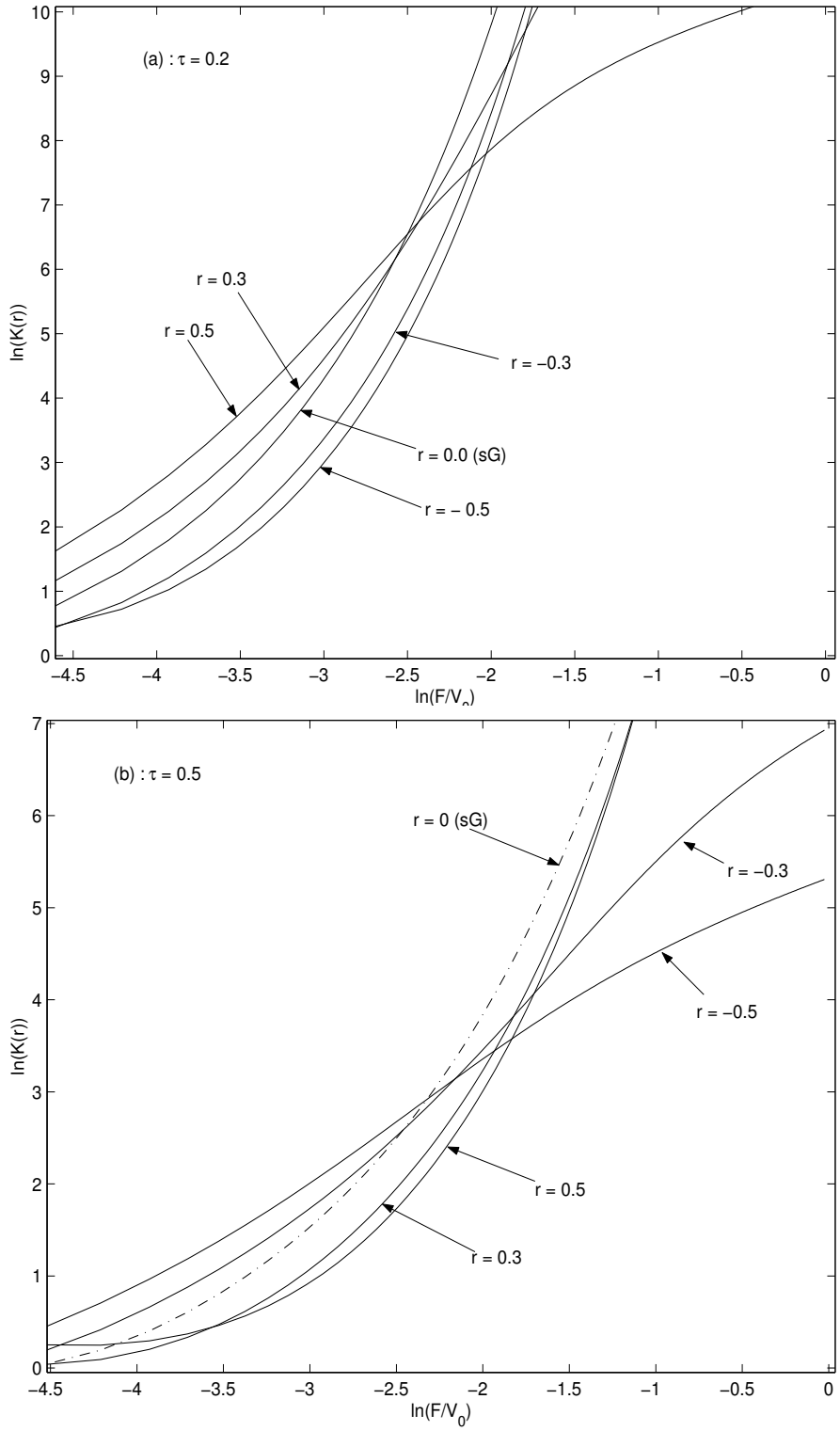

Figure 4: Non-Gaussian correction factor $\ln(K(r))$, given by Eq.(26), as a function of the applied field $ln(F/V_0)$ for different shape parameter r with: a) $\tau = 0.2$ and b) $\tau = 0.5$.

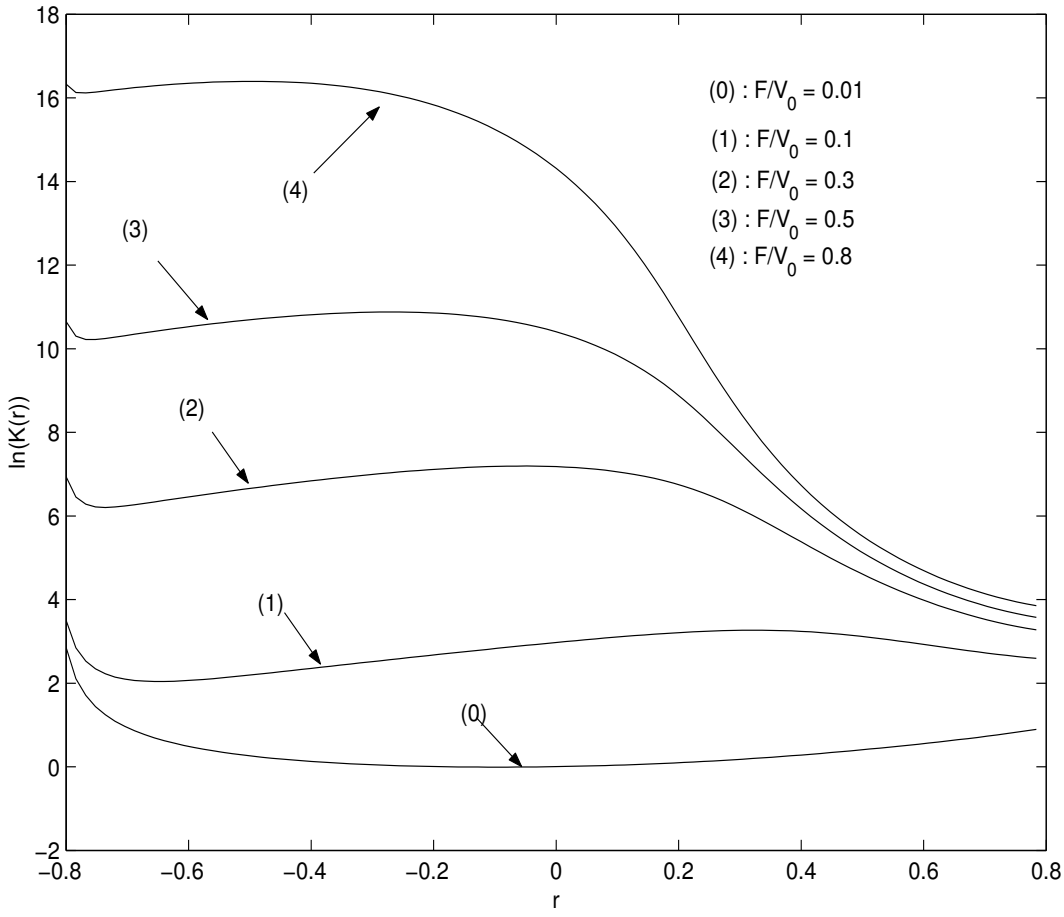

Figure 5: Non-Gaussian correction factor $\ln(K(r))$, given by Eq.(26), as a function of the shape parameter r for different values of the applied field and with the reduced temperature $\tau = 0.5$.