

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ELECTRO-ACOUSTIC SHOCK WAVES IN DUSTY PLASMAS

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Abstract

A rigorous theoretical investigation has been made of electro-acoustic [particularly, dust-ion acoustic (DIA) and dust-acoustic (DA)] shock waves in unmagnetized dusty plasmas. The reductive perturbation method has been employed for the study of the small but finite amplitude DIA and DA shock waves. It has been reported that the dust grain charge fluctuation can be one of the candidates for the source of dissipation, and can be responsible for the formation of DIA shock waves in an unmagnetized dusty plasma with static charged dust particles. It has also been reported that the strong co-relation among dust particles can be one of the candidates for the source of dissipation, and can be responsible for the formation of DA shock waves in an unmagnetized strongly coupled dusty plasma. The basic features and the underlying physics of DIA and DA shock waves, which are relevant to space and laboratory dusty plasmas, are briefly discussed.

MIRAMARE – TRIESTE

October 2005

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I. INTRODUCTION

Dusty plasmas are rather ubiquitous in space [1, 2]. There are a number of well known systems in space, such as interplanetary space, interstellar medium, interstellar or molecular clouds, circum-stellar clouds, comets, solar system, planetary rings, earth's environments, etc. where charged dust particles are always present. The interstellar space (the space between the stars) is filled with a vast medium of gas and dust. The gas content of the interstellar medium continually decreases with time as new generations of stars are formed during the collapse of giant molecular clouds. The collapse and fragmentation of these clouds give rise to the formation of stellar clusters. The presence of dust in interstellar or circum-stellar clouds has been known for a long time (from star reddening and infrared emission). The dust grains in interstellar or circum-stellar clouds are dielectric (ices, silicates, etc.) and metallic (graphite, magnetite, amorphous carbons, etc.). The solar system is also full of dust. The existence of dust in the early solar nebula has long been advocated by the Nobel Laureate Alfvén [3]. The coagulation of the dust grains in the solar nebula would have led to “planetesimal” from where comets and planets have been formed. The physical properties (such as size, mass, density, charge, etc.) of such dust grains vary depending on their origin and surroundings. The origins of the dust grains in the solar system are, for example, micro-meteoroids, space debris, man-made pollution, lunar ejecta, etc.

Dust particles do not only occur in space, but also they occur in laboratory devices, particularly in direct current (dc) and radio-frequency (rf) discharges, plasma processing reactors, fusion plasma devices, solid fuel combustion products, etc. The presence of dust particles in fusion devices has been known for a long time. However, their possible consequences for plasma operation and performances have become a topic of recent interest [4]. The plasma in fusion devices (for example, tokamaks, stellarator, etc.) are more or less contaminated by impurities (dust particles) heavier than the hydrogen isotopes which are the fuel in fusion reactors. The impurities (dust particles) are generated by a number of processes, such as desorption, arcing, sputtering, evaporation and sublimation of thermally overloaded wall material, etc. To know the details of the occurrence of dust in space and laboratories as well as different dusty plasma parameters, we refer to Shukla and Mamun [1], Verheest [2], Winter [4], Bouchoule [5], Hollenstein [6], etc.

Dust particles, which are invariably immersed in an ambient plasma with radiative background, are not neutral, but are positively or negatively charged depending on the plasma environment around the dust grains. The important elementary dust grain charging processes are i) interaction of dust grains with gaseous plasma particles, ii) interaction of dust grains with energetic particles (electrons and ions) and iii) interaction of dust grains with photons. When dust grains are immersed in a gaseous plasma, the plasma particles (electrons and ions) are collected by the dust grains which act as probes. The dust grains are, therefore, charged

by the collection of the plasma particles flowing onto their surfaces. The dust grain charge q_d is determined by $dq_d/dt = \sum_j I_j$, where q_d is the dust grain charge, j represents the plasma species (electrons and ions) and I_j is the current associated with the species j . At equilibrium the net current flowing onto the dust grain surface becomes zero, i.e. $\sum_j I_{j0} = 0$, where I_{j0} is the equilibrium current. This means that the dust grain surface acquires some potential ϕ_g which is $-2.5k_B T/e$ [where k_B is the Boltzmann constant, $T = T_e \simeq T_i$, and T_e (T_i) is the electron (ion) temperature] for a hydrogen plasma and $-3.6k_B T/e$ for an oxygen plasma [7]. It turns out that the dust grains immersed in a gaseous plasma are usually negatively charged. When energetic plasma particles (electrons or ions) are incident onto a dust grain surface, they are either backscattered/reflected by the dust grain or they pass through the dust grain material. During their passage they may lose their energy partially or fully. A portion of the lost energy can go into exciting other electrons that in turn may escape from the material. The emitted electrons are known as secondary electrons. The release of these secondary electrons from the dust grain tends to make the grain surface positive. The interaction of photons incident onto the dust grain surface causes photoemission of electrons from the dust grain surface. The dust grains, which emit photoelectrons, may become positively charged. The emitted electrons collide with other dust grains and are captured by some of these grains which may become negatively charged. There are, of course, a number of other dust grain charging mechanisms, namely thermionic emission, field emission, radioactivity, impact ionization, etc. These are significant only in some different special circumstances.

The charged dust grains can be static or mobile depending on the forces acting on them. The dynamics of dust grains in space attracted the main stream of interest of space physicists about 20 years ago, when Voyager 1 and 2 passed Saturn and sent back pictures of mysterious dark spokes sweeping around the B-ring [8, 9]. It had then been independently proposed by Hill and Mendis [10] and Goertz and Morfill [11] that the spokes might be charged dust and sculptured by electrostatic forces. The dynamical patterns of charged dust particles in interplanetary space observed by Voyager 1 and 2 also suggested to account for the combined effects of electromagnetic and gravitational forces acting on the dust particles. On the other hand, in laboratory plasmas, dust particles, which are subjected to various forces, often accumulate near the plasma boundaries (walls) and cause contamination to substrates and wafers [12]. It is, therefore, crucial to understand the behavior of macroscopic particles under the action of various forces, such as gravitational force, electric force, ion drag force, neutral drag force, thermophoretic force, etc., in order to control the dust transport.

Therefore, the physics of charged dust particles has become an outstanding and challenging research topic not only because dust particles are ubiquitous in most space [13–26] and laboratory plasmas [4–6], but also because it has introduced a great variety of new phenomena associated with waves and instabilities [27–37] in un-magnetized weakly coupled dusty plasma, and it plays a vital role in understanding different interesting phenomena in astrophysical and space

environments, such as interplanetary space, interstellar medium, interstellar or molecular clouds, comets, planetary rings, earth's environments, etc. [13–26].

The charged dust grains in a plasma does not only modify the existing plasma wave spectra [27–29] but also introduces a number of new novel eigenmodes in dusty plasmas. The most important classes of new novel acoustic waves, which are experimentally observed in un-magnetized weakly coupled dusty plasmas, are dust-ion acoustic (DIA) and dust-acoustic (DA) waves.

Shukla and Silin [30] have first theoretically shown that due to the conservation of equilibrium charge density $n_{e0} + n_{d0}Z_{d0} = n_{i0}$ [where n_{s0} is the equilibrium number density of the species $s = e, i, d$], and the strong inequality $n_{e0} \ll n_{i0}$ a dusty plasma (with negatively charged static dust grains) supports low-frequency DIA waves with phase speed much smaller (larger) than electron (ion) thermal speed. The dispersion relation (a relation between the wave frequency ω and the wave number k) of the linear DIA waves is [30] $\omega^2 = (n_{i0}/n_{e0})k^2C_i^2/[1 + k^2\lambda_{De}^2(1 + T_in_{e0}/T_en_{i0})]$, where $C_i = (k_B T_e/m_i)^{1/2}$ is the ion-acoustic speed, $\lambda_{De} = (k_B T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye-radius, and m_i is the ion mass. When we consider a long wavelength limit (viz. $k\lambda_{De} \ll 1$), and the dispersion relation for the DIA waves becomes $\omega = (n_{i0}/n_{e0})^{1/2}kC_i$. This form of spectrum is similar to the usual ion-acoustic wave spectrum for a plasma with $n_{i0} = n_{e0}$ and $T_i \ll T_e$. However, in dusty plasmas we usually have $n_{i0} \gg n_{e0}$ and $T_i \simeq T_e$. Therefore, a dusty plasma cannot support the usual ion-acoustic waves, but can do the DIA waves of Shukla and Silin [30]. The phase speed ω/k of the DIA waves is larger than C_i because of $n_{i0} \gg n_{e0}$ for negatively charged dust grains. The increase in the phase velocity is attributed to the electron density depletion in the background plasma, so that the electron Debye-radius becomes larger. As a result, there appears a stronger space charge electric field which is responsible for the enhanced phase velocity of the DIA waves. The DIA waves have been observed in laboratory experiments [31, 32].

Rao *et al.* [33] theoretically predicted the existence of extremely low phase velocity (in comparison with the electron and ion thermal speeds) DA waves in an un-magnetized dusty plasma whose constituents are an inertial charged dust fluid and Boltzmann ions and electrons. Thus, in the DA waves the inertia is provided by the dust particle mass and the restoring force comes from the pressures of electrons and ions. The dispersion relation for the DA waves with the phase speed ω/k much smaller (larger) than the ion (dust) thermal speed is given by [33] $\omega = kC_d(1 + k^2\lambda_D^2)^{-1/2}$, where $\lambda_D = (\lambda_{De}^{-2} + \lambda_{Di}^{-2})^{-1/2}$ is the global screening length, $\lambda_{Di} = (k_B T_i/4\pi n_{i0}e^2)^{1/2}$ is the ion Debye-radius, $C_d = \omega_{pd}\lambda_D$ is the dust-acoustic speed, $\omega_{pd} = (4\pi n_{d0}Z_{d0}^2 e^2/m_d)^{1/2}$, and m_d is the dust mass. It is obvious that one cannot obtain the DA mode without the consideration of the dynamics of the dust grains. The theoretical prediction of Rao *et al.* [33] has been conclusively verified by a number of laboratory experiments [34, 35].

The linear properties of the DIA and DA waves in dusty plasmas are now well understood and have been reported by a large number of regular and review articles or books during last few years [1, 2, 30–40]. The linear theory is valid only when the wave amplitude is so small that one may

neglect the nonlinearities. However, there are numerous processes via which unstable modes can saturate and attain large amplitudes. When the amplitudes of the waves are sufficiently large, nonlinearities can no longer be ignored. The nonlinearities come from the harmonic generation involving the fluid advection, nonlinear Lorentz force, trapping of particles in the wave potential, ponderomotive force, etc. The nonlinearities in plasmas contribute to the localization of waves, leading to different types of interesting nonlinear coherent structures (viz. solitary waves, shock waves, double layers, vortices, etc.) which are important from both theoretical and experimental points of view, and have received a great deal of attention for understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas.

Recently, a number of theoretical attempts have been made in order to study the properties of DIA shock [41–49] and, DA shock [46–50] waves in unmagnetized dusty plasmas. To the best of our knowledge, there is no regular/review article where a rigorous theoretical investigation on the basic features and the underlying physics of electro-acoustic DIA and DA shock waves is systematically presented. Therefore, in our present article, we have tried to provide systematically the basic features and the underlying physics of electro-acoustic DIA and DA shock waves in un-magnetized dusty plasmas.

The manuscript is organized as follows. We first consider an un-magnetized dusty plasma with static dust, and rigorously investigate the basic features and underlying physics of DIA shock waves in Sec. II. We then consider an un-magnetized dusty plasma with mobile dust, and rigorously investigate the basic features and underlying physics of DA shock waves in Sec. III. We, finally, provide a brief discussion in Sec. IV.

II. STATIC DUST: DIA SHOCK WAVES

DIA shock structures are associated with the DIA waves. To understand the formation of DIA shock waves, we first give a theoretical analysis and then provide their experimental observation.

A. Theoretical Analysis

Shukla [41] first presented an analytical model for DIA shock waves in an unmagnetized dusty plasma with a constant dust grain charge, and derived the KdV-Burgers equation, which admits shock waves solutions by including an arbitrary dissipative (kinematic viscosity) term on the right hand side of the momentum balance equation. There is an important question regarding the source/mechanism of this dissipative term which Shukla [41] considered in order to obtain DIA shock waves in a dusty plasma. Motivated by this question, recently, Mamun and Shukla [45] have considered an unmagnetized dusty plasma with charge fluctuating dust grains, and have proposed that dust grain charge fluctuations can arise this dissipative term and can be responsible for the formation of the DIA shock waves in an unmagnetized dusty plasma.

We study the propagation of the DIA shock waves in an unmagnetized dusty plasma consisting of charge-fluctuating static dust, mobile ions, and Maxwellian electrons. The nonlinear dynamics of one-dimensional DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed, is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu \exp(\phi) - n_i + (1 - \mu)z_d, \quad (3)$$

where n_i is the ion number density normalized by its equilibrium value n_{i0} , u_i is the ion fluid velocity normalized by the ion-acoustic speed C_i , ϕ is the ion-acoustic wave potential normalized by $K_B T_e / e$ and $\mu = n_{e0} / n_{i0}$, and z_d is the number of electrons residing onto the dust grain surface normalized by its equilibrium value Z_{d0} . We note that z_d is not constant, but varies with space and time. Thus, Eqs. (1), (2) and (3) are completed by the normalized dust grain charging equation [45]

$$\eta \frac{\partial z_d}{\partial t} = \mu \beta \exp(\phi - \alpha z_d) - \beta_i n_i u_i \left(1 + \frac{2\alpha z_d}{u_i^2} \right), \quad (4)$$

where $\eta = \sqrt{\alpha m_e (1 - \mu) / 2m_i}$, $\alpha = Z_{d0} e^2 / k_B T_e r_d$, $\beta = (r_d / a)^{3/2}$, $a = n_{d0}^{-1/3}$, and $\beta_i = \beta \sqrt{\pi m_e / 8m_i}$. We note that at equilibrium $\mu \beta \exp(-\alpha) = \beta_i u_0 (1 + 2\alpha / u_0^2)$, where u_0 is the ion streaming speed normalized by C_i .

To study the small but finite amplitude DIA shock waves, we first introduce stretched coordinates [51] $\xi = \epsilon^{1/2}(x - v_0 t)$, $\tau = \epsilon^{3/2} t$, and coefficient [41] $\eta_0 = \epsilon^{-1/2} \eta$, where ϵ is the expansion parameter, measuring the amplitude of the wave or the weakness of the wave dispersion. We then expand n_i , u_i , ϕ and z_d in a power series of ϵ

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \dots, \quad (5)$$

$$u_i = u_0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \dots, \quad (6)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots, \quad (7)$$

$$z_d = 1 + \epsilon z_d^{(1)} + \epsilon^2 z_d^{(2)} + \dots. \quad (8)$$

We develop the equations in various powers of ϵ . To the lowest order in ϵ , Eqs. (1)–(4) give

$$w_0 n_i^{(1)} = u_i^{(1)}, \quad (9)$$

$$w_0 u_i^{(1)} = \phi^{(1)}, \quad (10)$$

$$0 = \mu \phi^{(1)} - n_i^{(1)} + (1 - \mu) z_d^{(1)}, \quad (11)$$

$$0 = \beta_e \phi^{(1)} - \alpha u_\beta z_d^{(1)} - \beta_i u_1 u_i^{(1)} - u_0 \beta_i u_2 n_i^{(1)}, \quad (12)$$

where $w_0 = v_0 - u_0$, $u_1 = 1 - 2\alpha / u_0$, $u_2 = 1 + 2\alpha / u_0^2$, $u_\beta = \beta_e + 2\beta_i / u_0$, and $\beta_e = \mu \beta \exp(-\alpha)$. Now, substituting $n_i^{(1)}$, $u_i^{(1)}$, and $z_d^{(1)}$ [obtained from Eqs. (9)–(11)] into Eq. (12), we obtain the

dispersion relation

$$aw_0^2 - bw_0 - c = 0, \quad (13)$$

where

$$a = \mu + \frac{(1-\mu)\beta_e}{\alpha u_\beta}, \quad (14)$$

$$b = \frac{(1-\mu)u_1\beta_i}{\alpha u_\beta}, \quad (15)$$

$$c = 1 + \frac{(1-\mu)u_2u_0\beta_i}{\alpha u_\beta}. \quad (16)$$

To the next higher order in ϵ , from Eqs. (1)–(4) we obtain a set of equations

$$\frac{1}{w_0^2} \frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{2}{w_0^3} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = 0, \quad (17)$$

$$\frac{1}{w_0} \frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{1}{w_0^2} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = \frac{\partial \phi^{(2)}}{\partial \xi}, \quad (18)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \mu \phi^{(2)} + \frac{1}{2} \mu [\phi^{(1)}]^2 - n_i^{(2)} + (1-\mu)z_d^{(2)}, \quad (19)$$

$$-u_0 \eta_0 \frac{\partial z_d^{(1)}}{\partial \xi} = \beta_e \phi^{(2)} - \alpha u_\beta z_d^{(2)} + \beta_1 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} - \beta_i u_1 u_i^{(2)} - u_0 \beta_i u_2 n_i^{(2)}. \quad (20)$$

where

$$\beta_1 = \beta_e [1 + (\alpha \beta_0 - 2) \alpha \beta_0] - \frac{2\beta_i}{w_0^3} \left[1 + \frac{w_\alpha}{u_0^3} \right], \quad (21)$$

$$\beta_0 = \frac{1}{\alpha u_\beta} \left[\beta_e - \frac{\beta_i w_2}{w_0} + \frac{2\alpha \beta_i}{w_0 u_0} \left(1 - \frac{1}{w_0} \right) \right], \quad (22)$$

$w_1 = 1 - u_0/w_0$, $w_2 = 1 + u_0/w_0$, and $w_\alpha = 2\alpha w_0 w_1 (1 - \beta_0 w_0 u_0)$.

Now, using Eqs. (17)–(20), we obtain a KdV-Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A_c \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B_c \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = C_c \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}, \quad (23)$$

where

$$C_c = \frac{B_c v_0 \eta_0 \beta_0 (1-\mu)}{\alpha (\beta_e + 2\beta_i/u_0)}. \quad (24)$$

An exact analytical solution of Eq. (23) is not possible. However, we can deduce some approximate analytical solutions. Let us try to find an analytical solution which is valid for both space and laboratory dusty plasmas. We first transform the independent variables ξ to $\zeta = \xi - U_0 \tau$. Under the steady state condition we then find from Eq. (23) a third order ordinary differential equation for $\phi^{(1)}(\zeta) = \phi$. The latter can be integrated to obtain

$$B_c \frac{d^2 \phi}{d\zeta^2} - C_c \frac{d\phi}{d\zeta} + \frac{A_c}{2} \phi^2 - U_0 \phi = 0, \quad (25)$$

where we have imposed the appropriate boundary conditions, viz. $\phi \rightarrow 0$, $d\phi/d\zeta \rightarrow 0$, $d^2\phi/d\zeta^2 \rightarrow 0$ at $\zeta \rightarrow \infty$. We can use a simple mechanical analogy [52, 53] based on the fact that it has the form of an equation of motion for a pseudo particle of mass B_c , of pseudo time $-\zeta$, and pseudo position ϕ in a force field with the potential

$$V(\phi) = \frac{A_c}{6}\phi^3 - \frac{U_0}{2}\phi^2, \quad (26)$$

and a frictional force with coefficient C_c . If the frictional force were absent and if we plot $V(\phi)$ against ϕ , we can find that there is a potential well on positive ϕ -axis. This means that the quasi-particle entering from the left will go the right-hand-side of the well ($\phi < 0$), reflect, and return to $\phi = 0$, making a single transit. This corresponds to the DIA solitary waves. However, since a frictional force is present, i.e. the particle suffers a loss of energy, it will never return to $\phi = 0$, but will oscillate about some negative value of ϕ corresponding to the minimum of $V(\phi)$. We assume that at pseudo time $-\zeta = -\infty$, i.e. at $\zeta = \infty$, the quasi-particle is located at the coordinate origin, i.e. $\phi(\infty) = 0$ and at pseudo time $-\zeta = \infty$, i.e. at $\zeta = -\infty$, the quasi-particle is located at a point corresponding to the minimum of $V(\phi)$, i.e. $\phi(-\infty) = 2U_0/A_c$. Thus, the solution of Eq. (25) describes a shock wave whose speed U_0 is related to the extreme values $\phi(\infty) = 0$ and $\phi(-\infty) = 2U_0/A_c$ by $\phi(-\infty) - \phi(\infty) = 2U_0/A_c$.

The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B_c and C_c . If the value of C_c is very small, the energy of the particle decreases very slowly and the first few oscillations at the wave front will be close to solitary waves. However, if the value of C_c is larger than a certain critical value, the motion of the particle will be aperiodic and we obtain a shock wave with a monotonic structure.

We now determine the condition for monotonic or oscillating shock profiles by investigating the asymptotic behavior of the solutions of Eq. (25) for $\zeta \rightarrow -\infty$. We first substitute $\phi(\zeta) = \phi_0 + \tilde{\phi}(\zeta)$, where $\tilde{\phi} \ll \phi_0$, into Eq. (25), then linearize it with respect to $\tilde{\phi}$, and obtain

$$B_c \frac{d^2\tilde{\phi}}{d\zeta^2} - C_c \frac{\partial\tilde{\phi}}{\partial\zeta} + U_0\tilde{\phi} = 0. \quad (27)$$

The solutions of Eq. (27) are proportional to $\exp(p\zeta)$, where p is given by

$$p = \frac{C_c \pm \sqrt{C_c^2 - 4B_cU_0}}{2B_c}. \quad (28)$$

It implies that the shock wave has a monotonic profile for $S_c = C_c/2\sqrt{B_cU_0} > 1$ and an oscillating profile for $S_c < 1$. We have numerically analyzed S_c for space ($\alpha = 0.0288$ and $\beta = 3 \times 10^{-10}$) and laboratory ($\alpha = 1.44$ and $\beta = 3 \times 10^{-4}$) dusty plasma parameters and have found that $S_c \gg 1$ is always valid for $0 < \mu < 1$. This means that for typical space and laboratory dusty plasma parameters Eq. (27) [or Eq. (25) under steady state condition] exhibits a monotonic shock wave solution which is given by

$$\phi^{(1)} \simeq \psi_{sh} \left[1 - \tanh \left(\frac{\zeta}{\Delta_{sh}} \right) \right], \quad (29)$$

where $\psi_{sh} = U_0/A_c$ and $\Delta_{sh} = 2C_c/U_0$ represent the amplitude and the width of the shock waves, respectively. We note that since $S_c \gg 1$, we have neglected the dispersive term just to avoid the complexity of the mathematics which does not affect the properties of the shock structure that may exist in our space and laboratory dusty plasmas. We note that the shock structures predicted by Popel *et al.* [42] do not follow our shock solution (29), and look like the ion density steepening as observed by Luo *et al.* [54]. However, the shock structures found in our present investigation are very close to those observed by Nakamura *et al.* [55].

B. Experimental Observation

1. Experiment of Nakamura *et al.* [55]:

DIA shock waves were experimentally excited in a dusty double plasma (DP) device by Nakamura *et al.* [55]. We here briefly illustrate the formation of these experimentally excited DIA shock waves by summarizing the experimental work of Nakamura *et al.* [55]. The inner diameter of the dusty DP device is 40 cm and its length is 90 cm. The device is separated into a source and a target section by a fine mesh grid which is kept electrically floating. The chamber is evacuated down to 5×10^{-7} torr with a turbo-molecular pump. The argon gas is bled into the chamber at a partial pressure of about 5×10^{-4} torr. A dust dispersing setup fitted at the target section consisted of a dust reservoir coupled to an ultrasonic vibrator. The dust reservoir consists of a fine stainless steel mesh (118 lines per cm) of 10 cm (width) \times 16 cm (axial length) area at the bottom end and is placed horizontally closer to the anode wall. An ultrasonic vibrator is tuned at 27 kHz to vibrate the dust reservoir by using a signal generator and a power amplifier. Glass powder of average diameter 8.8 μm is used. The dust number density is easily controlled by adjusting the power of the signal applied to the vibrator and is measured from the intensity of the laser light which passes through the dust column and is collected by a photodiode array. A maximum dust density of the order of 10^5 cm^{-3} is obtained in this setup. The plasma parameters measured by a plane Langmuir probe of 6 mm diameter and a retarding potential analyzer are as follows: $n_e = 10^8 - 10^9 \text{ cm}^{-3}$, $T_e = (1 - 1.5) \times 10^4 \text{ K}$, $T_i \simeq 0.1 T_e$, $Z_d \simeq 10^5$ for $n_d < 10^3 \text{ cm}^{-3}$ and $Z_d \simeq 10^2$ for $n_d < 10^5 \text{ cm}^{-3}$.

The shock waves are excited in the plasma by applying a ramp signal with an amplitude of 2 V and a rise time of approximate 10 μs . The Langmuir probe is biased above the plasma potential in order to detect the signal as fluctuations in the electron saturation currents. Oscillatory shock waves are first excited in the plasma without the dust and the dust density is then increased in small steps, keeping the probe fixed at 12 cm measured from the grid. The corresponding time normalized by the ion plasma period (ω_{pi}^{-1}) for the signal at 12 cm is about 150. The observed signals reveal that the oscillatory wave structure behind the shock becomes less in number with increasing dust particle number density and finally completely disappears at a sufficiently high dust particle number density, leaving only the laminar shock front. The

shock speed also increases with increasing dust particle number density. It is also noted that the particle density behind the shock remains constant, although the amplitude of the shock front (steepened part) seems to decrease when the dust particle number density is increased.

2. Experiment of Luo *et al.* [54]:

The effect of the dust particle number density on the ion acoustic compressional pulses has also been experimentally studied by Luo *et al.* [54] who observed a steepening of the ion-acoustic pulses as they propagated through a dusty plasma if the percentage of the negative charge in the plasma on the dust grains was about 75% or more.

III. MOBILE DUST: DA SHOCK WAVES

DA shock structures are associated with the DA waves in a strongly coupled dusty plasma. To understand the formation of DA shock waves, we consider a strongly coupled dusty plasma where dust are strongly coupled because of their lower temperature and higher electric charge [56, 57], but electrons and ions are weakly coupled due to their higher temperatures and smaller electric charges, compared to those of dust grains. We first assume that both electron and ion species are Maxwellian. We next replace the Maxwellian ion distribution by the non-Maxwellian (trapped) ion distribution of Schammel [58–61] to study the effect of trapped ion distribution on DA shock waves in a strongly coupled dusty plasma.

A. Maxwellian Ion Distribution

We assume that the electrons and the ions are weakly coupled due to their higher temperatures and smaller electric charges. Thus, in the presence of low phase velocity (in comparison with the electron and ion thermal velocities) DA waves, the electron and ion number densities obey a Boltzmann distribution. On the other hand, we assume that the dust grains are strongly coupled because of their lower temperature and larger electric charge. The dynamics of the nonlinear DA waves in such a strongly coupled dusty plasma is governed by the well known generalized hydrodynamic (GH) equations [56, 57]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial z}(n_d u_d) = 0, \quad (30)$$

$$(1 + \tau_m D_t) \left[n_d \left(D_t u_d + \nu_{dn} u_d - \frac{\partial \varphi}{\partial z} \right) \right] = \eta_d \frac{\partial^2 u_d}{\partial z^2}, \quad (31)$$

$$\frac{\partial^2 \varphi}{\partial z^2} = n_d + \mu_e \exp(\sigma_i \varphi) - \mu_i \exp(-\varphi), \quad (32)$$

where n_d is the particle number density normalized by its equilibrium value n_{d0} , u_d is the dust fluid velocity normalized by $C_d = (Z_{d0} k_B T_i / m_d)^{1/2}$, φ is the electrostatic wave potential normalized by $k_B T_i / e$, $\mu_e = n_{e0} / Z_{d0} n_{d0} = \mu / (1 - \mu)$, $\mu_i = n_{i0} / Z_{d0} n_{d0} = 1 / (1 - \mu)$, and $\sigma_i = T_i / T_e$. The time (t) and space (z) variables are in units of the dust plasma period $\omega_{pd}^{-1} = (m_d / 4\pi n_{d0} Z_{d0}^2 e^2)^{1/2}$ and the Debye-radius $\lambda_{Dim} = (k_B T_i / 4\pi Z_{d0} n_{d0} e^2)^{1/2}$, respectively. $D_t = \partial / \partial t + u_d \partial / \partial z$, ν_{dn} is dust-neutral collision frequency normalized by the dust plasma frequency ω_{pd} , τ_m is the viscoelastic relaxation time normalized by the dust plasma period ω_{pd}^{-1} , and $\eta_d = (\tau_d / m_d n_{d0} \lambda_{Dim}^2) [\eta_b + (4/3)\zeta_b]$ is the normalized longitudinal viscosity coefficient with η_b , and ζ_b being the shear and bulk transport coefficients [56].

There are various approaches [56, 62] for calculating these transport coefficients. The viscoelastic relaxation time τ_m is given by [56]

$$\tau_m = \eta_l \frac{T_e}{T_d} \left[1 - \mu_d + \frac{4}{15} u(\Gamma) \right]^{-1}, \quad (33)$$

where

$$\mu_d = \frac{1}{k_B T_d} \frac{\partial P_d}{\partial n_d} = 1 + \frac{1}{3} u(\Gamma) + \frac{\Gamma}{9} \frac{\partial u(\Gamma)}{\partial \Gamma} \quad (34)$$

is the compressibility [62] and $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ($\Gamma < 1$) as [62] $u(\Gamma) \simeq -(\sqrt{3}/2)\Gamma^{3/2}$. To express $u(\Gamma)$ in terms of Γ for a range $1 < \Gamma < 100$, Slattery *et al.* [63] have analytically derived a relation

$$u(\Gamma) \simeq -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81, \quad (35)$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient η_l on Γ is somewhat more complex and cannot be expressed in such a closed analytical form. However, tabulated/graphical results of their functional behavior derived from molecular dynamic (MD) simulations and a variety of statistical schemes are available in the literature [62, 64]. Typical values [62] of η_l are $1.04a_d^2 / \lambda_{Dd}^2$ for $\Gamma = 1$, $0.08a_d^2 / \lambda_{Dd}^2$ for $\Gamma = 10$, and $0.3a_d^2 / \lambda_{Dd}^2$ for $\Gamma = 160$.

To derive a dynamical equation for DA shock waves from our basic equations (30)–(32), we employ the reductive perturbation technique. Thus, we introduce the stretched coordinates $\xi = \epsilon^{1/2}(z - u_0 t)$ and $\tau = \epsilon^{3/2} t$, and use the expansion of the variables n_d , u_d ,

and φ about the unperturbed states in power series of ϵ , viz.

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \dots, \quad (36)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \dots, \quad (37)$$

$$\varphi = \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + \dots \quad (38)$$

Now, substituting Eqs. (36)–(38) into Eqs. (30)–(32), from the equations of the lowest order in ϵ we obtain $u_d^{(1)} = -\varphi^{(1)}/u_0$, $n_d^{(1)} = -\varphi^{(1)}/u_0^2$, and $u_0 = (\mu_e + \sigma_i \mu_i)^{-1/2}$. To the next order in ϵ , we have

$$\frac{\partial n_d^{(1)}}{\partial \tau} - u_0 \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi} [n_d^{(1)} u_d^{(1)}] + \frac{\partial u_d^{(2)}}{\partial \xi} = 0, \quad (39)$$

$$(1 + \nu_{dn} \tau_m) \frac{\partial u_d^{(1)}}{\partial \tau} - u_0 \frac{\partial u_d^{(2)}}{\partial \xi} - \frac{\partial \varphi^{(2)}}{\partial \xi} + (1 - \nu_{dn} \tau_m) u_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi} - \eta_0 \frac{\partial^2 u_d^{(1)}}{\partial \xi^2} = 0, \quad (40)$$

$$\frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} - \frac{1}{u_0^2} \varphi^{(2)} - n_d^{(2)} = \frac{1}{2} (\mu_e - \mu_i \sigma_i^2) [\varphi^{(1)}]^2, \quad (41)$$

where we have assumed that $\eta_i \sim \epsilon^{1/2} \eta_0$ [41]. It may be noted that because of the ordering we have used, in Eq. (40) the terms containing $\nu_{dn} \tau_m$ survive and the terms that depend only on τ_m are factored out. By eliminating $n_d^{(2)}$, $u_d^{(2)}$, and $\varphi^{(2)}$ from Eqs. (39)–(41) we readily obtain the K-dV-Burgers equation

$$A_d^{-1} \frac{\partial \varphi^{(1)}}{\partial \tau} + \varphi^{(1)} \frac{\partial \varphi^{(1)}}{\partial \xi} + \beta_d \frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = \mu_d \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \quad (42)$$

where $A_d = (v_0^3 a_d / 2) (1 + \nu_{dn} \tau_m / 2)^{-1}$, $\beta_d = 1/a_d$, $\mu_d = \eta_{d0} / a_d v_0^3$, $a_d = (\nu_{dn} \tau_m - a_{\mu\sigma}) / v_0^4$, and $a_{\mu\sigma} = 2v_0^4 [1 + (3 + \sigma_i \mu) \sigma_i \mu + (1 + \sigma_i^2) \mu / 2] / (1 - \mu)^2$. Thus, for a weakly coupled or collisionless dusty plasma ($\nu_{dn} \tau_m \rightarrow 0$) we have $a_{\mu\sigma} < \nu_{dn} \tau_m$, i.e. $a_d < 0$, i.e. all coefficients (A_d , β_d and μ_d) are negative, whereas for a strongly coupled highly collisional dusty plasma satisfying $\nu_{dn} \tau_m > a_{\mu\sigma}$ we have $a_d > 0$, i.e. all coefficients (A_d , β_d and μ_d) are positive. We note that for the usual dusty plasma parameters (viz. $\mu = 0.1$ and $\sigma_i = 1$) we have $a_{\mu\sigma} \simeq 2.0$. This means that for $T_i \leq T_e$ and $n_{e0} \leq 0.1 n_{i0}$ all coefficients (A_d , β_d and μ_d) can be positive if $\nu_{dn} \tau_m \geq 2$.

We now find the shock solutions of the KdV-Burgers equation (42) which, after transformation of the space variable ξ to $\zeta = \xi - U_0 \tau$, where U_0 is the normalized velocity of the shock waves, can be integrated to obtain

$$\beta_d \frac{d^2 \varphi^{(1)}}{d\zeta^2} - \mu_d \frac{d\varphi^{(1)}}{d\zeta} + \frac{1}{2} [\varphi^{(1)}]^2 - \frac{U_0}{A_d} \varphi^{(1)} = 0, \quad (43)$$

where we have used the steady state condition and have imposed the appropriate boundary conditions, viz. $\varphi^{(1)} \rightarrow 0$, $d\varphi^{(1)}/d\zeta \rightarrow 0$, $d^2\varphi^{(1)}/d\zeta^2 \rightarrow 0$ at $\zeta \rightarrow \infty$. We can now easily show [52, 53] that Eq. (43) describes a shock wave whose speed U_0 is related to the extreme values $\varphi^{(1)}(-\infty) - \varphi^{(1)}(\infty) = Y$ by $U_0/A_d = Y/2$. Thus, in the rest frame the normalized speed of the shock waves is $(1 + A_d Y/2)$. The nature of these shock waves depends on the relative values of the dispersive and dissipative parameters β_d and μ_d .

We first consider a situation where the dissipative term is dominant over the dispersive term, i.e. we can express Eq. (43) as

$$\left(\varphi^{(1)} - \frac{U_0}{A_d}\right) \frac{d\varphi^{(1)}}{d\zeta} = \mu_d \frac{d^2\varphi^{(1)}}{d\zeta^2}. \quad (44)$$

Using the condition that y is bounded as $\zeta \rightarrow \pm\infty$, Eq. (44) can be integrated to obtain [52]

$$\varphi^{(1)} = \frac{U_0}{A_d} \left[1 - \tanh\left(\frac{U_0}{2\mu_d}(\xi - U_0\tau)\right) \right]. \quad (45)$$

Equation (45) represents a monotonic shock solution with the shock speed U_0 , the shock height U_0/A_d and the shock thickness $2\mu_d/U_0$. The shock solution appears because of the dissipative term, which is proportional to the viscosity coefficient.

Now, we discuss the combined effects of dissipation (μ_d) and dispersion (β_d). When μ_d is extremely small, the shock waves will have an oscillatory profile in which the first few oscillations at the wave front will be close to solitons moving with the speed U_0 [52]. If μ_d is increased and it is larger than a critical value μ_{dc} , the shock wave will have a monotonic behavior. To determine the value of the dissipation coefficient μ_d corresponding to monotonic or oscillating shock profiles, we investigate the asymptotic behavior of the solution of Eq. (43) for $\zeta \rightarrow -\infty$. We first substitute $\varphi^{(1)}(\zeta) = y_0 + y_1(\zeta)$, where $y_1 \ll y_0$, into equation Eq. (43) and then linearize it with respect to y_1 in order to obtain

$$\beta_d \frac{d^2 y_1}{d\zeta^2} - \mu_d \frac{dy_1}{d\zeta} + \frac{U_0}{A_d} y_1 = 0. \quad (46)$$

The solution of Eq. (46) is proportional to $\exp(p_s x)$, where p_s is

$$p_s = \frac{\mu_d}{2\beta_d} \pm \left(\frac{\mu_d^2}{4\beta_d^2} - \frac{U_0}{A_d\beta_d} \right)^{1/2}. \quad (47)$$

It turns out that the shock wave has a monotonic profile for $\mu_d > \mu_{dc}$ and an oscillatory profile for $\mu_d < \mu_{dc}$, where $\mu_{dc} = (4\beta_d U_0/A_d)^{1/2}$.

B. Trapped Ion Distribution

We consider the above strongly coupled dusty plasma model with two exceptions that the ion number density follows the vortex-like distribution representing the ion trapping in the DA wave potential, and the collision effect has been neglected (i.e. $\nu_{dn} = 0$). Thus, the dynamics of the nonlinear DA waves in our strongly coupled dusty plasma is governed by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \quad (48)$$

$$\left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[n_d \left(D_t u_d - \frac{\partial \varphi}{\partial x} \right) \right] = \eta \frac{\partial^2 u_d}{\partial x^2}, \quad (49)$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_d + \mu_e e^{\sigma_i \varphi} - \mu_i n_i, \quad (50)$$

To find n_i for the trapped ion distribution, we consider the trapped or vortex-like [58, 59] ion distribution $f_i = f_{if} + f_{it}$, where

$$f_{if} = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2}(v_i^2 + 2\varphi) \right] \quad (51)$$

for $|v_i| > \sqrt{-2\varphi}$ and

$$f_{it} = \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2}\sigma_{it}(v_i^2 + 2\varphi) \right] \quad (52)$$

for $|v_i| \leq \sqrt{-2\varphi}$. We note that the ion distribution function, as prescribed above, is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution [65]. Here the ion velocity v_i in Eqs. (51) and (52) is normalized by the ion thermal speed V_{Ti} , and $\sigma_{it} = T_i/T_{it}$ (the ratio of the free ion temperature T_i to the trapped ion temperature T_{it}) is a parameter determining the number of trapped ions. Now, integrating the ion distribution functions over velocity space, we readily obtain the ion number density n_i as [60]

$$n_i = I(-\varphi) + \frac{1}{\sqrt{\sigma_{it}}} \exp(-\sigma_{it}\varphi) \operatorname{erf}(\sqrt{-\sigma_{it}\varphi}) \quad (53)$$

for $\sigma_{it} > 0$ and

$$n_i = I(-\varphi) + \frac{1}{\sqrt{\pi|\sigma_{it}|}} W_D(\sqrt{\sigma_{it}\varphi}) \quad (54)$$

for $\sigma_{it} < 0$, where

$$I(z_0) = [1 - \operatorname{erf}(\sqrt{z_0})] \exp(z_0), \quad (55)$$

$$\operatorname{erf}(z_0) = \frac{2}{\sqrt{\pi}} \int_0^{z_0} \exp(-y^2) dy, \quad (56)$$

$$W_D(z_0) = \exp(-z_0^2) \int_0^{z_0} \exp(y^2) dy. \quad (57)$$

If we expand n_i in the small amplitude limit (viz. $\varphi \ll 1$) and keep the terms up to φ^2 , it is found that n_i is the same for both $\sigma_{it} < 0$ and $\sigma_{it} > 0$. It is expressed as

$$n_i \simeq 1 - \varphi - \frac{4(1 - \sigma_{it})}{3\sqrt{\pi}} (-\varphi)^{3/2} + \frac{1}{2}\varphi^2. \quad (58)$$

To derive the nonlinear evolution equation (K-dV-Burgers equation) we follow the reductive perturbation technique, and construct a weakly nonlinear theory by using the stretched coordinates [58] $\xi = \epsilon^{1/4}(x - v_0 t)$, $\tau = \epsilon^{3/4} t$, and the coefficient [41] $\eta_l = \epsilon^{1/4} \eta$, where ϵ is a small parameter measuring the weakness of the dispersion and v_0 is the wave phase speed. We expand the perturbed quantities about their equilibrium values in various powers of ϵ as $n_d = 1 + \epsilon n_d^{(1)} + \epsilon^{3/2} n_d^{(2)} + \dots$, $u_d = \epsilon u_d^{(1)} + \epsilon^{3/2} u_d^{(2)} + \dots$, and $\varphi = \epsilon \varphi^{(1)} + \epsilon^{3/2} \varphi^{(2)} + \dots$. Now, substituting the stretched coordinates and the expansion series into our basic equations, we develop equations in various powers of ϵ , and finally obtain a modified K-dV-Burgers equation [66]

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + A \sqrt{-\varphi^{(1)}} \frac{\partial \varphi^{(1)}}{\partial \xi} + B \frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = C \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2}, \quad (59)$$

where $A = v_0^3 \mu_i (1 - \sigma_{it}) / \sqrt{\pi}$, $B = v_0^3$, $C = \eta / 2$. Equation (59) is a new form of the KdV-Burgers equation containing a square root nonlinearity and dissipation. The square root nonlinearity arises due to the trapped ion distribution, whereas the dissipation arises due to the strongly correlated dust [56, 57]. We note that Eq. (59) does not contain the effect of τ_m , and that for any kind of ordering/stretching the terms containing τ_m vanish. It is not possible to solve Eq. (49) analytically. However, for $\eta_l = 0$ one can find its stationary solution [46] $\varphi^{(1)} = -\varphi_m^{(1)} \operatorname{sech}^4[(\xi - u_0 \tau) / \Delta]$, where the amplitude $\varphi_m^{(1)}$ and the width Δ are given by $(15u_0/8A)^2$ and $\sqrt{16B/u_0}$, respectively. As $u_0 > 0$ and $\delta > 1$, there exist solitary waves with $\varphi < 0$. It is observed that as $|\sigma_{it}|$ increases, the amplitude decreases.

To explore the evolution of ion holes, we have solved Eq. (59) numerically for different values of C [57]. We have used the initial condition $\varphi^{(1)} = -\varphi_m^{(1)} \operatorname{sech}^4(\xi/5\Delta)$, with $\varphi_m^{(1)} = 0.24$ and $\Delta = 4.6$. We have first numerically solved Eq. (59) with $C = 0$ (weakly coupled dusty plasma), and found that the initial (wide) pulse after some time breaks up into a group of narrower solitary ion holes. The latter with larger amplitudes have larger speeds. We have then solved Eq. (59) with $C = 0.2$, and have shown that the system

develops shock-like structures with oscillations close to the shock front. We have finally solved Eq. (59) with $C = 1.0$ (strongly coupled dusty plasma), and have found that the system develops a smooth shock front without any oscillations, and similar to the weakly coupled plasma, the pulse amplitude decreases with increasing time.

IV. DISCUSSION

We have presented a rigorous theoretical investigation of the acoustic (particularly DIA and DA) shock waves in unmagnetized dusty plasma. We have first considered a dusty plasma with charge fluctuating static dust, and have found the source of dissipation in the system. We have found that the dust grain charge fluctuations are responsible for the formation of DIA shock waves in both space and laboratory dusty plasmas. We have also found that the shock width decreases with μ (when $1 > \mu > 0.2$) for both space and laboratory dusty plasma parameters. However, for $0 < \mu < 0.2$ as we increase μ , the shock width increases for laboratory dusty plasma conditions, but decreases for space dusty plasma conditions. We briefly discussed the implications of our results to some experimental observations of DIA shock waves.

We next consider a strongly coupled (correlated) dusty plasma, where dust are strongly coupled because of their low temperature and higher electric charge, but electrons and ions are weakly coupled due to their higher temperatures and smaller electric charges, compared to the dust grains. We have found that the strong dust co-relation may support DA shock waves instead of DA solitary structures due to the dissipation effect μ_d . When μ_d is extremely small, shock waves will have an oscillatory profile in which first few oscillations will be close to solitary structures. If μ_d is increased and it is larger than a critical value μ_{dc} , shock waves will have a monotonic behavior.

We have also investigated how the effect of the trapped ion distribution can convert DA solitary waves into DA shock-like structures in an unmagnetized strongly coupled dusty plasma. We have derived a new form of the KdV-Burgers equation containing a square root nonlinearity and dissipation. The square root nonlinearity arises due to the trapped ion distribution, whereas the dissipation arises due to the effect of strong co-relation among dust particles. We have numerically solved this new form of K-dV-Burgers equation, and have shown how DA solitary waves can be converted into shock-like structures with oscillations close to the shock front.

We finally hope that the basic features and the underlying physics of electro-acoustic shock waves that we have presented in our present article should be useful for performing further laboratory experiments.

Acknowledgments

This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. The financial support from the Swedish International Development Cooperation Agency (SIDA) is acknowledged. A.A. Mamun is grateful to Prof. P.K. Shukla for his invaluable suggestions and ideas which are the basis of this article.

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