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ELECTRO-ACOUSTIC SOLITARY WAVES IN DUSTY PLASMAS

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Abstract

We present a rigorous theoretical investigation of electro-acoustic [particularly, dust-ion acoustic (DIA) and dust-acoustic (DA)] solitary waves in dusty plasmas. We employ the reductive perturbation method for small but finite amplitude solitary waves as well as the pseudo-potential approach for arbitrary amplitude ones. We also analyze the effects of non-planar geometry and dust charge fluctuations on both DIA and DA solitary waves, the effect of finite ion-temperature on DIA solitary waves, and the effects of dust-fluid temperature and non-isothermal ion distributions on DA solitary waves. It has been reported that these effects do not only significantly modify the basic features of DIA or DA solitary waves, but also introduce some important new features. The basic features and the underlying physics of DIA and DA solitary waves, which are relevant to space and laboratory dusty plasmas, are briefly discussed.

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I. INTRODUCTION

A dusty plasma is roughly defined as a normal electron-ion plasma with an additional charged component of micron or sub-micron sized dust particles. This extra component of such charged dust particles increases the complexity of the plasma system even further. This is why, a dusty plasma is also referred to as a "complex plasma". Dusty plasmas are low-temperature fully or partially ionized electrically conducting gases whose constituents are electrons, ions, charged dust grains, and neutral atoms. The dust grains are massive (billions times heavier than the protons) and their sizes range from nanometers to millimeters. Dust grains could be metallic, conducting, or made of ice particulate. The size and shape of dust grains could be different, unless these are man-made. However, when viewed from far away, they can be considered as point charges.

A plasma with dust particles or grains can be termed as either "dust in a plasma" or "a dusty plasma" depending on the ordering of a number of characteristic lengths. These are the dust grain radius (r_d), the average inter-grain distance (a_d), the plasma Debye radius (λ_D) and the dimension of the dusty plasma. The situation $r_d \ll \lambda_D < a_d$ (in which charged dust particles are considered as a collection of isolated screened grains) corresponds to "dust in a plasma", while the situation $r_d \ll a_d < \lambda_D$ (in which charged dust particles participate in the collective behavior) corresponds to "a dusty plasma". When we consider a plasma with isolated dust grains ($a_d \gg \lambda_D$), we should take into account the local plasma inhomogeneities. On the other hand, when we consider the opposite situation ($a_d \ll \lambda_D$), we should treat dust grains as massive charged particles similar to multiply charged negative or positive ions. However, in studies of collective dusty plasma behavior, we should also take into account the dust particle charging processes.

When no external disturbance is present, like an electron-ion plasma a dusty plasma is also macroscopically neutral. This means that in an equilibrium with no external force present, the net resulting electric charge in a dusty plasma is zero. Therefore, the equilibrium charge neutrality condition in a dusty plasma reads $\sum_s q_s n_{s0} = 0$, where n_{s0} is the unperturbed number density of the dusty plasma species s (s equals e for electrons, i for ions and d for dust grains), $q_i = Z_i e$ is the ion charge (we note that the ion charge state $Z_i = 1$ will be used in our present article), $q_d = Z_{d0}e$ ($-Z_{d0}e$) is the dust particle charge when the grains are positively (negatively) charged, e is the magnitude of the electron charge and Z_{d0} is the number of charges residing onto the dust grain surface. Typically, a dust grain acquires one thousand to several hundred thousands elementary charges and $Z_{d0}n_{d0}$ could be comparable to n_{i0}, even for $n_{d0} \ll n_{i0}$. However, in many laboratory and space plasma situations, a significant number of background electrons could stick onto the dust grain surface during the charging processes and as a result one might encounter a significant depletion of the electron number density in the ambient dusty plasma. It should be noted here that a complete depletion of the electrons is not possible, because the minimum value of n_{e0}/n_{i0} turns out to be $(m_e/m_i)^{1/2}$ when the electron temperature (T_e) is approximately equal to the ion temperature (T_i), and the grain surface potential approaches to zero, where m_e (m_i) is the electron (ion) mass.

The presence of charged dust particles significantly modifies the plasma Debye-radius. The dusty plasma Debye-radius λ_D is defined as [1] $\lambda_D = \lambda_{De}\lambda_{Di}/\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}$, where $\lambda_{De,i} = (k_B T_{e,i}/4\pi n_{e0,i0} e^2)^{1/2}$, and k_B is the Boltzmann constant. The quantity λ_D is a measure of the shielding distance or the thickness of the sheath. For a dusty plasma with negatively charged dust grains, we have $n_{e0} \ll n_{i0}$ and $T_e \geq T_i$, i.e. $\lambda_{De} \gg \lambda_{Di}$. Accordingly, we have $\lambda_D \simeq \lambda_{Di}$. This means that the shielding distance or the thickness of the sheath in a dusty plasma is mainly determined by the temperature and number density of the ions. However, when the dust particles are positively charged and most of the ions are attached onto the dust grain surface, i.e. when $T_e n_{i0} \ll T_i n_{e0}$, we have $\lambda_{De} \ll \lambda_{Di}$. This corresponds to $\lambda_D \simeq \lambda_{De}$. This means that in a dusty plasma with positively charged dust grains, the shielding distance or the thickness of the sheath is mainly determined by the temperature and density of the electrons.

Similar to the usual electron-ion plasma, an important dusty plasma property is the stability of its macroscopic space charge neutrality. When a plasma is instantaneously disturbed from its equilibrium, the resulting internal space charge field gives rise to collective particle motions which tend to restore the original charge neutrality. These collective motions are characterized by a natural frequency of oscillations known as the plasma frequency which is defined as $\omega_p^2 = \sum_s \omega_{ps}^2 = \sum_s (4\pi n_{s0} q_s^2/m_s)$, where m_s is the mass of the species s. The other important characteristic frequencies are associated the collisions of the plasma particles (electrons, ions and dust grains) with stationary neutrals. These are the electron-neutral collision frequency ν_{en}, the ion-neutral collision frequency ν_{in}, and the dust-neutral collision frequency ν_{dn}, respectively. The collision frequency ν_{sn} for scattering of the plasma species s by the neutrals is $\nu_{sn} = n_n \sigma_s^n V_{Ts}$ where n_n is the neutral number density, σ_s^n is the scattering cross section (which is typically of the order of 5×10^{-15} cm^2 and depends weakly on the temperature T_s of the species s, and $V_{Ts} = (k_B T_s/m_s)^{1/2}$ is the thermal speed of the species s. The collisions of the plasma particles with stationary neutrals tend to damp their collective oscillations and gradually diminish their amplitudes. The oscillations will be slightly damped only when the collision frequency ν_{sn} be smaller than the plasma frequency ω_p, i.e. $\nu_{en},\ \nu_{in},\ \nu_{dn} < \omega_p$.

Dusty plasmas are rather ubiquitous in space [1, 2]. There are a number of well known systems in space, such as interplanetary space, interstellar medium, interstellar or molecular clouds, circum-stellar clouds, comets, solar system, planetary rings, earth's environments, etc. where charged dust particles are always present. The interstellar space (the space between the stars) is filled with a vast medium of gas and dust. The gas content of the interstellar medium continually decreases with time as new generations of stars are formed during the collapse of giant molecular clouds. The collapse and fragmentation of these clouds give rise to the formation of stellar clusters. The presence of dust in interstellar or circum-stellar clouds has been known for a long time (from star reddening and infrared emission). The dust grains in interstellar or circum-stellar clouds are dielectric (ices, silicates, etc.) and metallic (graphite, magnetite, amorphous carbons, etc.). The solar system is also full of dust. The existence of dust in the early solar nebula has long been advocated by the Nobel Laureate Alfvén [3]. The coagulation of the dust grains in the solar nebula would have led to "planetesimal" from where comets and planets have been formed. The physical properties (such as size, mass, density, charge, etc.) of such dust grains vary depending on their origin and surroundings. The origins of the dust grains in the solar system are, for example, micro-meteoroids, space debris, man-made pollution, lunar ejecta, etc.

Dust particles do not only occur in space, but also they occur in laboratory devices, particularly in direct current (dc) and radio-frequency (rf) discharges, plasma processing reactors, fusion plasma devices, solid fuel combustion products, etc. The presence of dust particles in fusion devices has been known for a long time. However, their possible consequences for plasma operation and performances have become a topic of recent interest [4]. The plasma in fusion devices (for example, tokamaks, stellarator, etc.) are more or less contaminated by impurities (dust particles) heavier than the hydrogen isotopes which are the fuel in fusion reactors. The impurities (dust particles) are generated by a number of processes, such as desorption, arcing, sputtering, evaporation and sublimation of thermally overloaded wall material, etc. To know the details of the occurrence of dust in space and laboratories as well as different dusty plasma parameters, we refer to Shukla and Mamun [1], Verheest [2], Winter [4], Bouchoule [5], Hollenstein [6], etc.

As we already mentioned, dust particles, which are invariably immersed in an ambient plasma with radiative background, are not neutral, but are positively or negatively charged depending on on the plasma environment around the dust grains. The important elementary dust grain charging processes are i) interaction of dust grains with gaseous plasma particles, ii) interaction of dust grains with energetic particles (electrons and ions) and iii) interaction of dust grains with photons. When dust grains are immersed in a gaseous plasma, the plasma particles (electrons and ions) are collected by the dust grains which act as probes. The dust grains are, therefore, charged by the collection of the plasma particles flowing onto their surfaces. The dust grain charge q_d is determined by $dq_d/dt = \sum_j I_j$, where j represents the plasma species (electrons and ions) and I_j is the current associated with the species j. At equilibrium the net current flowing onto the dust grain surface becomes zero, i.e. $\sum_j I_{j0} = 0$, where I_{j0} is the equilibrium current. This means that the dust grain surface acquires some potential ϕ_g which is $-2.5k_BT/e$ (where $T = T_e \simeq T_i$) for a hydrogen plasma and $-3.6k_BT/e$ for an oxygen plasma [7]. It turns out that the dust grains immersed in a gaseous plasma are usually negatively charged. When energetic plasma particles (electrons or ions) are incident onto a dust grain surface, they are either backscattered/reflected by the dust grain or they pass through the dust grain material. During their passage they may lose their energy partially or fully. A portion of the lost energy can go into exciting other electrons that in turn may escape from the material. The emitted electrons are known as secondary electrons. The release of these secondary electrons from the dust grain tends to make the grain surface positive. The interaction of photons incident onto the dust grain surface causes photoemission of electrons from the dust grain surface. The dust grains, which emit photoelectrons, may become positively charged. The emitted electrons collide with other dust grains and are captured by some of these grains which may become negatively charged. There are, of course, a number of other dust grain charging mechanisms, namely thermionic emission, field emission, radioactivity, impact ionization, etc. These are significant only in some different special circumstances.

The charged dust grains are not always static, but they can also be mobile. The dynamics of dust grains in space attracted the main stream of interest of space physicists about 20 years ago, when Voyager 1 and 2 passed Saturn and sent back pictures of mysterious dark spokes sweeping around the B-ring [8, 9]. It had then been independently proposed by Hill and Mendis [10] and Goertz and Morfill [11] that the spokes might be charged dust and sculptured by electrostatic forces. The dynamical patterns of charged dust particles in interplanetary space observed by Voyager 1 and 2 also suggested to account for the combined effects of electromagnetic and gravitational forces acting on the dust particles. On the other hand, in laboratory plasmas, dust particles, which are subjected to various forces, often accumulate near the plasma boundaries (walls) and cause contamination to substrates and wafers [12]. It is, therefore, crucial to understand the behavior of macroscopic particles under the action of various forces, such as gravitational force, electric force, ion drag force, neutral drag force, thermophoretic force, etc., in order to control the dust transport.

Therefore, the physics of charged dust particles has become an outstanding and challenging research topic not only because dust particles are ubiquitous in most space [13–26] and laboratory plasmas [4–6], but also because it has introduced a great variety of new phenomena associated with waves and instabilities [27–37] in un-magnetized weakly coupled dusty plasma, and it plays a vital role in understanding different interesting phenomena in astrophysical and space environments, such as interplanetary space, interstellar medium, interstellar or molecular clouds, comets, planetary rings, earth's environments, etc. [13–26].

The charged dust grains in a plasma does not only modify the existing plasma wave spectra [27–29] but also introduces a number of new novel eigenmodes in dusty plasmas. The most important classes of new novel acoustic waves, which are experimentally observed in un-magnetized weakly coupled duty plasmas, are dust-ion acoustic (DIA) and dust-acoustic (DA).

Shukla and Silin [30] have first theoretically shown that due to the conservation of equilibrium charge density $n_{e0} + n_{d0}Z_{d0} = n_{i0}$ and the strong inequality $n_{e0} \ll n_{i0}$ a dusty plasma (with negatively charged static dust grains) supports low-frequency DIA waves with phase speed much smaller (larger) than electron (ion) thermal speed. The dispersion relation (a relation between the wave frequency ω and the wave number k) of the linear DIA waves is [30]

$\omega^2 = (n_{i0}/n_{e0})k^2C_i^2/[1 + k^2\lambda_{De}^2(1 + T_in_{e0}/T_en_{i0})]$, where $C_i = (k_BT_e/m_i)^{1/2}$ is the ion-acoustic speed. When we consider a long wavelength limit (viz. $k\lambda_{De} \ll 1$), and the dispersion relation for the DIA waves becomes $\omega = (n_{i0}/n_{e0})^{1/2}kC_i$. This form of spectrum is similar to the usual ion-acoustic wave spectrum [38] for a plasma with $n_{i0} = n_{e0}$ and $T_i \ll T_e$. However, in dusty plasmas we usually have $n_{i0} \gg n_{e0}$ and $T_i \simeq T_e$. Therefore, a dusty plasma cannot support the usual ion-acoustic waves, but can do the DIA waves of Shukla and Silin [30]. The phase speed ω/k of the DIA waves is larger than C_i because of $n_{i0} \gg n_{e0}$ for negatively charged dust grains. The increase in the phase velocity is attributed to the electron density depletion in the background plasma, so that the electron Debye-radius becomes larger. As a result, there appears a stronger space charge electric field which is responsible for the enhanced phase velocity of the DIA waves. The DIA waves have been observed in laboratory experiments [31, 32].

Rao *et al.* [33] theoretically predicted the existence of extremely low phase velocity (in comparison with the electron and ion thermal speeds) DA waves in an un-magnetized dusty plasma whose constituents are an inertial charged dust fluid and Boltzmann ions and electrons. Thus, in the DA waves the inertia is provided by the dust particle mass and the restoring force comes from the pressures of electrons and ions. The dispersion relation for the DA waves with the phase speed ω/k much smaller (larger) than the ion (dust) thermal speed is given by [33] $\omega = kC_d(1 + k^2\lambda_D^2)^{-1/2}$, where $C_d = \omega_{pd}\lambda_D$ is the dust-acoustic speed. It is obvious that one cannot obtain the DA mode without the consideration of the dust dynamics. The theoretical prediction of Rao *et al.* [33] has been conclusively verified by a number of laboratory experiments [34, 35].

The linear properties of the DIA and DA waves in dusty plasmas are now well understood and have been reported by a large number of regular and review articles or books during last few years [1, 2, 30–37, 39–41]. The linear theory is valid only when the wave amplitude is so small that one may neglect the nonlinearities. However, there are numerous processes via which unstable modes can saturate and attain large amplitudes. When the amplitudes of the waves are sufficiently large, nonlinearities can no longer be ignored. The nonlinearities come from the harmonic generation involving the fluid advection, nonlinear Lorentz force, trapping of particles in the wave potential, ponderomotive force, etc. The nonlinearities in plasmas contribute to the localization of waves, leading to different types of interesting nonlinear coherent structures (viz. solitary waves, shock waves, double layers, vortices, etc.) which are important from both theoretical and experimental points of view, and have received a great deal of attention for understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas.

Recently, a number of theoretical attempts have been made in order to study the properties of DIA [42–46] and, DA [1, 33, 46–57] solitary waves in unmagnetized dusty plasmas. To the best of our knowledge, there is no regular/review article where a rigorous theoretical investigation on the basic features and the underlying physics of small as well as arbitrary amplitude electro-acoustic DIA and DA solitary waves is systematically presented. Therefore, in our present article, we have tried to provide a detailed information about the basic features and the underlying physics of small as well as arbitrary amplitude electro-acoustic DIA and DA solitary waves in un-magnetized dusty plasmas.

The manuscript is organized as follows. We first consider an un-magnetized dusty plasma with static dust, and rigorously investigate the basic features and underlying physics of small as well as arbitrary amplitude DIA solitary waves in Sec. II. We then consider an un-magnetized dusty plasma with mobile dust, and rigorously investigate the basic features and underlying physics of small as well as arbitrary amplitude DA solitary waves in Sec. III. We, finally, provide a brief discussion in Sec. IV.

II. STATIC DUST: DIA SOLITARY WAVES

We confine ourself, in this section, to the DIA solitary waves in an unmagnetized dusty plasma in which dust particles are stationary and provide only the background charge-neutrality. The basic equations governing the dynamics of one-dimensional DIA waves, whose phase speed V_p is much larger (smaller) than the electron (ion) thermal speed, viz. $V_{Ti} \ll V_p \ll V_{Te}, V_{Ti}$, can be expressed in terms of normalized variables as

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu \exp(\phi) - n_i + (1-\mu), \tag{3}$$

where n_i is the ion number density normalized by its equilibrium value n_{i0}, u_i is the ion fluid velocity normalized by the ion-acoustic speed C_i, ϕ is the ion-acoustic wave potential normalized by $K_B T_e/e$, and $\mu = n_{e0}/n_{i0}$. We have neglected the ion thermal pressure term and assumed a Boltzmannean electron density response, which are valid as long as $V_{Ti} \ll V_p \ll V_{Te}$. The space variable x is normalized by the modified electron Debye-radius $\lambda_{Dem} = (K_B T_e/4\pi n_{i0} e^2)^{1/2}$ and the time variable t is normalized by the ion plasma period $\omega_{pi}^{-1} = (m_i/4\pi n_{i0} e^2)^{1/2}$.

To obtain a solitary wave solution, we assume that all the dependent variables depend on a single independent variable $\xi = x - Mt$, where M is the Mach number (the velocity of the solitary wave normalized by the ion-acoustic speed C_i). Considering the steady state condition, i.e. $\partial/\partial t = 0$, we can write Eqs. (1)–(3) as

$$M\frac{\partial n_i}{\partial \xi} - \frac{\partial}{\partial \xi}(n_i u_i) = 0, \tag{4}$$

$$M\frac{\partial u_i}{\partial \xi} - u_i \frac{\partial u_i}{\partial \xi} = \frac{\partial \phi}{\partial \xi}, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = \mu \exp(\phi) - n_i + (1-\mu). \tag{6}$$

Now, under the appropriate boundary conditions, viz. $\phi \to 0$, $u \to 0$, and $n \to 1$ at $\xi \to \pm\infty$, Eqs. (4) and (5) can be integrated to give

$$n_i = \frac{1}{\sqrt{1 - 2\phi/M^2}}, \tag{7}$$

Substituting Eq. (7) into Eq. (6) and integrating the resultant equation, we have an energy integral [58, 59]

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{8}$$

where the pseudo-potential $V(\phi)$ is given by

$$V(\phi) = \mu[1 - \exp(\phi)] - (1 - \mu)\phi + M^2\left(1 - \sqrt{1 - \frac{2\phi}{M^2}}\right). \tag{9}$$

It is obvious from Eq. (9) that $V(\phi) = dV/d\phi = 0$ at $\phi = 0$. Therefore, solitary wave solutions of Eq. (9) exist if (i) $(d^2V/d\phi^2)_{\phi=0} < 0$ so that the fixed point at the origin is unstable and (ii) $(d^3V/d\phi^3)_{\phi=0} > (<) 0$ for solitary waves with $\phi > (<) 0$. The nature of these solitary waves, whose amplitude tends to zero as the Mach number M tends to its critical value, can be found by expanding the pseudo-potential $V(\phi)$ to third order in a Taylor series in ϕ. The critical Mach number is that which corresponds to the vanishing of the quadratic term. At the same time, if the cubic term is negative, there is a potential well on the negative side and if the cubic term is positive, there is a potential well on the positive side. Therefore, by expanding the pseudo-potential $V(\phi)$ around the origin, the critical Mach number at which the second derivative changes sign can be found as $M_c = \mu^{-1/2}$. At this critical value of M the cubic term of $V(\phi)$ can be expressed as $(3\mu - 1)\mu^{-1/2}/2$ which reveals that the cubic term is positive (negative) for $\mu > (<)1/3$. This means that depending on the value of μ, DIA solitary waves with both positive and negative potential can exist with $\phi > (<)0$ when $\mu > (<)1/3$. Since in most space and laboratory dusty plasmas $\mu < 1/3$, unlike the usual ion-acoustic solitary waves (associated with a positive potential) in a two component electron-ion plasma [38], DIA solitary waves have a new feature in that these are associated with a negative potential which is due to the presence of negatively charged dust grains.

We have discussed the properties of the DIA solitary waves by assuming a planar geometry, constant dust grain charge, cold ion fluid. However, it is shown that the effects of nonplanar geometry, dust grain charge fluctuations, and finite ion fluid temperature introduce new features or significantly modify the properties of DIA solitary waves [43–45]. We now investigate the effects of nonplanar geometry, dust grain charge fluctuations, and finite ion fluid temperature on the properties of the DIA solitary waves.

A. Nonplanar Geometry

The nonlinear dynamics of the DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed (viz. $V_{Ti} \ll V_p \ll V_{Te}$), in nonplanar cylindrical and spherical geometries is governed by

$$\frac{\partial n_i}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_i u_i) = 0, \tag{10}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} = -\frac{\partial \phi}{\partial r}, \tag{11}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \phi}{\partial r}\right) = \mu \exp(\phi) - n_i + (1-\mu), \tag{12}$$

where $\nu = 0$ for a one-dimensional geometry and $\nu = 1$ (2) for a non-planar cylindrical (spherical) geometry, We assumed here a constant dust grain charge (i.e. $Z_d = Z_{d0}$).

To investigate ingoing solutions of Eqs. (10)–(12), we introduce stretched coordinates [60, 61] $\zeta = -\epsilon^{1/2}(r + v_0 t)$ and $\tau = \epsilon^{3/2} t$, expand n_i, u_i, and ϕ in a power series of ϵ

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{13}$$

$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{14}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{15}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, Eqs. (10)–(15), give $n_i^{(1)} = -u_i^{(1)}/v_0$, $u_i^{(1)} = -\phi^{(1)}/v_0$ and $v_0 = \mu^{-1/2}$. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_i^{(1)}}{\partial \tau} - v_0 \frac{\partial n_i^{(2)}}{\partial \zeta} - \frac{\partial u_i^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}\left[n_i^{(1)} u_i^{(1)}\right] - \frac{\nu u_i^{(1)}}{v_0 \tau} = 0, \tag{16}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - v_0 \frac{\partial u_i^{(2)}}{\partial \zeta} - u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} = 0, \tag{17}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} - \mu \phi^{(2)} + n_i^{(2)} - \frac{1}{2}\mu \left[\phi^{(1)}\right]^2 = 0. \tag{18}$$

Combining Eqs. (16)–(18) we deduce a modified Kortweg-de Vries (KdV) equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{\nu}{2\tau}\phi^{(1)} + A_g \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B_g \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \tag{19}$$

where $A_g = (3\mu - 1)/\sqrt{2\mu}$ and $B_g = 1/2\mu^{3/2}$.

We have already mentioned that $\nu = 0$ corresponds to a one-dimensional geometry. Thus, for a one-dimensional geometry ($\nu = 0$) and for a moving frame moving with a speed u_0, the stationary solitary wave solution of Eq. (19) is

$$\phi^{(1)}(\nu = 0) = \left(\frac{3u_0}{A_g}\right) \text{sech}^2 \left[\sqrt{\frac{u_0}{4B_g}}(\zeta - u_0 \tau)\right]. \tag{20}$$

It is obvious from Eq. (20) that for $\mu > (<)\ 1/3$, a dusty plasma supports compressive (rarefactive) DIA solitary waves which are associated with a positive (negative) potential. We now turn to Eq. (19) with the term $(\nu/2\tau)\phi^{(1)}$ which is due to the effect of the nonplanar (cylindrical or spherical) geometry. An exact analytic solution of Eq. (19) is not possible.

Therefore, we have numerically solved Eq. (19) and have studied the effects of cylindrical and spherical geometries on time-dependent DIA solitary waves. The initial condition that we have used in our numerical analysis is in the form of the stationary solution of Eq. (20) without the term $(\nu/2\tau)\phi$, i.e. in the form $\phi^{(1)} = (3u_0/A_g)\text{sech}^2(\zeta/\sqrt{4B_g})$.

The numerical solutions of Eq. (19) reveal [43] that for a large (negative) value of τ the spherical and cylindrical solitary waves are similar to one-dimensional solitary waves. This is because for a large value of τ the term $(\nu/2\tau)\phi^{(1)}$, which is due to the effect of the cylindrical or spherical geometry, is no longer dominant. However, as the value of τ decreases, the term $(\nu/2\tau)\phi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves differ from one-dimensional solitary waves. It is found that as the value of τ decreases, the amplitude of these localized pulses increases. It is also found that the amplitude of cylindrical solitary waves is larger than that of the one-dimensional solitary waves but smaller than that of the spherical ones.

B. Dust Charge Fluctuation

We study the propagation of the DIA solitary waves in an unmagnetized dusty plasma where the dust charge is not constant but varies with space and time. The nonlinear dynamics of one-dimensional DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed, is governed by Eq. (1) and

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{21}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu \exp(\phi) - n_i + (1-\mu) z_d, \tag{22}$$

where z_d is the number of electrons residing onto the dust grain surface normalized by its equilibrium value Z_{d0}. We note that z_d is not constant but varies with space and time. Thus, Eqs. (1), (21) and (22) are completed by the normalized dust grain charging equation [45]

$$\eta \frac{\partial z_d}{\partial t} = \mu\beta \exp(\phi - \alpha z_d) - \beta_i n_i u_i \left(1 + \frac{2\alpha z_d}{u_i^2}\right), \tag{23}$$

where $\eta = \sqrt{\alpha m_e(1-\mu)/2m_i}$, $\alpha = Z_{d0}e^2/k_B T_e r_d$, $\beta = (r_d/a)^{3/2}$, $a = n_{d0}^{-1/3}$, and $\beta_i = \beta\sqrt{\pi m_e/8m_i}$. We note that at equilibrium $\mu\beta\exp(-\alpha) = \beta_i u_0(1 + 2\alpha/u_0^2)$, where u_0 is the ion streaming speed normalized by C_i.

To study small but finite amplitude DIA solitary waves, we first introduce stretched coordinates [60] $\xi = \epsilon^{1/2}(x - v_0 t)$, $\tau = \epsilon^{3/2} t$, and $\eta = \epsilon\eta_0$, where ϵ is the expansion parameter, measuring the amplitude of the wave or the weakness of the wave dispersion. We then expand n_i, u_i, ϕ and z_d in a power series of ϵ

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{24}$$

$$u_i = u_0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{25}$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots, \tag{26}$$

$$z_d = 1 + \epsilon z_d^{(1)} + \epsilon^2 z_d^{(2)} + \cdots. \tag{27}$$

We develop the equations in powers of ϵ. To the lowest order in ϵ, Eqs. (1), (21)–(27) give

$$w_0 n_i^{(1)} = u_i^{(1)}, \tag{28}$$

$$w_0 u_i^{(1)} = \phi^{(1)}, \tag{29}$$

$$0 = \mu\phi^{(1)} - n_i^{(1)} + (1-\mu) z_d^{(1)}, \tag{30}$$

$$0 = \beta_e \phi^{(1)} - \alpha u_\beta z_d^{(1)} - \beta_i u_1 u_i^{(1)} - u_0 \beta_i u_2 n_i^{(1)}, \tag{31}$$

where $w_0 = v_0 - u_0$, $u_1 = 1 - 2\alpha/u_0$, $u_2 = 1 + 2\alpha/u_0^2$, $u_\beta = \beta_e + 2\beta_i/u_0$, and $\beta_e = \mu\beta\exp(-\alpha)$. Now, substituting $n_i^{(1)}$, $u_i^{(1)}$, and $z_d^{(1)}$ [obtained from Eqs. (28), (29) and (30)] into Eq. (31), we obtain the dispersion relation

$$a w_0^2 - b w_0 - c = 0, \tag{32}$$

where

$$a = \mu + \frac{(1-\mu)\beta_e}{\alpha u_\beta}, \tag{33}$$

$$b = \frac{(1-\mu) u_1 \beta_i}{\alpha u_\beta}, \tag{34}$$

$$c = 1 + \frac{(1-\mu) u_2 u_0 \beta_i}{\alpha u_\beta}. \tag{35}$$

To the next higher order in ϵ, from Eqs. (1) and (21)–(23) we obtain a set of equations

$$\frac{1}{w_0^2}\frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{2}{w_0^3}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} = 0, \tag{36}$$

$$\frac{1}{w_0}\frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{1}{w_0^2}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} = \frac{\partial \phi^{(2)}}{\partial \xi}, \tag{37}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \mu\phi^{(2)} + \frac{1}{2}\mu\left[\phi^{(1)}\right]^2 - n_i^{(2)} + (1-\mu) z_d^{(2)}, \tag{38}$$

$$0 = \beta_e \phi^{(2)} - \alpha u_\beta z_d^{(2)} - \beta_i u_1 u_i^{(2)} - u_0 \beta_i u_2 n_i^{(2)} + \beta_1 \phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi}, \tag{39}$$

where

$$\beta_1 = \beta_e[1 + (\alpha\beta_0 - 2)\alpha\beta_0] - \frac{2\beta_i}{w_0^3}\left[1 + \frac{w_\alpha}{u_0^3}\right], \tag{40}$$

$$\beta_0 = \frac{1}{\alpha u_\beta}\left[\beta_e - \frac{\beta_i w_2}{w_0} + \frac{2\alpha\beta_i}{w_0 u_0}\left(1 - \frac{1}{w_0}\right)\right], \tag{41}$$

$w_1 = 1 - u_0/w_0$, $w_2 = 1 + u_0/w_0$, and $w_\alpha = 2\alpha w_0 w_1 (1 - \beta_0 w_0 u_0)$. Now, combining equations Eqs. (36)–(39) we deduce a KdV equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A_c \phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B_c \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{42}$$

where

$$A_c = \frac{3c + bw_0 - \beta_2 w_0^4}{w_0(2c + bw_0)}, \tag{43}$$

$$B_c = \frac{w_0^3}{2c + bw_0}, \tag{44}$$

$$\beta_2 = \mu + \frac{\beta_1(1-\mu)}{\alpha u_\beta}. \tag{45}$$

The stationary solution of the KdV equation (42) is obtained by transforming the independent variables ξ to $\zeta = \xi - U_0\tau$, where U_0 is a constant speed normalized by C_i, and imposing the appropriate boundary conditions for localized perturbations, viz. $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\zeta \to 0$, $d^2\phi^{(1)}/d\zeta^2 \to 0$ at $\zeta \to \pm\infty$. Thus, the stationary solitary wave solution of Eq. (42) is

$$\phi^{(1)} = \psi \;\mathrm{sech}^2\left[(\xi - U_0\tau)/\Delta\right], \tag{46}$$

where $\psi = 3U_0/A_c$ and $\Delta = \sqrt{4B_c/U_0}$ represent the amplitude and the width of the solitary waves, respectively. It is obvious from (46) that there exists compressive (rarefactive) solitary waves if $A_c > 0$ ($A_c < 0$). We have numerically analyzed ψ and Δ for the parameters corresponding to space dusty plasma parameters [1, 16]: $n_{d0} = 10^{-7}$ cm^{-3}, $r_d = 1$ μm, $k_BT_e = 50$ eV, and $Z_{d0} = 10^3$, which correspond to $\alpha = 0.0288$ and $\beta = 3 \times 10^{-10}$, as well as laboratory dusty plasma parameters [31, 32]: $n_{d0} = 10^5$ cm^{-3}, $r_d = 5$ μm, $k_BT_e = 0.2$ eV and $Z_{d0} = 10^3$, which correspond to $\alpha = 1.44$ and $\beta = 3 \times 10^{-4}$. We found that ψ is always positive. This means that our present dusty plasma model can support only compressive solitary waves (solitary waves with $\phi > 0$).

C. Ion Fluid Temperature

To examine the effects of ion fluid temperature on one-dimensional DIA waves, we start with Eqs. (1), (3) and

$$\frac{\partial u_i}{\partial t} + u_i\frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x}, \tag{47}$$

$$\frac{\partial p_i}{\partial t} + u_i\frac{\partial p_i}{\partial x} + 3p_i\frac{\partial u_i}{\partial x} = 0, \tag{48}$$

where $\sigma_i = T_i/T_e$ and p_i is the ion pressure normalized by $n_{i0}k_BT_i$.

Again using $\xi = x - Mt$ and $\partial/\partial t = 0$, we can write Eqs. (1) and (3) as Eqs. (4) and (6), and we can write Eqs. (47) and (48) as

$$M\frac{\partial u_i}{\partial \xi} - u_i\frac{\partial u_i}{\partial \xi} - \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial \xi} = \frac{\partial \phi}{\partial \xi}, \tag{49}$$

$$M\frac{\partial p_i}{\partial \xi} - u_i\frac{\partial p_i}{\partial \xi} - 3p_i\frac{\partial u_i}{\partial \xi} = 0, \tag{50}$$

Now using Eqs. (4) and (50) and using the appropriate boundary conditions, viz. $u_i \to 0$, $p_i \to 1$ and $n_i \to 1$ at $\xi \to \pm\infty$, we have

$$p_i = n_i^3. \tag{51}$$

On the other hand, using Eqs. (4) and (49) we obtain

$$M\frac{\partial u_i}{\partial \xi} - u_i\frac{\partial u_i}{\partial \xi} - \sigma_i\frac{\partial p_i}{\partial \xi} + \frac{\sigma_i}{M}u_i\frac{\partial p_i}{\partial \xi} = \frac{\partial \phi}{\partial \xi}. \tag{52}$$

Multiplying Eq. (50) by σ_i/M one can write

$$\sigma_i\frac{\partial p_i}{\partial \xi} - \frac{\sigma_i}{M}u_i\frac{\partial p_i}{\partial \xi} - 3p_i\frac{\sigma_i}{M}\frac{\partial u_i}{\partial \xi} = 0. \tag{53}$$

Now, using Eqs. (52) and (53), one can get

$$3\sigma_i\frac{\partial p_i}{\partial \xi} - 3\frac{\sigma_i}{M}\frac{\partial}{\partial \xi}(p_i u_i) - 2M\frac{\partial u_i}{\partial \xi} + 2u_i\frac{\partial u_i}{\partial \xi} + 2\frac{\partial \phi}{\partial \xi} = 0. \tag{54}$$

The integration of this equation yields

$$3\frac{\sigma_i}{M}p_i u_i - 3\sigma_i(p_i - 1) + 2Mu_i - u_i^2 - 2\phi = 0, \tag{55}$$

where we have again used the appropriate boundary conditions, viz. $\phi \to 0$, $u_i \to 0$, $p_i \to 1$ and $n_i \to 1$ at $\xi \to \pm\infty$.

Now, substituting u_i [obtained from Eq. (4)] and p_i [obtained from Eq. (51)] into Eq. (55) we obtain a quadratic equation for n_i^2

$$3\sigma_i n_i^4 - (3\sigma_i + M^2 - 2\phi)n_i^2 + M^2 = 0. \tag{56}$$

The solution of Eq. (56) is given by

$$n_i = \frac{\sigma_{i1}}{\sqrt{2}\sigma_{i0}}\left[1 - \frac{2\phi}{M^2\sigma_{i1}^2} - \sqrt{\left(1 - \frac{2\phi}{M^2\sigma_{i1}^2}\right)^2 - 4\frac{\sigma_{i0}^2}{\sigma_{i1}^4}}\right]^{1/2}, \tag{57}$$

where $\sigma_{i0} = \sqrt{3\sigma_i/M^2}$ and $\sigma_{i1} = \sqrt{1+\sigma_{i0}^2}$. Substituting Eq. (57) into Eq. (6) and integrating the resultant equation, we have an energy integral

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{58}$$

where the pseudo-potential $V(\phi)$ is given by

$$\begin{aligned} V(\phi) &= -\mu\exp(\phi) - -(1-\mu)\phi \\ &\quad -\frac{M^2\sigma_{i1}}{\sqrt{2}}\left[1 - \frac{2\phi}{M^2\sigma_{i1}^2} + \sqrt{\left(1 - \frac{2\phi}{M^2\sigma_{i1}^2}\right)^2 - 4\frac{\sigma_{i0}^2}{\sigma_1^4}}\right]^{1/2} \\ &\quad -\frac{2\sqrt{2}\sigma}{\sigma_{i1}^3}\left[1 - \frac{2\phi}{M^2\sigma_{i1}^2} + \sqrt{\left(1 - \frac{2\phi}{M^2\sigma_{i1}^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{-3/2} + C_1. \end{aligned} \tag{59}$$

Here C_1 is the integration constant which we will choose in such a manner that $V(\phi) = 0$ at $\phi = 0$. It is important to mention that in our study the condition for ion density to be real, $|1 - 2\phi/M^2\sigma_1^2| \geq 2\sigma_0/\sigma_1^2$, must always be satisfied.

To examine how the ion fluid temperature (σ_i) modifies the properties of the arbitrary amplitude DIA solitary waves, we have analyzed the pseudo-potential $V(\phi)$ given by Eq. (59), and found that as we increase σ_i, (i) we need higher values of the critical Mach number (corresponding to a lower value of μ) in order to have the DIA solitary waves, and (ii) the amplitude of both the positive and negative solitary waves decreases, but their width increases. We have also found the same results by numerical analysis of the energy integral given by Eq. (58).

III. MOBILE DUST: DA SOLITARY WAVES

We, in this section, consider the dynamics of negatively charged dust particles in an unmagnetized dusty plasma, and study the basic features and the underlying physics of the DA solitary waves. The nonlinear dynamics of one dimensional DA waves, whose phase speed is much smaller (larger) than (dust thermal speed) electron and ion thermal speeds, viz. $V_{Td} \ll V_p \ll V_{Te}, V_{Ti}$, is governed by [33, 50]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial z}(n_d u_d) = 0, \tag{60}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial z} = \frac{\partial \varphi}{\partial z}, \tag{61}$$

$$\frac{\partial^2 \varphi}{\partial z^2} = n_d + \mu_e \exp(\sigma_i \varphi) - \mu_i \exp(-\varphi), \tag{62}$$

where n_d is the particle number density normalized by its equilibrium value n_{d0}, u_d is the dust fluid velocity normalized by $C_d = (Z_{d0} k_B T_i / m_d)^{1/2}$ and φ is the electrostatic wave potential normalized by $k_B T_i / e$, $\mu_e = n_{e0}/Z_{d0} n_{d0} = \mu/(1-\mu)$, and $\mu_i = n_{i0}/Z_{d0} n_{d0} = 1/(1-\mu)$. The time ($t$) and space ($z$) variables are in units of the dust plasma period ${\omega_{pd}}^{-1} = (m_d / 4\pi n_{d0} Z_{d0}^2 e^2)^{1/2}$ and the Debye-radius $\lambda_{Dim} = (k_B T_i / 4\pi Z_{d0} n_{d0} e^2)^{1/2}$, respectively.

To study arbitrary amplitude DA solitary waves in such a dusty plasma system, we assume that all the dependent variables depend on a single independent variable $\xi = z - Mt$ and reduce Eq. (60)–(62) to an energy integral (by following the mathematical procedure presented in Sec. II)

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = 0, \tag{63}$$

where the pseudo-potential potential $V(\varphi)$ is given by [50]

$$V(\varphi) = \mu_i[1 - \exp(-\varphi)] + \frac{\mu_e}{\sigma_i}[1 - \exp(\sigma_i \varphi)] + M^2 \left[1 - \left(1 + \frac{2\varphi}{M^2}\right)^{1/2}\right]. \tag{64}$$

It is obvious from Eq. (64) that $V(\varphi) = dV/d\varphi = 0$ at $\varphi = 0$. Therefore, solitary wave solutions of Eq. (63) exist if (i) $(d^2V/d\varphi^2)_{\varphi=0} < 0$ so that the fixed point at the origin is unstable and (ii) $(d^3V/d\varphi^3)_{\varphi=0} > (<)\ 0$ for solitary waves with $\varphi > (<)\ 0$. The nature of these solitary waves, whose amplitude tends to zero as the Mach number M tends to its critical value, can be found by expanding the pseudo-potential $V(\varphi)$ to third order in a Taylor series in φ. The critical Mach number is that which corresponds to the vanishing of the quadratic term. At the same time, if the cubic term is negative, there is a potential well on the negative side, and if the cubic term is positive, there is a potential well on the positive side. Therefore, by expanding the pseudo-potential $V(\varphi)$ around the origin, the critical Mach number at which the second derivative changes sign can be found as $M_c = \sqrt{(1-\mu)/(1+\sigma_i\mu)}$. We have numerically calculated the critical Mach number M_c for different values of μ σ_i, and observed [50] that the critical Mach number increases with σ_i, but decreases with μ. At the critical value of M the cubic term of $V(\varphi)$ can be expressed as $-[1+(3+\sigma_i\mu)\sigma_i\mu+\mu(1+\sigma_i^2)/2]/[3(1-\mu)^2]$. This clearly reveals that the cubic term is always negative for any value of σ_i and μ, i.e. DA solitary waves with $\varphi < 0$ can only exist.

It is of interest to examine whether or not there exists an upper limit of M for which DA solitary waves can exist. This upper limit of M can be found by the condition $V(\varphi_c) \geq 0$, where $\varphi_c = -M^2/2$ is the minimum value of φ for which the dust number density n_d is real. Thus, the upper limit of M is that maximum value of M for which $S_m \geq 0$, where $S_m = \mu_i + \mu_e/\sigma_i + M^2 - \mu_i \exp(M^2/2) - (\mu_e/\sigma_i)\exp(-\sigma_i M^2/2)$. By numerical analysis of this expression, we have estimated the variation of S_m with M for different values of μ, and found that as we increase μ, the upper limit of M decreases. We note that for $\sigma_i = 0.05$ and $\mu = 0.1$, there exists DA solitary waves with $\varphi < 0$ for $0.95 < M < 1.52$. We have also numerically analyzed $V(\varphi)$ and have found the same results [50] that for $\sigma_i = 0.05$ and $\mu = 0.1$ there exists a potential well on the negative φ-axis for $0.95 < M < 1.52$, i.e. there exists DA solitary waves with $\varphi < 0$ for $0.95 < M < 1.52$.

We have discussed the properties of the DA solitary waves in an unmagnetized dusty plasma by assuming a planar geometry, Maxwellian ion distribution and cold dust fluid. However, it can be shown shown that the effects of non-planar geometry, non-Maxwellian ion distribution introduce new features or significantly modify the properties of the DA solitary waves [48, 51, 52, 56, 57]. We now investigate the effects of non-planar geometry, non-Maxwellian ion distributions, finite dust-fluid temperature, external magnetic field on the properties of the DA solitary waves.

A. Nonplanar Geometry

It is well known that in laboratory devices one may encounter multi-dimensional DA solitary waves. We are, therefore, interested in examining radially ingoing DA solitary waves in nonplanar (cylindrical and spherical) geometries. The dynamics of low phase velocity nonlinear DA waves in cylindrical and spherical geometries are governed by [51]

$$\frac{\partial n_d}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_d u_d) = 0, \tag{65}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial r} = \frac{\partial \varphi}{\partial r}, \tag{66}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \varphi}{\partial r}\right) = n_d + \mu_e \exp(\sigma_i \varphi) - \mu_i \exp(-\varphi), \tag{67}$$

where $\nu = 1, 2$ for cylindrical and spherical geometries, respectively. The space variable r is normalized by the Debye-radius λ_{Ddm}.

To investigate ingoing solutions of Eqs. (65)–(67), we introduce the stretched coordinates [61] $\zeta = -\epsilon^{1/2}(r + v_0 t)$ and $\tau = \epsilon^{3/2} t$, expand n_d, u_d and φ in powers of ϵ, and develop equations in various powers of ϵ. Thus, performing all the mathematical steps as we did in the case of cylindrical or spherical dust ion-acoustic solitary waves, one can deduce a modified Kortweg-de Vries (KdV) equation

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + \frac{\nu}{2\tau}\varphi^{(1)} - A_{gd}\varphi^{(1)}\frac{\partial \varphi^{(1)}}{\partial \zeta} + B_{gd}\frac{\partial^3 \varphi^{(1)}}{\partial \zeta^3} = 0, \tag{68}$$

where

$$A_{gd} = \frac{v_0^3}{(1-\mu)^2}[1 + (3 + \sigma_i \mu)\sigma_i \mu + \frac{1}{2}\mu(1 + \sigma_i^2)], \tag{69}$$

and $B_{gd} = v_0^3/2$. If we compare Eq. (68) with equation (13) of Mamun [50], it is obvious that the term $(\nu/2\tau)\varphi^{(1)}$ in Eq. (68) is due to the effect of the cylindrical or spherical geometry. We numerically solved Eq. (68) and studied the effects of cylindrical and spherical geometries on time-dependent DA solitary waves. The numerical results [51] reveal that for a large value of τ (e.g. $\tau = -31.6$) the spherical and cylindrical solitary waves are similar to one-dimensional solitary waves. This is because for a large value of τ the term $(\nu/2\tau)\varphi^{(1)}$, which is due to the effect of the cylindrical or spherical geometry, is no longer dominant. However, as the value of τ decreases, the term $(\nu/2\tau)\varphi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves differ from one-dimensional solitary waves. It is also found that as the value of τ decreases, the amplitude of these localized pulses increases.

B. Trapped Ion Distribution

It is well known [62–65] that the electron and ion distribution functions can be significantly modified in the presence of large amplitude waves that are excited by the two-stream instability [67]. Accordingly, the electron and ion number densities depart from a Boltzmann distribution when a phase space vortex distribution appears in a plasma. For the DA waves, the ion trapping in the wave potential is of our interest. To study the effects of non-isothermal ions on the DA solitary waves, we consider the trapped or vortex-like [62, 63] ion distribution $f_i = f_{if} + f_{it}$, where

$$f_{if} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(v_i^2 + 2\varphi)\right] \tag{70}$$

for $|v_i| > \sqrt{-2\varphi}$ and

$$f_{it} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\sigma_{it}(v_i^2 + 2\varphi)\right] \tag{71}$$

for $|v_i| \leq \sqrt{-2\varphi}$. We note that the ion distribution function, as prescribed above, is continuous in velocity space and satisfies the regularity requirements for an admissible BGK solution [58]. Here the ion velocity v_i in Eqs. (70) and (71) is normalized by the ion thermal speed V_{Ti}, and $\sigma_{it} = T_i/T_{it}$, the ratio of the free ion temperature T_i to the trapped ion temperature T_{it}, is a parameter determining the number of trapped ions. Integrating the ion distribution functions over velocity space we readily obtain the ion number density n_i as [64]

$$n_i = I(-\varphi) + \frac{1}{\sqrt{\sigma_{it}}} \exp(-\sigma_{it}\varphi)\ \mathrm{erf}(\sqrt{-\sigma_{it}\varphi}) \tag{72}$$

for $\sigma_{it} > 0$ and

$$n_i = I(-\varphi) + \frac{1}{\sqrt{\pi|\sigma_{it}|}} W_D(\sqrt{\sigma_{it}\varphi}) \tag{73}$$

for $\sigma_{it} < 0$, where

$$I(z_0) = [1 - \mathrm{erf}(\sqrt{z_0})]\exp(z_0), \tag{74}$$

$$\mathrm{erf}(z_0) = \frac{2}{\sqrt{\pi}} \int_0^{z_0} \exp(-y^2)dy, \tag{75}$$

$$W_D(z_0) = \exp(-z_0^2) \int_0^{z_0} \exp(y^2)dy. \tag{76}$$

If we expand n_i in the small amplitude limit (viz. $\varphi \ll 1$) and keep terms up to φ^2, it is found that n_i is the same for both $\sigma_{it} < 0$ and $\sigma_{it} > 0$. It is expressed as

$$n_i \simeq 1 - \varphi - \frac{4(1-\sigma_{it})}{3\sqrt{\pi}}(-\varphi)^{3/2} + \frac{1}{2}\varphi^2. \tag{77}$$

We now follow the reductive perturbation technique of Schamel [63] and construct a weakly nonlinear theory for DA solitary waves by introducing the stretched coordinates $\zeta = \epsilon^{1/4}(z - v_0 t)$ and $\tau = \epsilon^{3/4}t$. Using Eqs. (60), (62) and (70) with the replacement of $\mu_i \exp(-\varphi)$ by right hand side of Eq. (77), and applying the reductive perturbation technique of Schamel [63] we can derive

$$\frac{\partial\varphi^{(1)}}{\partial\tau} + a_t\sqrt{-\varphi^{(1)}}\frac{\partial\varphi^{(1)}}{\partial\zeta} + b_s\frac{\partial^3\varphi^{(1)}}{\partial\zeta^3} = 0, \tag{78}$$

where

$$a_t = \frac{v_0^3(1-\sigma_{it})}{\sqrt{\pi}(1-\mu)}. \tag{79}$$

Equation (78) is a modified KdV equation exhibiting a stronger nonlinearity.

The stationary soliton-like solution of the modified KdV equation (78) can be obtained by transforming the space variable ζ to $\eta = \zeta - u_0\tau$ and by imposing the appropriate boundary conditions, viz. $\varphi \to 0$, $d\varphi^{(1)}/d\eta \to 0$, $d^2\varphi^{(1)}/d\eta^2 \to 0$ at $\eta \to \pm\infty$. Thus, the stationary solution of Eq. (78) can be expressed as

$$\varphi^{(1)} = -\varphi_m^{(1)}\mathrm{sech}^4[(\zeta - u_0\tau)/\Delta_t], \tag{80}$$

where the amplitude $\varphi_m^{(1)}$ and the width Δ_t are given by $\varphi_m^{(1)} = (15u_0/8a_t)^2$ and $\Delta_t = \sqrt{16b_s/u_0}$, respectively. As $u_0 > 0$ and $\mu < 1$, Eq. (80) reveals that there exist solitary waves with negative potential only. It is found that the effect of the trapped ion distribution causes the solitary waves of smaller width and larger propagation speed. It is also observed that as u_0 increases, the amplitude increases while the width decreases.

C. Nonthermal Ion distribution

To study the effect of a non-thermal ion distribution on the properties of DA solitary waves, we choose a more general class of the ion distribution which includes the population of nonthermal (fast) ions [66]. Thus, we take [48]

$$f_i(v_i) = \frac{1+\alpha_i v_i^4}{(1+3\alpha_i)\sqrt{2\pi}}\exp(-v_i^2/2), \tag{81}$$

where v_i is the ion speed normalized by the ion thermal speed V_{Ti} and α_i is a parameter determining the population of nonthermal (fast) ions in our dusty plasma model. The effect of electrostatic disturbance on the equilibrium ion distribution can easily be introduced by replacing v_i^2 with $v_i^2 + 2\varphi$. The resulting distribution function is then integrated over velocity space, yielding [48]

$$n_i = (1 + \alpha_0\varphi + \alpha_0\varphi^2)\exp(-\varphi), \tag{82}$$

where $\alpha_0 = 4\alpha_i/(1+3\alpha_i)$. We now make all the dependent variables depend only on a single variable $\xi = z - Mt$, use the steady state condition, impose the appropriate boundary conditions (namely $n_d \to 1$, $u_d \to 0$, $\varphi \to 0$ and $d\varphi/d\xi \to 0$ at $\xi \to \pm\infty$) and reduce to an energy integral: $(1/2)(d\varphi/d\xi)^2 + V(\varphi) = 0$, where $V(\varphi)$ is given by [48]

$$\begin{aligned} V(\varphi) &= \mu_i[1 + 3\alpha_0 - (1 + 3\alpha_0 + 3\alpha_0\varphi + \alpha_0\varphi^2)\exp(-\varphi)] \\ &\quad + \frac{\mu_e}{\sigma_i}[1 - \exp(\sigma_i\varphi)] + M^2\left[1 - \left(1 + \frac{2\varphi}{M^2}\right)^{1/2}\right]. \end{aligned} \tag{83}$$

Now, following the analytical steps or numerical analysis of the pseudo-potential $V(\varphi)$ described as before, we can show that when $\alpha_i > 0.155$ and $M > 1.41$, the potential well develops on both the positive and negative φ-axis [48]. This means that the presence of nonthermal ions ($\alpha_i > 0.155$) supports the coexistence of compressive and rarefactive DA solitary waves (DA solitary waves with $\varphi < 0$ and $\varphi > 0$).

D. Dust Fluid Temperature

To examine the effects of dust fluid temperature on one-dimensional DA solitary waves, we start with Eqs. (60), (62) and

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial z} = \frac{\partial \varphi}{\partial z} - \frac{\sigma_d}{n_d} \frac{\partial p_d}{\partial z}, \tag{84}$$

$$\frac{\partial p_d}{\partial t} + u_d \frac{\partial p_d}{\partial z} + 3p_d \frac{\partial u_d}{\partial z} = 0, \tag{85}$$

where p_d is the dust fluid pressure normalized by $n_{d0}k_BT_d$, $\sigma_d = T_d/Z_dT_i$. As before, assuming $\xi = z - Mt$ and $\partial/\partial t = 0$ and following the mathematical procedure presented in Sec. II, we can reduce Eq. (60), (62), (84) and (85) to an energy integral

$$\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + V(\varphi) = 0, \tag{86}$$

where the pseudo-potential $V(\varphi)$ is

$$V(\varphi) = -\left(\frac{\mu_e}{\sigma_i}\right)\exp(\sigma_i\varphi) - \mu_i\exp(-\varphi) - M^2\sqrt{\sigma_{d0}}\left[e^{\theta/2} + \frac{1}{3}e^{-3\theta/2}\right] + C_1, \tag{87}$$

with

$$\theta = \cosh^{-1}\left[\frac{\sigma_{d1}^2}{2\sigma_{d0}}\left(1 + \frac{2\varphi}{M^2\sigma_{d1}^2}\right)\right], \tag{88}$$

$\sigma_{d0} = \sqrt{3\sigma_d/M^2}$, $\sigma_{d1} = \sqrt{1+\sigma_{d0}^2}$ and C_1 is an integration constant. The latter is determined from $V(\varphi) = 0$ at $\varphi = 0$. It is important to note that we cannot consider the limit $\sigma_d \to 0$ in the pseudo-potential $V(\varphi, M, \sigma_d)$ in its present form. To consider the limit $\sigma_d \to 0$, we express θ as

$$\theta = \ln\left[\frac{\sigma_{d1}^2}{2\sigma_{d0}}\left(1 + \frac{2\varphi}{M^2\sigma_{d1}^2}\right) + \sqrt{\frac{\sigma_{d1}^4}{4\sigma_{d0}^2}\left(1 + \frac{2\varphi}{M^2\sigma_{d1}^2}\right)^2 - 1}\,\right]. \tag{89}$$

We note that in our study the condition for the dust density to be real requires $|1+2\varphi/M^2\sigma_{d1}^2| \geq 2\sigma_{d0}/\sigma_{d1}^2$. To examine how the dust fluid temperature (σ_d) modifies the properties of the arbitrary amplitude DA solitary waves, we analyze the pseudo-potential $V(\varphi)$ given by Eq. (87), and found that as we increase σ_d, (i) we need higher value of the Mach number in order to have solitary wave solutions of Eq. (86), and (ii) the amplitude of the solitary waves decreases, but their width increases. We have also found the same results by the numerical analysis of the energy integral given by Eq. (86).

IV. DISCUSSION

We have presented a rigorous theoretical investigation of the acoustic (particularly DIA and DA) solitary waves in unmagnetized dusty plasma. The results, which have been obtained from these studies, can be summarized as follows.

1. The DIA solitary waves with a positive (negative) potential can exist for $\mu > (<)\ 1/3$. To examine how the ion fluid temperature (σ_i) modifies the properties of the arbitrary amplitude DIA solitary waves, we found that as we increase σ_i, (i) we need higher value of the cortical Mach number (corresponding to a lower value of μ) in order to have DIA solitary waves with negative potential, and (ii) the amplitude of both positive and negative the solitary waves decreases, but their width increases.

2. To examine the effect of a non-planar geometry on DIA solitary waves, we found that the term $(\nu/2\tau)\phi^{(1)}$ in Eq. (19) is due to a non-planar (cylindrical or spherical) geometry. The numerical solutions of Eq. (19) reveal that for a large value of τ the spherical and cylindrical solitary waves are similar to one-dimensional solitary waves. This is because for a large value of τ, the term $(\nu/2\tau)\phi^{(1)}$, which is due to the effect of the cylindrical or spherical geometry, is no longer dominant. However, as the value of τ decreases, the term $(\nu/2\tau)\phi^{(1)}$ becomes dominant and both spherical and cylindrical solitary waves differ from one-dimensional solitary waves. It is found that as the value of τ decreases, the amplitude of these localized pulses increases. We also note that the amplitude of cylindrical solitary waves is larger than that of the one-dimensional solitary waves, but smaller than that of the spherical ones.

3. We have shown that the effects of dust grain charge fluctuations modify the properties of the DIA solitary waves. It has been found that the effects of dust grain charge fluctuations reduce the speed of the DIA solitary waves. The characteristics of these DIA solitary waves in the space dusty plasma condition are found to be different from those in laboratory dusty plasma condition. It is seen that for the parameters corresponding to space dusty plasma situations [1, 16], as we increase μ, both the amplitude and the width of the DIA solitary waves remains constant for $\mu < 0.5$, but increases very rapidly for $\mu > 0.5$. On the other hand, for laboratory dusty plasma parameters [31, 32], as we increase μ, the amplitude increases, but the width decreases.

4. The dusty plasma containing mobile dust particles and Boltzmann electrons and ions can support DA solitary waves with a negative potential only, corresponding to a hump in the dust number density. We found that as we increase the dust fluid temperature, the amplitude of DA solitary waves decreases, but their width increases.

5. We have shown that due to the effects of the trapped and non-thermal ion distributions, a dusty plasma admits a modified KdV equation, exhibiting a stronger nonlinearity, smaller width and larger propagation speed. On the other hand, we found that the presence of nonthermal ions ($\alpha_i > 0.155$) supports the coexistence of compressive and rarefactive DA solitary waves (DA solitary waves with $\varphi > 0$ and $\varphi < 0$)

6. We also considered a nonplanar geometry (cylindrical or spherical), we find that a dusty plasma admits a modified KdV equation containing $(\nu/2\tau)\varphi^{(1)}$. This extra term is due to the effect of cylindrical ($\nu = 1$) or spherical ($\nu = 2$) geometry. It is seen that as the value of τ decreases, the amplitude of these localized pulses increases. It is also found that the cylindrical solitary waves travel faster than the one-dimensional solitary waves but slower than the spherical ones, and that the amplitude of cylindrical solitary waves is larger than that of the one-dimensional solitary waves, but slightly smaller than that of spherical ones.

We finally hope that the basic features and the underlying physics of electro-acoustic (DIA and DA) solitary waves that we have presented here should be useful for understanding the localized electro-acoustic disturbances in space and laboratory dusty plasmas.

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