Available at: http://www.ictp.it/~pub_off IC/2005/097

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THERORETICAL PHYSICS

IRREVERSIBLITY ANALYSIS FOR GRAVITY DRIVEN NON-NEWTONIAN LIQUID FILM ALONG AN INCLINED ISOTHERMAL PLATE

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Abstract

In this paper, the first and second law of thermodynamics are employed in order to study the inherent irreversibility for a gravity driven non-Newtonian Ostwald-de Waele power law liquid film along an inclined isothermal plate. Based on some simplified assumptions, the governing equations are obtained and solved analytically. Expressions for fluid velocity, temperature, volumetric entropy generation numbers, irreversibility distribution ratio and the Bejan number are also determined.

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October 2005

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1. Introduction

The steady flow of a gravity driven non-Newtonian liquid film along an inclined isothermal plate finds application in industry, particularly in heating and cooling applications. For instance, the flowing liquid can be a mixture of two non-condensable liquids. In order to model such flow situations, the flow governing equations must take into account the flow behaviour of each liquid; consequently, the coupled equations for the liquids should be introduced. In most cases, the resulting mathematical problems become intractable analytically or may lead to extensive computational problems with the result being only valid for the flow parameters used in the simulations. Hence, generalizing the solution is difficult to achieve. Meanwhile, the liquids mixture can be considered as homogeneous medium with the assumption that the flow behaviour is non-Newtonian. In this case, the error associated with the analysis could be acceptably small, Johnson et al. (1991). Moreover, the analytical solution to the problem becomes possible, which in turn gives a general solution to flow and temperature fields. In modelling the non-Newtonian flow situations, the power-law model was used widely to characterize the rheological properties of the fluid e.g. Ekmann et al. (1986), Wang and Chukwu (1996). In this non-Newtonian model, the diffusion term in momentum equation is replaced by the Ostwald-de Waele power law viscous stress (i.e. $\tau = K(du/dy)^n$). For pseudoplastic fluids like fine particle suspensions $n < 1$, for dilatant fluids like ultrafine irregular particle suspensions $n > 1$, and for Newtonian fluids like air $n = 1$, (Andersson and Shang, 1998).

Meanwhile, the thermodynamic irreversibility in any fluid flow process can be quantified through entropy analysis. The first law of thermodynamics is simply an expression of the conservation of energy principle. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated with the real processes. As entropy generation takes place, the quality of energy (i.e. exergy) decreases. In order to preserve the quality of energy in a fluid flow process or at least to reduce the entropy generation, it is important to study the distribution of the entropy generation within the fluid volume. The optimal design for any thermal system can be achieved by minimizing entropy generation in the systems. Entropy generation in thermal engineering systems destroys available work and thus reduces its efficiency. This issue has been the topic of great importance in many engineering fields such as heat exchangers, cooling of nuclear reactors, energy storage systems, cooling of electronic devises, etc. Different sources of irreversibility such as heat transfer effect, viscous dissipation effect, etc., are responsible for entropy generation in thermal systems. Bejan (1980, 1994, 1996) illustrated that the flow parameter could be selected in order to minimize the entropy generation associated with specific convective heat transfer processes. The inherent irreversibility of fluid flow and heat transfer for non-Newtonian power law fluids in a pipe and channel made of two parallel plates was investigated by Mahmud and Fraser (2002). In particular, they reported that the entropy generation rate is higher in psuedoplastic fluid than that of dilatant with an increase in Brinkman number. Yilbas et al. (2004) presented an analytical solution for entropy generation in a constant viscosity non-Newtonian third grade fluid flow in a annular pipe. They show that entropy generation number attains high values in the region close to the inner wall of the annular pipe and increasing Brinkman number enhances entropy generation. Other applications of second law analysis for some steady flow devices can be found in references, Nag and Kumar (1989), Narusawa (1998), Sahin (1999) Latife et al. (2003), Makinde and Osalusi (2005), etc.

The main objective of the present study is to analyse the fluid flow and heat transfer irreversibility in a gravity-driven viscous incompressible non-Newtonian power law liquid film along an inclined plate that is subjected to a prescribed uniform wall temperature. In the following sections, the problem is formulated, analyzed, solved and discussed.

2. Analysis

Consider the laminar flow of a gravity-driven viscous incompressible non-Newtonian Ostwald-de-Waele power law liquid film along an inclined isothermal plate as shown in figure (1). It is assumed that the flow is fully developed and the liquid surface is free.

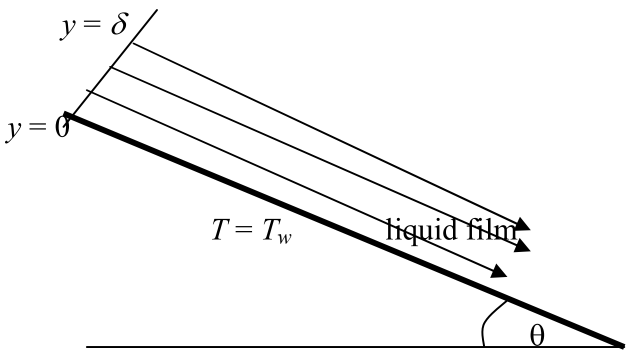

Figure (1): Schematic diagram of the problem

For steady flow, the governing equations for momentum and energy balance within the viscous boundary layer, together with their corresponding boundary conditions are:

$$K\frac{d}{dy}\left(\frac{du}{dy}\right)^{n}+\rho g\sin(\theta)=0, \qquad (1)$$

$$\frac{d^{2}T}{dy^{2}}+\frac{K}{\alpha}\left(\frac{du}{dy}\right)^{n+1}=0, \qquad (2)$$

No slip condition: $u(0)=0$, (3)

Free surface: $\frac{du}{dy}(\delta)=0$, (4)

Inclined plate temperature: $T(0)=T_w$, (5)

Free surface temperature: $T(\delta)=T_f$, (6)

where T is the absolute temperature and α the thermal conductivity, ρ the fluid density, δ the liquid film height, K the coefficient of consistency, n the power law index, g the acceleration due to gravity, u the fluid axial velocity and θ is the inclined angle. Introducing the following dimensionless variables and parameters into the governing equations

$$\bar{u} = \frac{u}{\left(\frac{n}{n+1}\right)\left(\frac{\rho g \sin(\theta)}{K}\right)^{\frac{1}{n}} \delta^{\frac{n+1}{n}}}, \ \bar{y} = \frac{y}{\delta}, \ \bar{T} = \frac{T - T_f}{T_w - T_f},$$

$$Br = \frac{\left(\frac{n}{(1+n)}\right)^{n+1} \delta^{\frac{3n+1}{n}} \left(\rho g \sin(\theta)\right)^{\frac{n+1}{n}}}{\alpha K^{\frac{1}{n}} (T_w - T_f)}. \quad (7)$$

We neglect the bar symbol for clarity and the dimensionless governing equations are obtained as

$$\frac{d}{dy}\left(\frac{du}{dy}\right)^n = -\left(\frac{n+1}{n}\right)^n, \ \frac{d^2T}{dy^2} + Br\left(\frac{du}{dy}\right)^{n+1} = 0, \quad (8)$$

with

$$u(0)=0, \ \frac{du}{dy}(1) = 0, \ T(0) = 1, \ T(1)=0, \quad (9)$$

where Br is the Brinkman number. After solving equations (8)-(9), we obtain the exact solutions for the velocity and temperature profiles as

$$u(y) = \left[1 - (1-y)^{\frac{n+1}{n}}\right], \quad (10)$$

$$T(y) = (1-y)\left[1 + \frac{n^2 Br}{(2n+1)(3n+1)}\left(\frac{n+1}{n}\right)^{n+1}\right] - \frac{n^2 Br (1-y)^{\frac{3n+1}{n}}}{(2n+1)(3n+1)}\left(\frac{n+1}{n}\right)^{n+1}. \quad (11)$$

3. Entropy Generation Rate

Following Mahmud and Fraser (2002), the dimensional volumetric entropy generation for power law fluid is defined as

$$S_G = \frac{\alpha}{T_f^2}\left(\frac{dT}{dy}\right)^2 + \frac{K}{T_f}\left(\frac{du}{dy}\right)^{n+1}. \quad (12)$$

The first term in equation (12) is the irreversibility due to heat transfer and the second term is the entropy generation due to viscous dissipation. Using equation (7), we express the entropy generation number in dimensionless form as (neglecting bar for clarity),

$$Ns = \frac{\delta^2 T_f^2 S_G}{\alpha (T_w - T_f)^2} = \left(\frac{dT}{dy}\right)^2 + \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^{n+1}, \tag{13}$$

and we obtain

$$Ns = \left[\frac{nBr(1-y)^{\frac{2n+1}{n}}}{(2n+1)}\left(\frac{n+1}{n}\right)^{n+1} - \frac{n^2 Br}{(2n+1)(3n+1)}\left(\frac{n+1}{n}\right)^{n+1} - 1\right]^2 + \frac{Br}{\Omega}\left(\frac{(n+1)}{n}(1-y)^{\frac{1}{n}}\right)^{n+1} \tag{14}$$

where $\Omega = (T_w - T_f)/T_f$ is the temperature different parameter. In equation (13), the first term can be assigned as N_1 and the second term due to viscous dissipation as N_2, i.e.

$$N_1 = \left(\frac{dT}{dy}\right)^2, \quad N_2 = \frac{Br}{\Omega}\left(\frac{du}{dy}\right)^{n+1}. \tag{15}$$

In order to have an idea whether fluid friction dominates over heat transfer irreversibility or vice-versa, Bejan (1996) defined the irreversibility distribution ratio as $\Phi = N_2/N_1$. Heat transfer dominates for $0 \le \Phi < 1$ and fluid friction dominates when $\Phi > 1$. The contribution of both heat transfer and fluid friction to entropy generation are equal when $\Phi = 1$. In many engineering designs and energy optimisation problems, the contribution of heat transfer entropy N_1 to overall entropy generation rate Ns is needed. As an alternative to irreversibility parameter, the Bejan number (Be) is define mathematically as

$$Be = \frac{N_1}{Ns} = \frac{1}{1+\Phi}. \tag{16}$$

Clearly, the Bejan number can only take values from 0 to 1. $Be = 1$ is the limit at which heat transfer irreversibility dominates while $Be = 0$ is the limit at which fluid friction irreversibility dominates. The contribution of both heat transfer and fluid friction to entropy generation are equal when $Be = 1/2$.

4. Results and Discussions

Depending on the magnitude of fluid behaviour index (*n)*, our results are valid for a wide variety of non-Newtonian fluids. Figure (2) shows the dimensionless velocity profiles, generally, the maximum velocity moves towards the liquid free surface. For pseudoplastic fluids (n<1), the velocity profile becomes flat near the liquid free surface and this flatness decreases with an increase in the magnitude of fluid behaviour index. Figures (3) and (4) show the dimensionless temperature profile. The temperature increases rapidly and attains its maximum value near the wall, then gradually decreases to its minimum value at the free surface. It is noteworthy that dilatant fluids (n>1) tend to get warmer comparing to pseudoplastc fluids. An increase in the Brinkman number (*Br*) also causes the fluid temperature to increase further as shown in figure(4). In figure (5), we observed that the volumetric entropy generation rate is significantly higher at the wall and sharply decreases towards the free surface for pseudoplastic fluids. Similar behaviour with lower entropy generation rate is noticed also for both Newtonian (n=1) and dilatant fluids. Entropy generation rate is higher in psuedoplastic fluids than that of dilatant fluids near the wall, however, near the free surface, the case is the opposite. Increasing values of group parameter ($Br\Omega^{-1}$) causes higher entropy generation number due to viscous dissipation effect as shown in figure (6). Figures (7) and (8) illustrate the irreversibility distribution ratio and Bejan number. For all values of flow behaviour index, it is quite obvious that the fluid friction irreversibility dominates near the wall while at the free surface heat transfer irreversibility dominates. In figure (9), we observed that increasing values of group parameter enhances further domination of fluid friction irreversibility near the wall.

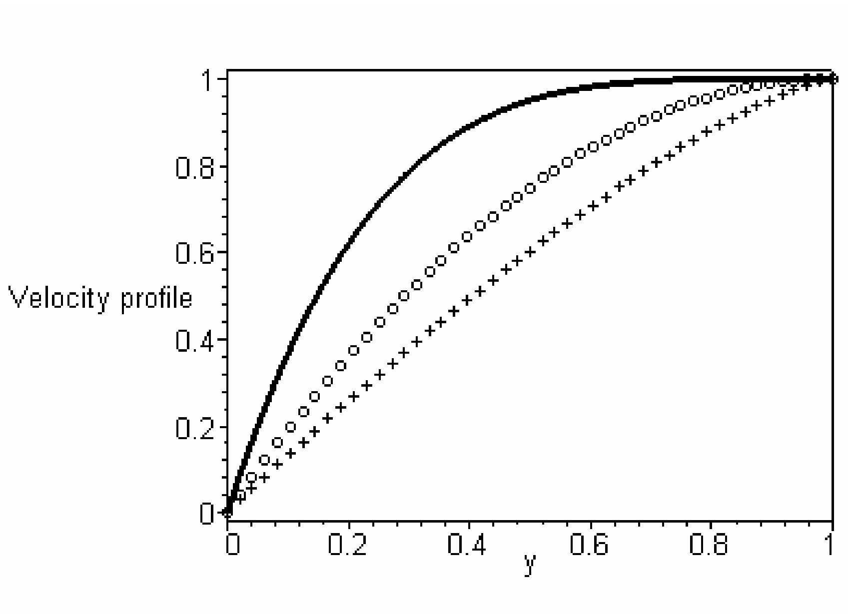

Figure (2): Velocity profile, _______n=0.3; ooooooooo n=1.0; ++++++++ n=3.0

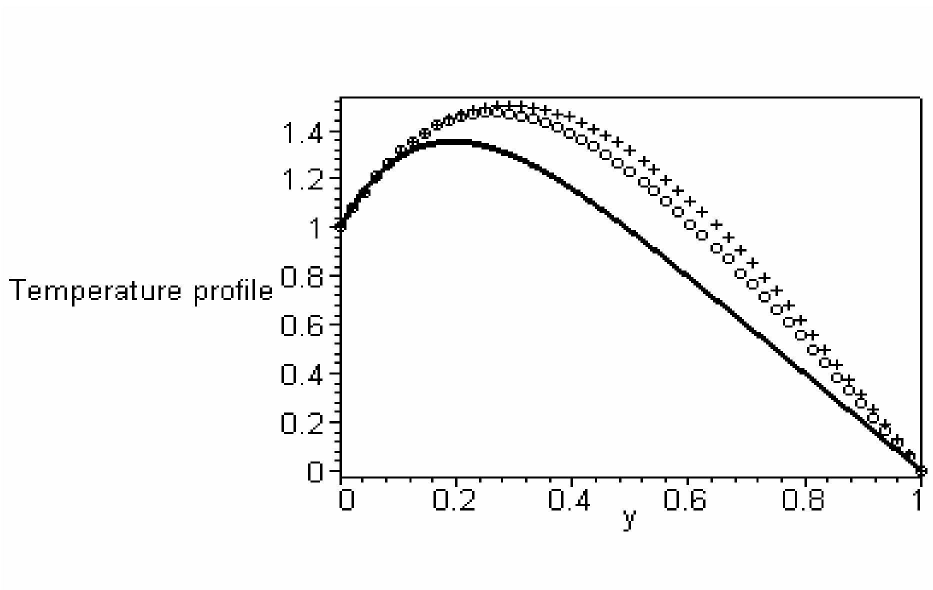

Figure (3): Temperature profile, Br = 5.0; _______n=0.3; ooooooooo n=1.0; ++++++++ n=3.0

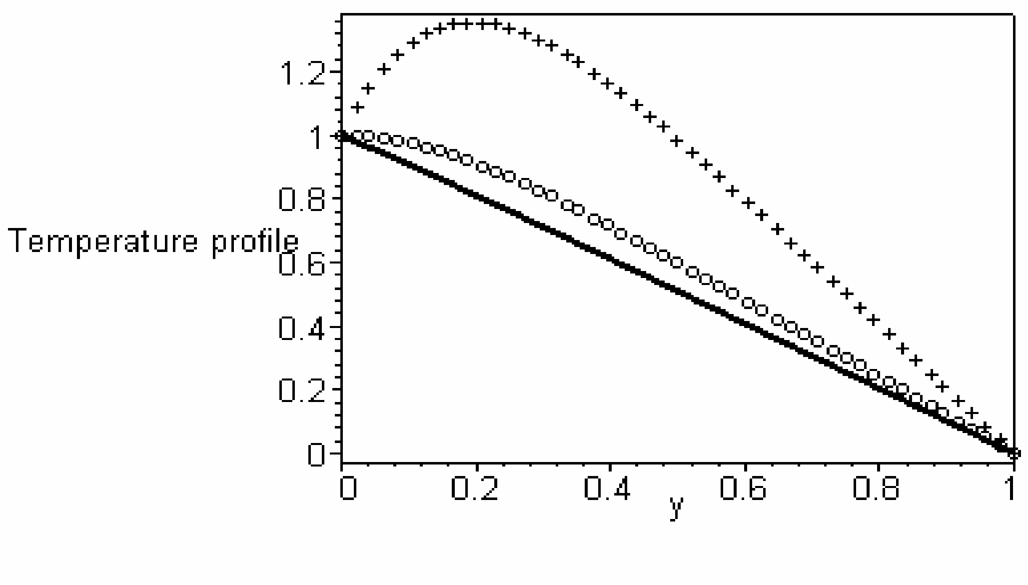

Figure (4): Temperature profile, n = 0.3; _______ Br =0.1; ooooooo Br=1.0; +++++++ Br =5.0

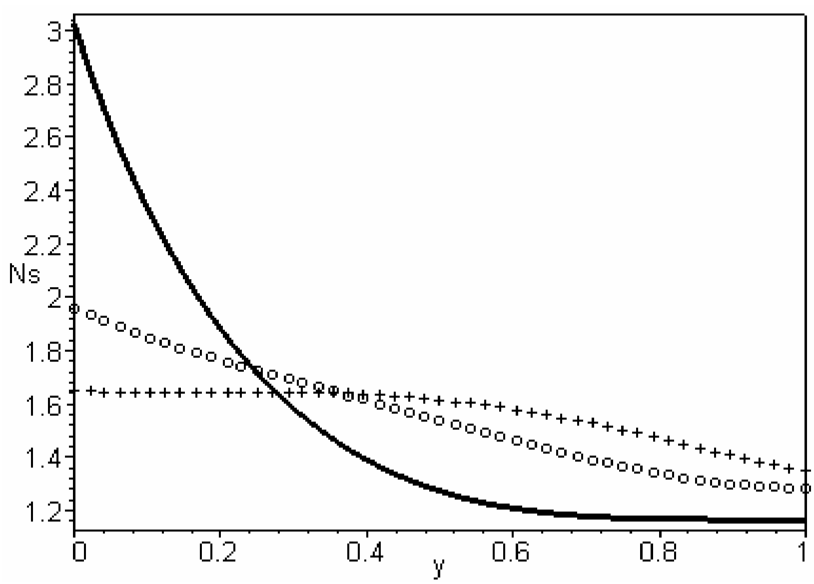

Figure (5): Entropy generation rate, $Br\Omega^{-1}$ = 0.4; _______ n=0.3; ooooooooo n=1; ++++++++ n=3

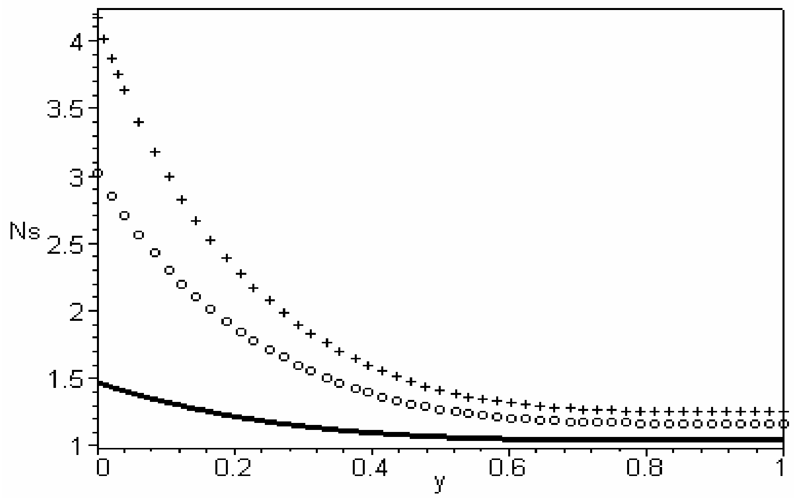

Figure (6): Entropy generation rate, n=0.3; ______$Br\Omega^{-1}$ = 0.1; ooooooo $Br\Omega^{-1}$ =0.4; +++++++ $Br\Omega^{-1}$= 0.6

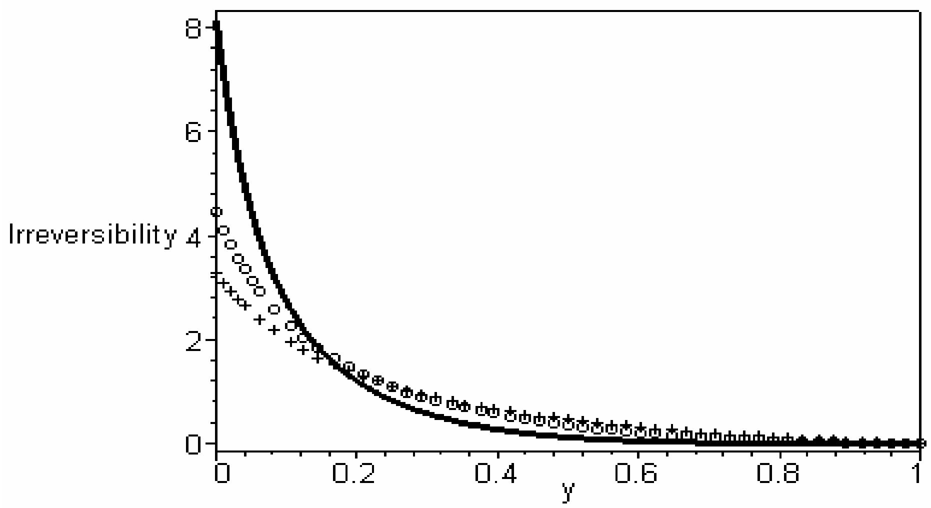

Figure (7): Irreversibility ratio, $Br\Omega^{-1}$ = 0.4; _______n=0.3; ooooooooo n=1.0; ++++++ n=3.0

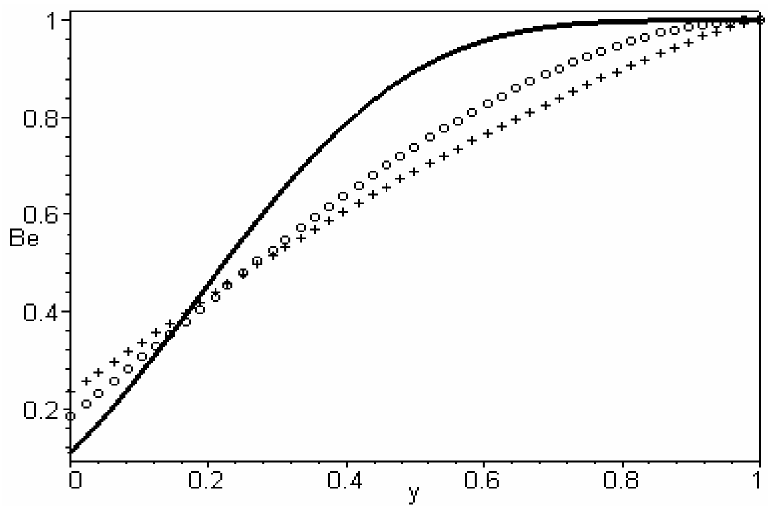

Figure (8): Bejan number, $Br\Omega^{-1}$ = 0.4; _______n=0.3; ooooooooo n=1.0; ++++++++ n=3.0

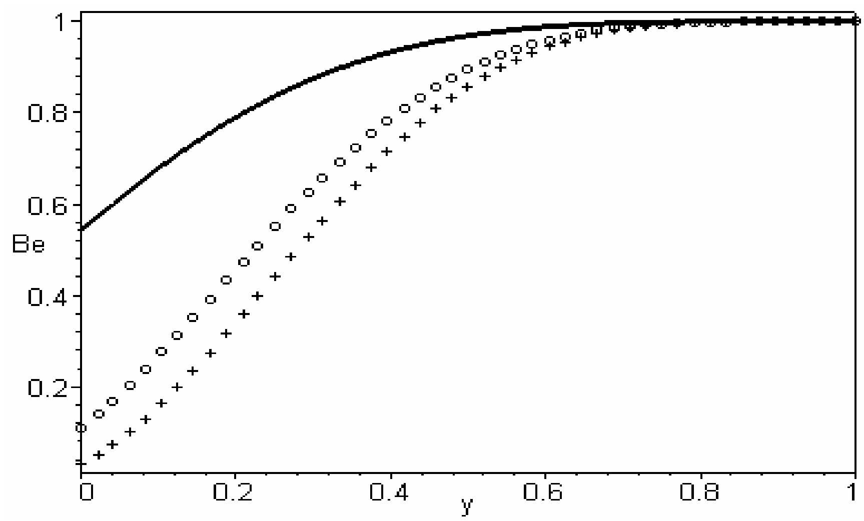

Figure (9): Bejan number, n=0.3; ______$Br\Omega^{-1}$ = 0.1; ooooooo $Br\Omega^{-1}$ =0.4; +++++++ $Br\Omega^{-1}$= 0.6

5. Conclusion

This study was focused on irreversibility analysis for gravity-driven non-Newtonian power law liquid film along an inclined plate with a prescribed uniform temperature. We obtained both the velocity and temperature profiles analytically. The volumetric entropy generation rate, the irreversibility distribution ratio and the Bejan number depend on group parameter ($Br\Omega^{-1}$) and fluid behaviour index (*n*). Our results reveal that for all values of fluid behaviour index, viscous dissipation irreversibility dominates at the inclined plate wall while near the liquid free surface the heat transfer irreversibility dominates. Increasing values of group parameter ($Br\Omega^{-1}$) enhances both entropy generation rate and the dominant effect of viscous dissipation irreversibility.

Acknowledgements

This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is acknowledged.

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