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EFFECTS OF INHOMOGENEITY ON THE SHUKLA-NAMBU-SALIMULLAH AND WAKE POTENTIALS IN A STREAMING DUSTY MAGNETOPLASMA

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Abstract

Detailed properties of the electrostatic Shukla-Nambu-Salimullah and the dynamical oscillatory wake potentials in an inhomogeneous dusty magnetoplasma in the presence of ion streaming, as in a laboratory discharge plasma, have been examined analytically. The potentials become sensitive functions of the external static magnetic field, the scalelength of inhomogeneity, and the deviation from the linear ion streaming velocity. For a decreasing ion density gradient, there is a limit of existence of the static modified shielding potential. For a strongly inhomogeneous dusty plasma, the effective length of the oscillatory wake potential increases with increasing deviation of the ion streaming velocity (u_{i0y}), but it does not depend on the external magnetic field.

I. Introduction

The most fundamental properties of a dusty plasma at per real laboratory experimental conditions, such as dust Coulomb crystal formation are related to the existence of static electrostatic shielding and the consequent dynamical wake potentials. The modified shielding potential in the most realistic magnetized plasmas is known as the Shukla-Nambu-Salimullah (SNS) potential [1]. It may give rise to many interesting properties of dusty plasmas in the presence of an external static magnetic field. In an earlier paper [2], we showed how such a new electrostatic potential affects the three dimensional dust crystal formation in a homogeneous streaming dusty magnetoplasma. Recently, the exact behavior of this SNS potential has been shown numerically in a homogeneous streaming dusty magnetoplasma [3]. Besides this, the formation of Coulomb crystals and subsequent phase transitions in dusty plasmas are attributed to novel attractive forces that may come dominantly from the oscillatory wake potential [4]. The theoretical idea of wake potential [4-7] has been supported by hybrid-code simulations [8] as well as experimental verification [9]. However, most of the dust crystal experiments involve inhomogeneous plasmas with inhomogeneity of the dusty plasma mainly along the ion flow direction (let us say $\hat{x}$-direction). The role of the magnetic field aligned ion and dust flows have been examined earlier [3]. But, an external transverse magnetic field ($\mathbf{B_0} \parallel \hat{z}$) and the inhomogeneous plasma background might play a significant role in the SNS and the oscillatory wake potentials.

Moreover, in a laboratory condition of an rf-discharge electron-ion plasma embedded with negatively charged dust grains, as-formed plasmas will be nonuniform with inhomogeneity plasma gradient along the ion flow direction ($\hat{x}$-axis). Also, the large negative charge on a dust grain will cause deviation of the ion flow velocity from linearity [10].

In this paper, we make a theoretical investigation on the SNS and the wake potentials in a dusty plasma having an inhomogeneous and flowing ion distribution with Boltzmann electrons, and static and immobile dust particles in the presence of an external magnetic field. We give attention to the effects of the transverse magnetic field, the deviation of the linear flow velocity of ions, and inhomogeneity scale length on these potentials.

In Sec. II, we first derive the dielectric response function of the dusty plasma appropriate for laboratory dust crystal experiments with a transversely magnetized, nonlinear ion flow, and inhomogeneous plasma density. Then, the expressions for SNS and the wake potentials have been derived in sub-sections A and B. Finally, a brief discussion of the results is given in Sec. III.

II. SNS and Wake potentials

In the presence of low-frequency (in comparison with the electron gyrofrequency ω_{ce}) electrostatic waves with parallel (to $\hat{\mathbf{z}}$) phase speed much smaller than the electron thermal speed v_{te}, hot electrons rapidly thermalize along $\hat{\mathbf{z}}$-direction and establish a Boltzmann distribution. The corresponding dielectric susceptibility is $1/k^2\lambda_{De}^2$, where k is the wavenumber and λ_{De} is the electron Debye radius.

In a laboratory condition of an rf-discharge plasma, a plasma sheath will be formed and an static electric field ($\mathbf{E}$) will be established because of the fast motion of lighter electrons compared to the motion of ions. In the presence of this electric field, ions will be flowing nearly transverse to the external static magnetic field $\mathbf{B_0}$. But, since dust grains will be negatively charged, the path of ions will not be linear because of the presence of highly charged static dust grains [10].

For $kV_{td}, kV_{ti}, \omega_{cd} \ll \omega \ll \omega_{ci}$, the ions suffer the $\mathbf{E} \times \mathbf{B}_0$ drift, in addition to streaming along $\hat{\mathbf{x}}$, where ω is the wave frequency. The susceptibility for strongly magnetized ions with inhomogeneous distribution is given by [11-14]

$$\chi_i(\omega, \mathbf{k}) = \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2 \lambda_{Di}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2}, \tag{1}$$

where $k_\perp(k_\parallel)$ is the component of $\mathbf{k}$ across (parallel) $\hat{\mathbf{z}} \parallel \mathbf{B}_0$, $\omega_{pi} = (4\pi e^2 n_{i0}/m_i)^{1/2}$ is the ion plasma frequency, $\omega_{ci} = eB_0/m_i c$ is the gyrofrequency, $\omega_i^* = -k_y v_{ti}^2/L_{ni}\omega_{ci}$ is the drift frequency, k_y is the component of the wave vector $\mathbf{k}$ along the y-axis which is transverse to $\hat{\mathbf{z}}$, $v_{ti} = (T_i/m_i)$ is the thermal velocity, $L_{ni}(= -n_{i0}(x)/n'_{i0}(x)$, $n'_{i0}(x) = dn_{i0}(x)/dx)$ is the scalelength of the density inhomogeneity of the ions, $\omega' = (\omega - \mathbf{k}\cdot\mathbf{u}_{i0})$, is the Doppler shifted frequency and $\lambda_{Di} = (T_i/4\pi e^2 n_{i0})^{1/2} = v_{ti}/\omega_{pi}$ is the ion Debye length. In general, L_{ni} will be a function of x. However, for a particular choice of the ion density inhomogeneity in the $x-$direction, viz., $n_{i0}(x) = n_{i0}^0(x)(1 \mp x/L_{ni})$, the scalelength of inhomogeneity is independent of x. Here, $-e$, n_{i0}^0, m_i, c and T_i are the electronic charge, equilibrium number density of ions, mass of an ion, light speed and ion temperature respectively. Further, since the frequency of the perturbation $\omega \gg \omega_{pd}$, ω_{cd} where ω_{pd} and ω_{cd} are the dust plasma and dust gyrofrequency, the dust grains can be considered unmagnetized and immobile.

Thus, the appropriate dielectric constant of such electrostatic waves in the magnetoplasma is given by

$$\begin{aligned}\epsilon(\omega, \mathbf{k}) &= 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2\lambda_{Di}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2}, \\ &= \frac{1}{k^2}\left[k^2 + k_e^2 + k_\perp^2 f - \frac{k_\parallel^2\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{u}_{i0})^2} + \frac{k_y\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{u}_{i0})L_{ni}\omega_{ci}}\right].\end{aligned} \tag{2}$$

where $k_e = 1/\lambda_{De}$ and $f = \omega_{pi}^2/\omega_{ci}^2$.

The electrostatic potential around a test dust particulate in the presence of electrostatic mode $(\omega, \mathbf{k})$ in the uniform dusty magnetoplasma, whose dielectric response function is given by Eq. (1), is

$$\Phi(\mathbf{x}, t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega, \mathbf{k})}\exp(i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{k}\, d\omega, \tag{3}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test dust particulate, and q_t is its charge. In the following we study the SNS potential in Sec. A and the oscillatory wakefield in Sec. B.

Sec. A

Using the cylindrical coordinates (ρ, θ, z) and evaluating the θ-integration, we obtain the electrostatic potential as

$$\phi(\mathbf{x}, t) = \frac{q_t}{\pi} \int \frac{J_0(k_\perp \rho) \exp(ik_\parallel z)\, \delta(\omega)}{k^2 \epsilon(\omega, \mathbf{k})}\ k_\perp\, dk_\perp\, dk_\parallel\, d\omega, \tag{4}$$

where $v_t \approx 0$ for the static test dust particulate.

For a drift wave $\mathbf{k} \simeq \hat{\mathbf{y}} k_y + \hat{\mathbf{z}} k_\parallel$ with $k_y \gg k_\parallel$ and performing the ω-integration, we obtain

$$\phi(\mathbf{x}, t) = \frac{q_t}{\pi} \int \frac{J_0(k_\perp \rho) \exp(ik_\parallel z) k_\perp\, dk_\perp\, dk_\parallel}{k_\parallel^2 + k_\perp^2(1+f) + k_e^2(1 - u_{ioy} M^{-2}/L_{ni}\omega_{ci})}, \tag{5}$$

where $M = u_{i0y}/C_s$ is the Mach number, $C_s = \omega_{pi}\lambda_{De}$ being the ion sound velocity and u_{i0y} is the component of $\mathbf{u_{i0}}$ along $\hat{\mathbf{y}}$-direction. Here, we have assumed the condition $k_\perp^2 \gg k_\parallel^2$. Following Ref.[1], we finally obtain performing $k_\parallel$-and $k_\perp$ integrations in Eq. (5)

$$\phi(\rho, z) = \frac{q_t}{\sqrt{1+f}} \frac{\exp[-\sqrt{\rho^2 + z^2(1+f)}/L_s]}{\sqrt{\rho^2 + z^2(1+f)}}, \tag{6}$$

where

$$L_s = \frac{\lambda_{De}\sqrt{1+f}}{\sqrt{1 - C_s^2/u_{ioy} L_{ni}\omega_{ci}}}. \tag{7}$$

This is the static asymmetric SNS potential in an inhomogeneous magnetoplasma. It is clear that if we neglect the ion inhomogeneity effect ($L_{ni} \approx \infty$) in the dielectric function, we can retrieve the usual SNS potential [1-3,11,12]. In the direction perpendicular to the magnetic field ($z = 0$), the amplitude of the potential decreases by a factor $\omega_{pi}/\omega_{ci} \gg 1$ and the effective shielding length of the potential increases by the same factor. However, in the direction parallel to the magnetic field ($\rho = 0$), the amplitude of the SNS potential decreases faster by the factor $\omega_{pi}^2/\omega_{ci}^2$ but the effective length is not affected by the magnetic field significantly [16].

Since $L_{ni} < 0$ for positive density gradient ($\partial n_{io}/\partial x > 0$) and if $1 < C_s^2/u_{i0y}|L_{ni}|\omega_{ci}$ for stronger inhomogeneity (smaller $|L_{ni}|$), the effective shielding length of the SNS potential reduces to

$$L_s \simeq \lambda_{De}\sqrt{1+f} \cdot \sqrt{\frac{u_{i0y}|L_{ni}|\omega_{ci}}{C_s^2}}. \tag{8}$$

L_s increases with u_{i0y}, ω_{ci}, and L_{ni}. Hence, the effective shielding length L_s is a sensitive function of the scale length L_{ni}, the nonlinear motion of streaming ions u_{i0y}, Mach number M and magnetic field $\mathbf{B_0}$. The scale length of inhomogeneity L_{ni} can be positive as well as negative depending upon the decreasing or increasing density gradient $\partial n_{i0}(x)/\partial x$.

For $L_{ni} > 0$ for the decreasing ion density gradient, the ion dynamics may prohibit the shielding mechanism, and the shielding becomes oscillatory [cf. Eq.(7)] for $1 < |C_s^2/u_{i0y}L_{ni}\omega_{ci}|$. Consequently, the strong inhomogeneity with smaller $|L_{ni}|$ can even destroy the shielding effect. Thus, the plasma approximation is broken for this situation. However, for $1 > |C_s^2/u_{i0y}L_{ni}\omega_{ci}|$, the SNS potential is retrieved with modified asymmetric screening length in different directions.

Sec. B

For a low frequency electrostatic mode propagating nearly perpendicular to $\hat{z} \parallel \mathbf{B_0}$ and involving the dust dynamics, the drift wave phase velocity can be much smaller than the component of the ion streaming velocity in the direction of propagation of the wave, i.e., $\omega/k_y \ll u_{i0y}$. Thus, the dielectric function, Eq. (2) can be rewritten as

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{k_y v_{ti}^2}{\omega' k^2 \lambda_{Di}^2 L_{ni}\omega_{ci}} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2} - \frac{\omega_{pd}^2}{\omega^2},$$

$$\simeq \frac{(1+f)k^2\lambda_D^2 + 1}{k^2\lambda_D^2}\left(1 - \frac{\omega_k^2}{\omega^2}\right), \tag{9}$$

where

$$\omega_k^2 = \frac{k^2 C_d^2}{1 + (1+f)k^2\lambda_D^2},$$
$$C_d^2 = \omega_{pd}^2\lambda_D^2, \tag{10}$$
$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2}\left(1 - \frac{C_s^2}{u_{i0y}L_{ni}\omega_{ci}}\right).$$

In writing Eq. (9), we assumed $k_\perp^2 \gg k_\parallel^2$ and $\omega \ll k_y u_{i0y}$, ω_{ci}.

Then the inverse of the dielectric function is

$$\frac{1}{\epsilon(\omega, \mathbf{k})} = \frac{k^2\lambda_D^2}{1 + (1+f)k^2\lambda_D^2}\left(1 + \frac{\omega_k^2}{\omega^2 - \omega_k^2}\right). \tag{11}$$

Substituting the first part of Eq. (9) into Eq. (3), we can derive the SNS potential, Eq. (6). Substituting the second part of Eq.(9) into Eq. (3), we obtain the dynamical potential as

$$\Phi_{II}(\mathbf{x}, t) = \left(\frac{q_t\lambda_D^2}{2\pi^2}\right) \int \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)\ \omega_k^2\ \exp(i\mathbf{k}\cdot\mathbf{r})}{[1 + (1+f)k^2\lambda_D^2](\omega^2 - \omega_k^2)}\ d\mathbf{k}\ d\omega. \tag{12}$$

Assuming $\mathbf{v_t} || \hat{\mathbf{z}}$ and $\mathbf{k} \approx \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_\parallel$ with $k_y \gg k_\parallel$, and carrying out the θ- and ω- integrations in Eq. (10), we obtain

$$\Phi_{II}(\rho, \xi) = \left(\frac{q_t C_d^2 \lambda_D^2}{\pi v_t^2}\right) \int \frac{J_0(k_\perp\rho)\ \exp(ik_\parallel\xi)\ \ k^2\ k_\perp\ dk_\perp\ dk_\parallel}{[1 + (1+f)k^2\lambda_D^2]\left\{[1 + (1+f)k^2\lambda_D^2]k_\parallel^2 - k^2 M'^{-2}\right\}}, \tag{13}$$

where $M' \equiv v_t/C_d$ is the Mach number and $\xi \equiv z - v_t t$.

An exact form of the three-dimensional wake potential can be found by solving Eq. (13) numerically. However, following the procedures of Ref. [15], we finally obtain the one-dimensional wake potential in the inhomogeneous dusty magnetoplasma in the presence of ion streaming

$$\Phi_{II}(\rho \approx 0, \xi) \approx \frac{2q_t(1 + M'^{-2})}{(1+f)}\ \frac{\cos(|\xi|/L_s')}{|\xi|}, \tag{14}$$

where $L_s' = M'\sqrt{1+f}\,\lambda_D$. The effect of inhomogeneity is contained in the expression of $\lambda_D = \lambda_{De}/(1 - C_s^2/u_{i0y}L_{ni}\omega_{ci})$.

We have studied the effects of ion inhomogeneity, external magnetic field, and the streaming of ions on the formation of the SNS and the oscillatory wake potentials in a nonuniform dusty magnetoplasma. From Eq. (14), we notice that when the ion inhomogeneity is neglected, the effective length of the wake potential reduces to $L'_s = M'\sqrt{1+f}\lambda_{De}$. Hence, the wake potential differs significantly from the unmagnetized case [4] in this transversely magnetized dusty plasma.

For the stronger inhomogeneity, $1 < C_s^2/u_{i0y}|L_{ni}|\omega_{ci}$, the effective length of the wake potential reduces to $L'_s = \lambda_{De}M'\omega_{pi}u_{i0y}L_{ni}/C_s^2$ which is independent of the external magnetic field. In this case, for a given L_{ni}, L'_s increases with M' and u_{i0y}. But for the appropriate limit of homogeneous plasmas $(1 > C_s^2/u_{i0y}L_{ni}\omega_{ci})$, $L'_S = \lambda_{De}M'\omega_{pi}/\omega_{ci}$ for $\omega_{pi} \gg \omega_{ci}$.

III. Discussion

In this paper, we have investigated analytically the detailed properties of the electrostatic SNS potential and the oscillatory wake potential in an inhomogeneous and transversely magnetized dusty plasma in the presence of ion streaming as in a laboratory discharge plasma. The potentials become sensitive functions of the external static magnetic field, the scalelength of inhomogeneity, and the deviation of linear ion streaming velocity. For a positive ion gradient ($L_{ni} < 0$), there is a limit of existence of the modified static SNS potential [cf. Eq. (6)]. For the strong inhomogeneous dusty plasma, the effective length of the dynamical wake potential increases with increasing deviation of the ion streaming velocity, but it is independent of the external magnetic field.

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