APPLICATION OF THE ENTROPY CONCEPT TO DESCRIBE THE QUALITY OF A X -RAY BEAM

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INTRODUCTION

In a previous work (Oliveira and Pedroso de Lima, 2000), we introduced the concept of entropy to describe the degradation of the energy of a photon beam incident in a material. We pointed out the relation between both, the entropy of the distribution of the energy deposited in matter and the traditional quality factor of the radiation protection.

From the point of view of the entropy, we showed that, in what concerns to the monoenergetic approximation to a polyenergetic beam, we can hardly speak in an approximation (Oliveira 2001). In fact, the behaviour of the entropy is very different for both, an X-ray and a monoenergetic beam.

In this work we developed further the study of the degradation of the energy of X-ray photons, from the point of view of the entropy, introducing the concepts of surface entropy, S_{surf}, equilibrium entropy, S_{eq}, and concepts of entropy variation, namely, primary to surface variation, S_{ps}, and total entropy variation, S_T. These are characteristic parameters of the X-ray beams describing the degradation of the primary photons while they interact with matter.

METHODS

We considered water as scatter medium and X-ray beams with perpendicular incidence in a half-extended geometry. The primary beams used are in range 50–140 kV_{cp} (where cp means constant potential. For simplicity we drop the index cp), and are from the catalogue Birch et al (1979).

In the Monte Carlo code we simulate three interactions: photoelectric, coherent and incoherent scattering. For the photoelectric effect we considered the deposition energy in the interaction point. In the Compton scattering we used the klein-Nishina cross-section modified by the incoherent scattering function. In the coherent scattering we used the Thomsom cross-section modified by the atomic form factor. The code was implemented following Chan and Doi (1988), the mass attenuation coefficients are from Ouellet and Schreiner (1991), the atomic form factors and incoherent scattering functions are from Hubbell et al (1975) and interaction data for water are from ICRU (1992).

We divided the water in plane-parallel layers of 1 cm and for each one we obtained a sample of the distribution of the energy deposited. We applied the concept of entropy to this energy distribution. For the layer m, with $m=0,1,2,\ldots$, we define the entropy as

$$S_m = -\sum_{i=1}^{N_m} p(E_i) \ln p(E_i) \ , \qquad (1)$$

where N_m is the number of different values of the energy in the spectrum (we used steps of 1 keV) and $p(E_i)$ is the probability of finding a photon with energy E_i.

ENTROPY OF PRIMARY BEAMS

Nowadays is still in use to express the quality of an X-ray beam with the half value layer (HVL), together with the voltage (kV) and the total filtration of the tube. From the catalogue of Birch et al (1979) we have, for example, the relation between kV and HVL, with the filtration as a parameter (Figure 1).

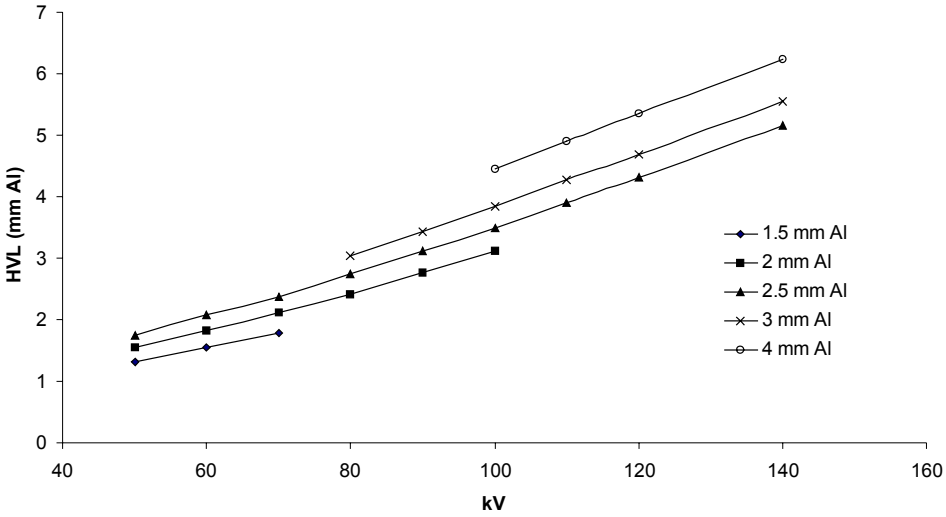

Figure 1 – The half value layer (HVL) as a function of the tube potential. Data from Birch et al (1979)

It is well known that the HVL have serious limitations in specifying beam quality (Carlsson and Carlsson, 1984). The HVL suggest the existence of an effective energy related with an effective linear attenuation coefficient, $\mu=\ln 2/HVL$. Applying the entropy concept to describe the quality of the energy deposited in water, as we pointed out above, we can hardly speak in monoenergetic approximation (Oliveira, 2001). This is because we have rather different behaviour of the entropy for mono and polyenergetic beams.

The application of equation (1) to the primary X-ray spectra leads to the results presented in Figure 2.

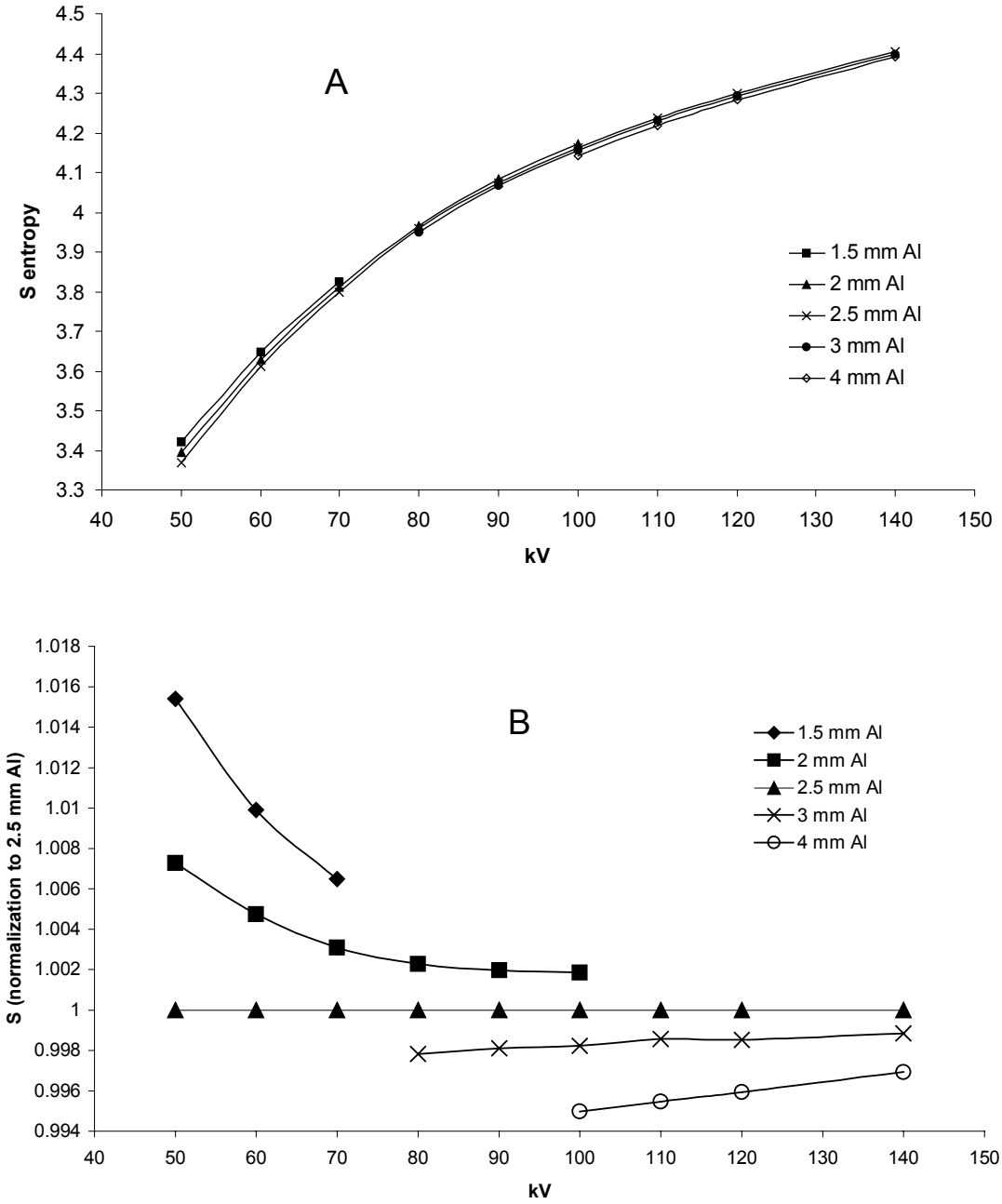

Figure 2 – A) The entropy, S, as a function of the tube voltage and with tube filtration as parameter. B) The same data using the result for 2.5 mm Al as normalisation.

We have two main remarks to Figure 2. First, if the tube voltage increases then the entropy of the spectrum increases, as we see in Figure 2A. Second, if the filtration increases then entropy decreases, which is seen better with the normalisation in Figure 2B. Both of the previous remarks are in agreement with our expectations. Higher tube voltages add and higher filtration removes spectral lines, which leads to the observed results of the entropy values. Furthermore,

the voltage has the strongest influence in the entropy value compared with the influence of the filtration.

We compared the entropy of Figure 2 with the HVL of Figure 1, by plotting one parameter as a function of the other (Figure 3).

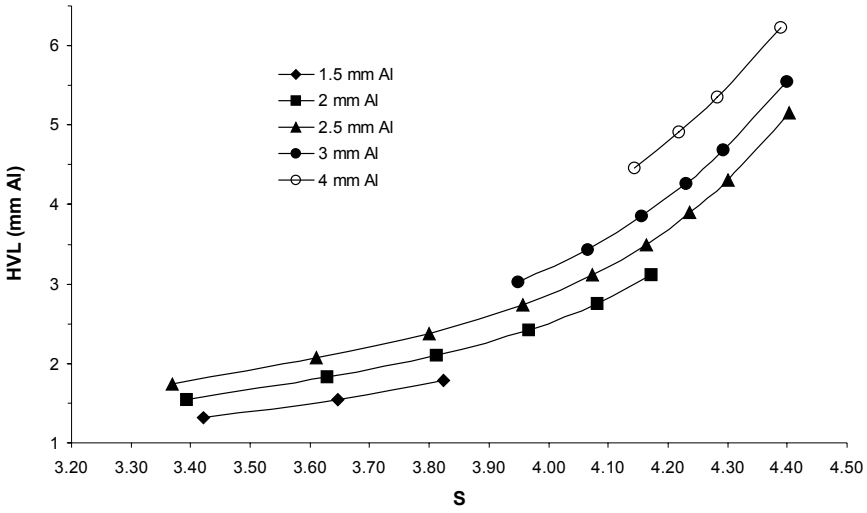

Figure 3 – Comparison between half value layer (HVL) and Entropy (S).

If the relationship between both, S and HVL (Figure 3), was linear, then the entropy merely reproduces the results obtained already with the parameter HVL, but that doesn't happen. A crucial difference exists between S and HVL, which is: the entropy are obtained only with the data of the primary spectra, while the HVL depends on what happen to photons as they interact in matter. In general the relationship between S and HVL is non-linear with higher HVL corresponding to higher entropy.

THE DEGRADATION OF THE ENERGY OF X-RAY PRIMARY BEAM.

In Figure 4 we present the main result of this work which is the entropy of the energy deposited in water as a function of depth.

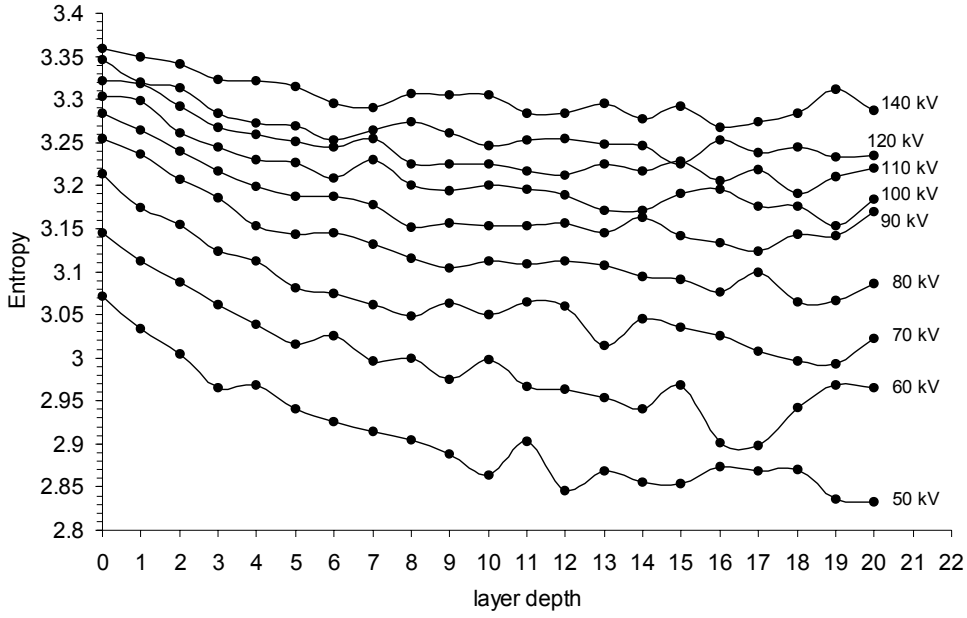

Figure 4 – Entropy as a function of depth in kilovoltage range 50 – 140 kVcp with filtration of 2.5 mm Al.

Each curve was obtained with 150000 primary photons, which is enough because of the simplicity of the geometry, allowing the determination of the entropy value with two decimal places. We obtained the primary spectra applying a rejection technique, leading to a typical mean square error of 0.007 for 50 kV or 0.003 for 140 kV, for example.

To analyse the results we used the function $y = a(b - \exp(-cx))$ to fit the results of the Figure 4. As an example in Figure 5 we present the result for 80 kV.

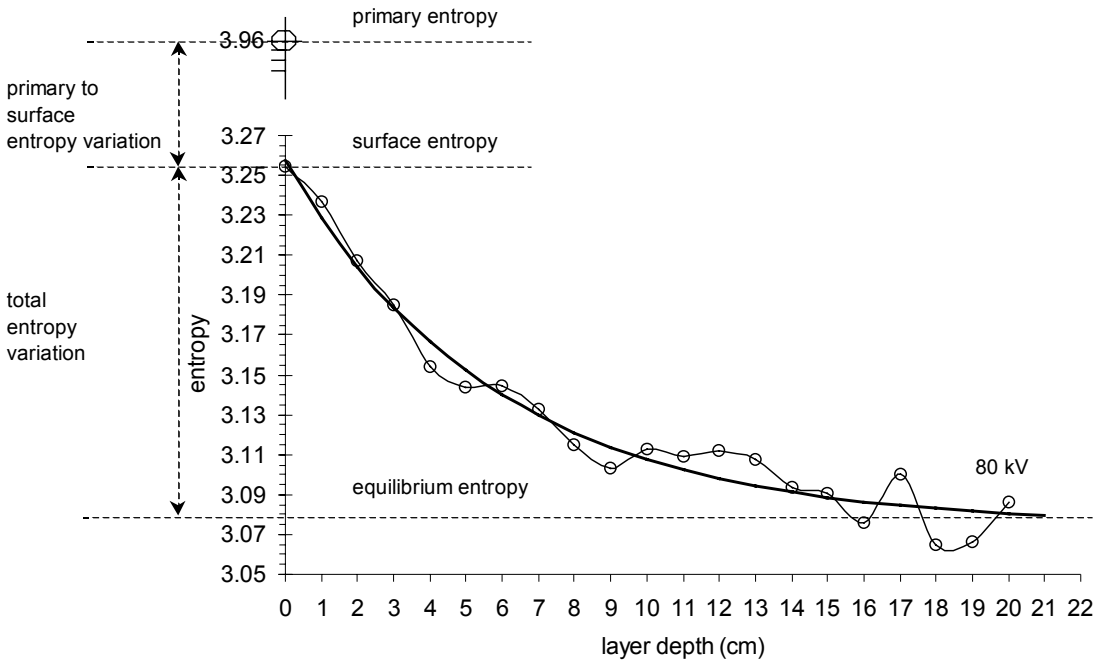

Figure 5 – Entropy for 80 kV with the fit and the definitions of primary entropy, surface entropy, equilibrium entropy, primary to surface entropy variation and total entropy variation.

From the function fit parameters we can obtain entropy values presented in Figure 5, as follows:

- Primary entropy, $S_{primary}$, is the entropy of the primary spectrum (Figure 2).
- Surface entropy = $S_{surf} = (ab - a)$
- Equilibrium entropy = $S_{eq} = (ab)$
- Total entropy variation = $S_T = (-a)$
- Primary to surface variation, S_{ps}, is obtained from the data of Figure 2, the entropy of primary spectra subtracting the value of S_{surf}.

Following the example of Figure 5 we present next the primary spectrum for 80 kV together with the distribution of energy deposited at the surface layer with 1 cm thickness and the equilibrium spectrum in a layer also with 1 cm thickness at 20 cm depth in water. The entropy values are, $S_{primary} = 3.96$, $S_{surf} = 3.26$ and $S_{eq} = 3.08$.

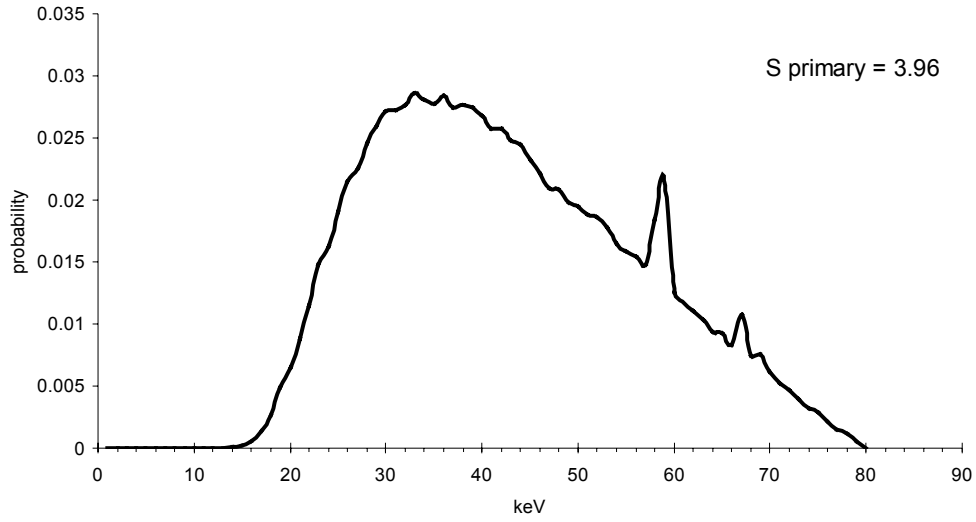

Figure 6 – Primary spectra for 80 kVcp with 2.5 mm Al of filtration, obtained using the rejection technique with the catalogue of Birch et al (1979). The entropy of the distribution is 3.96.

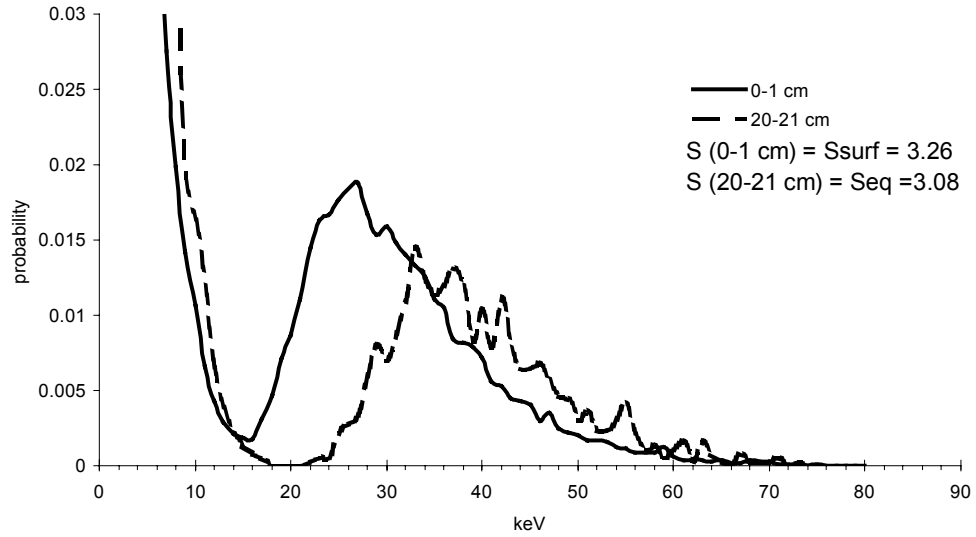

Figure 7 – Energy deposited in water in layers with 1 cm thickness, at surface and 20 cm depth from a 80 kV primary beam. The surface entropy is 3.26 and the equilibrium entropy is 3.08. The lower region of the spectra due to Compton is truncated in this figure but, in fact they extends until the origin of the axis

Fitting the data of Figure 4 we obtain the entropy values presented in Figure 8.

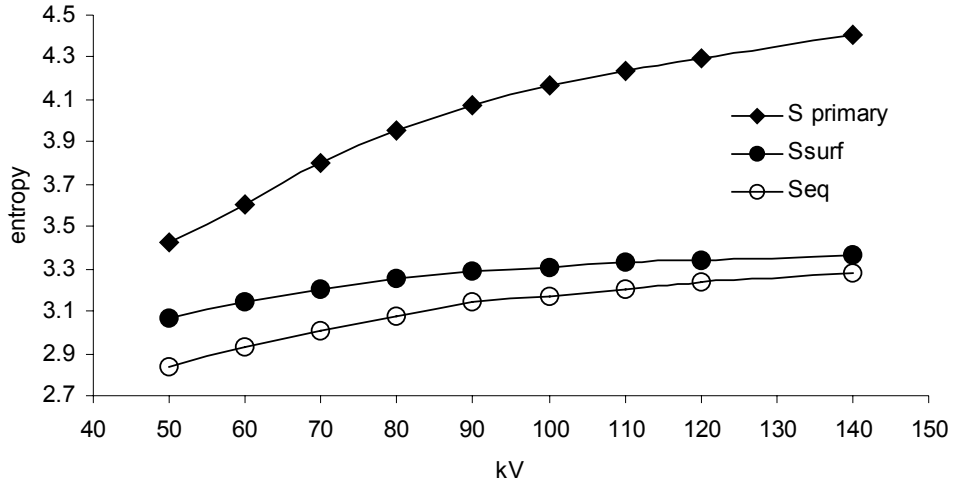

Figure 8 – Entropy values in the range 50 – 140 kVcp. S primary: entropy of the primary spectra, Ssurf: entropy at the water surface, Seq: entropy when the energy distribution reach equilibrium.

In general this entropy values increases when voltage increases. The entropy variation from primary to surface, S_{ps}, and from surface to the equilibrium, named S_T, are presented in **Figure 9**.

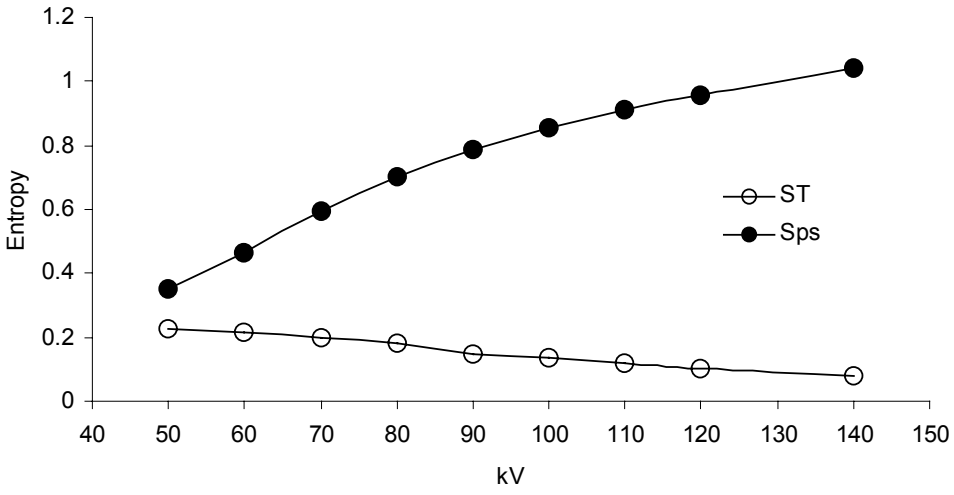

Figure 9 – Entropy variations in range 50 – 140 kVcp, from primary to surface, Sps, and from surface to equilibrium, ST.

As the voltage increases the variation from primary to surface increases while the total entropy variation decreases.

DISCUSSION

The decrease of the entropy with depth in water needs to be discussed. The entropy of the energy distribution depends on the number of spectral lines and their relative weight in the spectrum.

In radiation protection our concern goes mainly to the energy deposited in matter instead of the primary x-ray beam. If we focus on the energy deposited in matter and if we use purely qualitative arguments we can reach to erroneous conclusions.

From the literature it is well known the concept of beam hardening due to the removal of lower energy photons from the primary beam, leading to an increase in the average energy. However, as a consequence of the hardening of the primary beam, in what concerns to the energy deposited, we can expect a kind of softening phenomenon leading to the decrease of the average energy. As the primary beam interacts with matter, the Compton interaction adds a lot of new spectral lines to the lowest energy region of the distribution of the energy deposited in water, so together with the decrease of the average energy we expected an increase in the entropy value. The results we obtained don't confirm this expectation based in purely qualitative arguments.

From the point of view of the distribution of energy deposited our results show that the photons of high-energy region are removed by interactions without replacement, as we can see in Figure 8 and Figure 9. This may be the justification for the decrease of the entropy which we can associate a kind of softening process that is, however, not enough to increase the entropy.

Our question is what is the role of addition and removal of spectral lines due to the interaction of radiation with matter in the deposition of energy?

We conclude that, to the concept of hardening we should joint a concept of softening to the energy deposited in matter. These concepts are quantitatively well expressed by the value of the entropy of the energy spectrum.

The decrease of the entropy we found for X-ray beams (Figure 4) are the result of the hardening and softening processes we mention above. Qualitatively we are unable to distinguish, from the Figure 7, which is the hardest or the most softened spectrum. The entropy value allows the conclusion that the hardest is the surface spectrum and the softened is the spectrum at 20 cm depth.

CONCLUSION

In this work we apply the concept of entropy to describe the distribution of energy deposited in water from X-rays beams in range 50 – 140 kVcp. We found that the entropy of primary beams increases as the voltage increases. On the contrary, as the tube filtration increases the entropy decreases. This is an expected result because voltage adds and filtration removes spectral lines of the primary spectra.

The entropy of the energy deposited in layers with 1 cm thickness reveals always the same behaviour in the range studied, 50–140 kVcp. The entropy always decreases from primary to surface of water. Likewise inside water the entropy decreases always, showing a trend

towards an equilibrium value. We defined several parameters, for example, surface entropy, equilibrium entropy and total variation of entropy from surface to the equilibrium. As the voltage increases we found that the total variation of entropy decreases. This is due to the fact that for higher voltages the high energy photons have less and less weight comparing with the lower energy region that is continually reinforced due to Compton interaction.

The decrease of the entropy with depth in water, leads to the conclusion that exist a kind of hardening process in the energy deposition in matter due to the removal of high energy photons. This is in opposition with the hardening of the primary beam caused by the removal of low energy photons.

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