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ESTIMATION OF MONIN-OBUKHOV LENGTH USING RICHARDSON AND BULK RICHARDSON NUMBER

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ABSTRACT

The 1996 NOVA atmospheric boundary layer data from North Carolina are used in 30 minute's averages for five days. Because of missing data of friction velocity "u_*" and sensible heat flux "H", it is urgent to calculate "u_*" and "H" using the equations of logarithmic wind speed and net radiation (Briggs [7]), which are considered in this work. It is found that the correlation between the predicted and observed values of "u_*" and "H" is 0.88 and 0.86 respectively. A comparison is made of the Monin-Obukhov length scale "L" estimated using Richardson number "R_i" and bulk Richardson number "Ri_b" with L-value computed using formula of "L", it is found that the agreement between the predicted and observed values of "L" is better in the case "L" is estimated from the bulk Richardson number "Ri_b", rather than from the gradient Richarson number "R_i".

Keywords: *"Monin-Obukhov length scale, Bulk Richardson number, Friction velocity, Sensible heat flux*

INTRODUCTION

Numerical models often employ the Monin-Obukhov length scale, L, for characterizing the turbulence within the surface boundary layer, which is defined by

$$L = \frac{-\rho \; C_p T \; u_*^3}{kgH} \tag{1}$$

where ρ is the density of air at temperature T, C_p is specific heat capacity at constant pressure, u_* is the friction velocity, k is the Von Karman constant and H is the sensible heat flux. Because of the difficulty of characterizing u_* and H by direct measurements, the Monin-Obukhov length is often estimated from routine meteorological measurements, see Golder [1]. We compared the estimates of L, obtained using the gradient Richardson number and bulk Richardson number, with values of the Monin-Obukhov length, L, compared directly from the field measurements of u_* and H. The formulations used to estimate L using profile measurements of wind speed and temperature gradient are analogous to those described by Binkowski [2], and Barker and Baxter [3]. From a practical viewpoint, the bulk Richardson number is easy to compute requiring only a single wind speed measurements and a temperature difference measurement; while the gradient Richardson number requires measurements of both temperature and wind speed gradients. The bulk Richardson number may be more useful than the gradient Richardson number in routine measurement programs where estimates of the Monin-Obukhov length are desired.

THEORY

The minimum information needed to compute a gradient Richardson number, Ri, are wind speed and temperature at two levels, allowing Ri to be approximated as

$$Ri = \left(\frac{g \Delta z \Delta \theta}{\theta_1 \Delta u^2} \right) \tag{2}$$

where $\Delta\theta = \theta_2 - \theta_1$, $\Delta u = u_2 - u_1$ and $\Delta z = z_2 - z_1$ in which z_2 and z_1 are the upper and lower levels at which temperature and wind speed are measured, θ_2 and θ_1 are the mean potential temperature at the two levels, and u_1 and u_2 are the wind speeds at the two levels. A bulk Richardson number can be computed with a measurement of wind speed only at the upper level, see Barker and Baxter [3].

$$Ri_b = \left(\frac{g z_2 \Delta \theta}{\theta_2 \, u_2^2} \right) \tag{3}$$

Businger et al [4] suggested

$$\zeta = \frac{\phi_m^2}{\phi_h} Ri \tag{4}$$

where $\bar{z}/L = \zeta$, $\bar{z}$ is the geometric mean height of z_2 and z_1 used in the computation of Ri, and ϕ_m and ϕ_h are the nondimensional functions for wind shear and temperature gradient ϕ_m and ϕ_h have the form (Dyer [5])

$$\phi_m = \begin{cases} (1-\gamma\zeta)^{-1/4} & \zeta<0 \\ (1+\beta\zeta) & \zeta\geq 0 \end{cases} \quad (5)$$

$$\phi_h = \begin{cases} R(1-\gamma'\zeta)^{-1/2} & \zeta<0 \\ (R+\beta'\zeta) & \zeta\geq 0 \end{cases} \quad (6)$$

Dyer [5] suggested k=0.4, R=1.0, $\beta = \beta' = 5$ and $\gamma = \gamma' = 16$. Substituting equations (5) and (6) into equation (4) yields an empirical relationship between Ri and L, which can be solved by iteration for L, given some value for Ri, is the approximation (Binkowski [2]), yielding:

$$\zeta = \begin{cases} (Ri/R)(1-\gamma' Ri)^{1/2}/(1-\gamma Ri)^{1/2} & Ri \leq 0 \\ (Ri/R)/(1-Ri\beta^2/\beta') & 0<Ri\beta^2/\beta'<1 \end{cases} \quad (7)$$

Barker and Baxter [3] discussed how the bulk Richardson number is related to the Monin-Obukhov length. The relationship between the Monin-Obukhov length and the bulk Richardson number is then

$$\bar{z} = (z_1 + z_2)/2$$

in which

$$z_2 / L = kRi_b F^2 / G \quad (8)$$

where

$$F = \frac{u}{u_*} = \begin{cases} \ln\left[\left(\frac{z_2}{z_o}\right)\left(\frac{\eta_o^2+1}{\eta_2^2+1}\right)\left(\frac{\eta_o+1}{\eta_2+1}\right)^2\right] + 2\tan^{-1}\left(\frac{\eta_o-\eta_2}{1+\eta_o\eta_2}\right) & L\leq 0, \\ \ln(z_2/z_o)+\beta z_2/L, & L\geq 0, \end{cases} \quad (9)$$

$$G = \frac{\Delta\theta \ u_*}{(-\overline{w'\theta'})} = \begin{cases} R\ln\left[\left(\frac{z_2}{z_1}\right)\left(\frac{\lambda_1+1}{\lambda_2+1}\right)^2\right], & L\leq 0, \\ R\left[\ln(z_2/z_1)\right]+\beta'(z_2-z_1)/L, & L>0 \end{cases} \quad (10)$$

and

$$\eta_2 = (1-\gamma z_2 / L)^{1/4},$$
$$\eta_o = (1-\gamma z_o / L)^{1/4},$$
$$\lambda_1 = (1-\gamma' z_1 / L)^{1/2},$$
$$\lambda_2 = (1-\gamma' z_2 / L)^{1/2}.$$

Where z_0 is the roughness height. The expression for F and G given in equations (9) and (10) for unstable conditions are equivalent to those recommended by Benoit [6]. A comparison is made of the L-values estimated using Equations (7) and (8) with the L-values computed directly using equation (1).

COMPARISON

Using the 1996 NOVA atmospheric boundary layer data (unpublished) from North Carolina a comparison was made of the L-values estimated using equations (7) and (8), with L-values computed using equation (1). The NOVA data are reported in 30 minute averages for five days. The surface roughness was assumed to be 0.06 m, the displacement length was equal to 0.5 m. The half hour temperatures at 2 m were used with equation (1) to compute the Monin-Obukhov length. The wind speed and temperature data at the 2 and 10 m height are used to compute Ri and Ri_b in view of equations (7) and (8) to estimate values for the Monin -Obukhov length.

Because of missing data from the NOVA tower for friction velocity "u_*" and sensible heat flux "H", the friction velocity was calculated from the formula

$$u_* = \bar{u}k / \ln(\frac{z-d}{z_o}) \tag{11}$$

where $\bar{u}$ is the mean wind speed, k the Von Karman constant equal to 0.4, z the height in meter, d the displacement length equal to 0.5 m, and z_o the roughness height equal to 0.06 m. We used another formula for the sensible heat flux which is very near to fact when using the form (Briggs [7])

$$H = c_o N \tag{12}$$

where c_o is a constant equal to the value 0.17 and N is the net radiation in units of (W/m^2).

Figure (1) shows comparison between calculated and observed values of friction velocity which agree very well, with correlation coefficient 0.88, mean of calculated and observed value equal 0.17 and 0.16 respectively, and standard deviation of calculated and observed value equal 0.09 and 0.10 respectively. Note the missing data in the observations. Figure (2) shows the relation between calculated and observed sensible heat flux at 2 and 10 m, the correlation between them at 10 m equals 0.86. As shown from these statistics, the predicted and observed values agree well.

Fig. (1) Comparison between calculated and observed friction velocity at 10m.

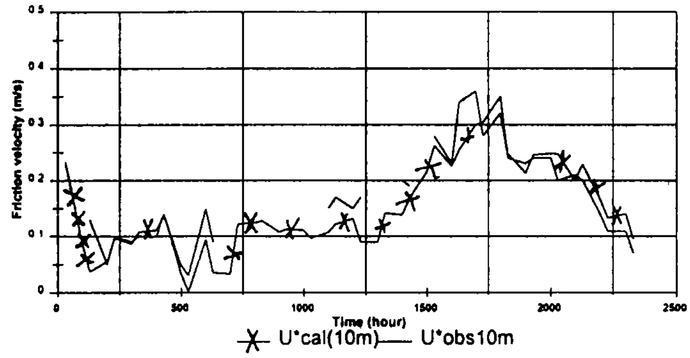
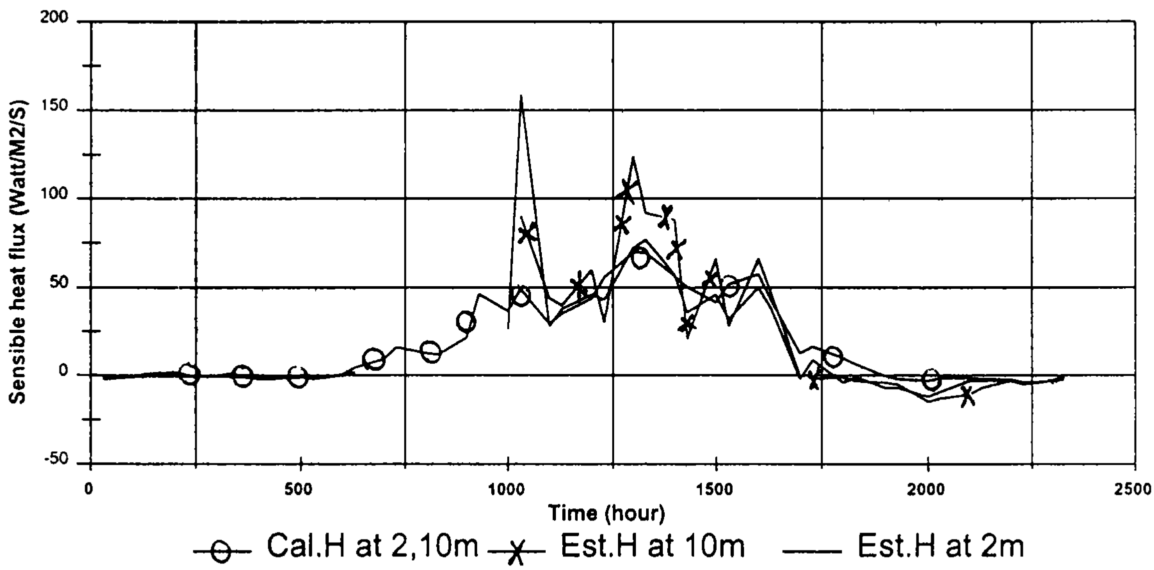


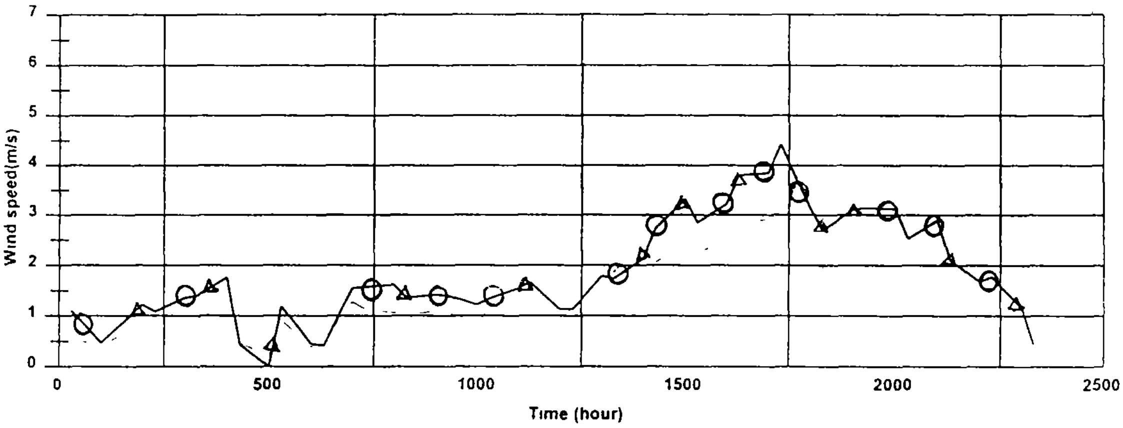
Fig. (2) Comparison between calculated and observed sensible heat flux at 2 and 10 m



We have calculated the wind speed from Businger [8], with this formula

$$\bar{u} = \frac{u_*}{k} \ln(\frac{z + z_o}{z_o}) \quad (13)$$

Fig. (3) Comparison between observed and calculated wind speed with time



We get the best agreement between the observed and calculated values, as shown in Fig (3) The non-dimensional profiles of wind (u) and potential temperature (θ) can be introduced in the following forms:

$$\Phi_m = \frac{kz}{u_*}\frac{\partial u}{\partial z} \tag{14}$$

$$\Phi_h = \frac{kz}{\theta_*}\frac{\partial \theta}{\partial z} \tag{15}$$

where θ_* is a scaling temperature. These may be integrated following Panofsky [9] from z_o to z in the surface layer, yielding wind and potential temperature profiles

$$u(z) = \frac{u_*}{k}\left[\ln\frac{z-d}{z_o} - \psi_m(\frac{z}{L},\frac{z_o}{L})\right] \tag{16}$$

$$\theta(z) = \theta_o + r\frac{\theta_*}{k}\left[\ln\frac{z-d}{z_o} - \psi_h(\frac{z}{L},\frac{z_o}{L})\right] \tag{17}$$

Lee [10] took the value of z/L in unstable and stable cases as:

$$z/L = (\frac{z-d}{z-z_o})\ln(\frac{z-d}{z_o})\frac{Ri_b}{r}(\frac{1}{1-\beta' Ri_b})$$

and

$$z/L = \frac{\left(\frac{z-d}{z-z_o}\right)\ln\left(\frac{z-d}{z_o}\right)}{2\left(\frac{\beta}{r}\right)(\beta Ri_b - 1)}\left[-2\left(\frac{\beta}{r}\right)Ri_b + 1 - \left(1 + \frac{4\beta(1-r)Ri_b}{r^2}\right)^{1/2}\right]$$

respectively, where the values of r and β are constants equal to 1 and 5 for Dyer [5], 0.74 and 4.7 for Businger [4], and the constant β' is equal to 0.077 and 0.367 when we used the Dyer and Businger formulas, respectively. After substituting the values of z/L in the two equations (14) and (15), we obtain the values of u and θ plotted below. Figures (4) and (5) show the best result between observed and calculated wind speed and potential temperature at 2 meters.

Figures (6) and (7) show the variation of estimated values of absolute L from gradient Richardson number and bulk Richardson number and computed values of absolute L with time using Dyer's [5] suggestions of k=0.4, r=1.0, $\gamma = \gamma' = 16$ and $\beta = \beta' = 5$. The result between the computed and estimated values of absolute L is better in the case where absolute L is estimated from the bulk Richardson number rather than the gradient Richardson number.

Fig. (4) Comparisons between the values of observed and calculated wind speed, using the Dyer's (1974) and Businger's (1971) constants

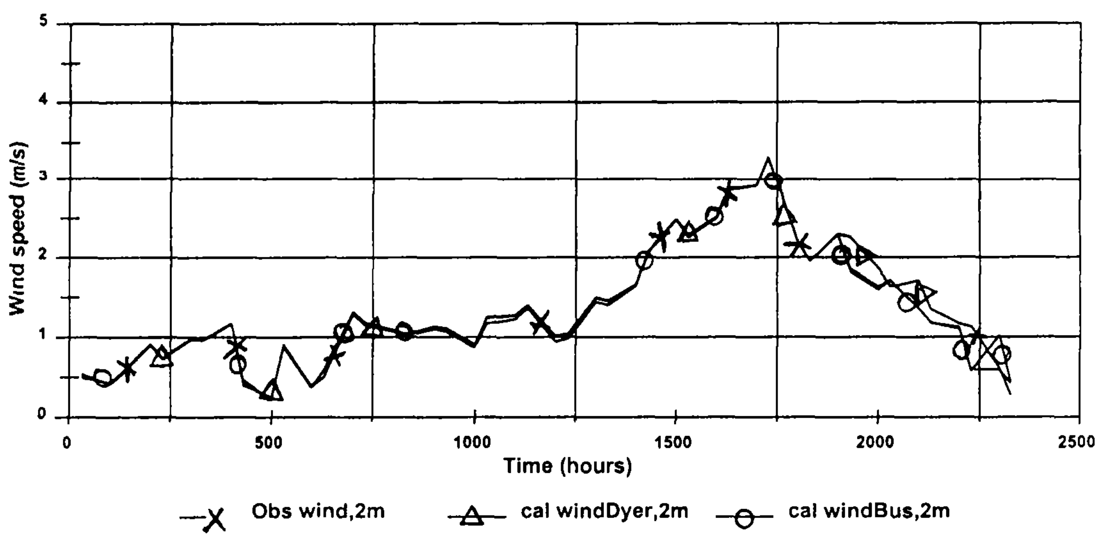


Fig. (5) Comparison between the values of observed and calculated potential temperature, using the Dyer's (1974) and Businger's (1971) constants

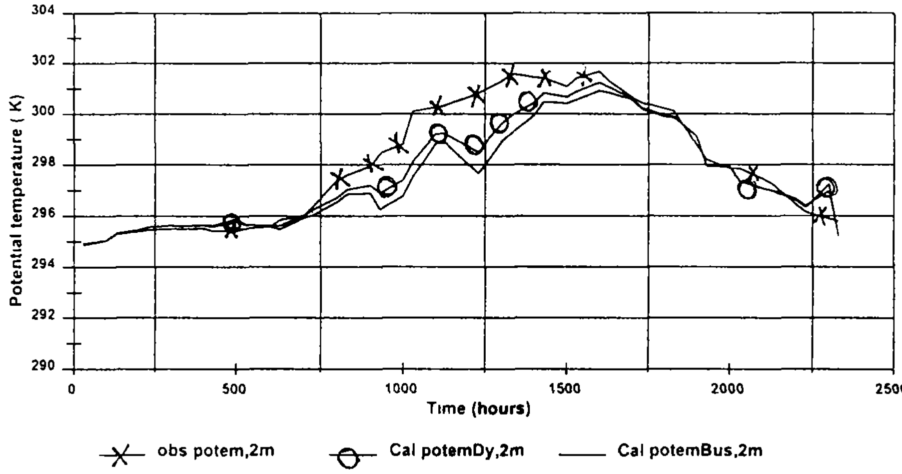


Fig. (6) Comparison of estimated M-O scaling length absolute L using local gradient Richardson number, computed values of absolute L with time using Dyer's constants

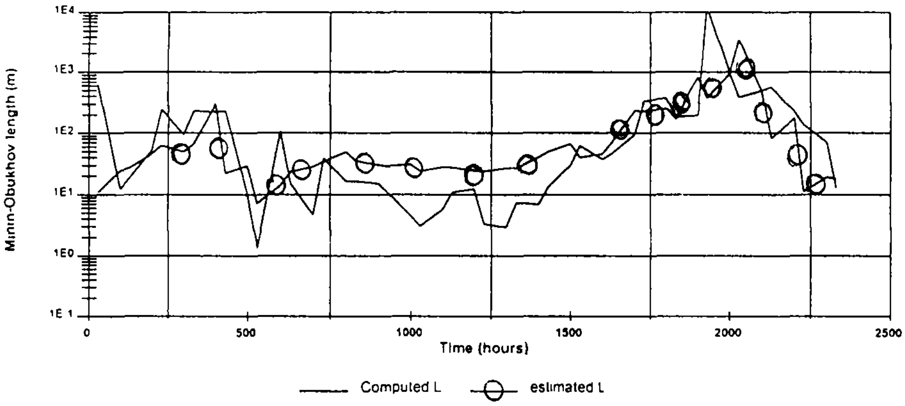
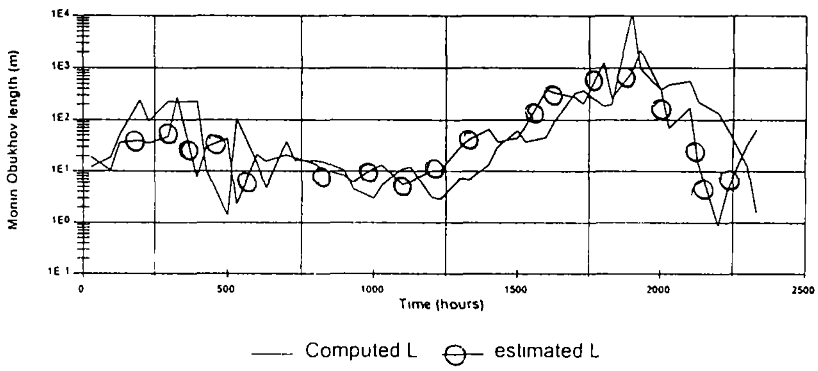


Fig. (7) Comparison of estimated M-O scaling length absolute L using local bulk Richardson number, computed values of absolute L with time using Dyer's constants



Figures (8) and (9) show the same thing except using another constant for Businger's et al. [4], k=0.4, r=0.74, $\beta = \beta' = 4.7$, $\gamma' = 9$, $\gamma = 15$. Also we find that the estimated values of absolute L using bulk Richardson number has good agreement with computed values of absolute L. Both gradient Richardson number approach and bulk Richardson number for estimating L from equations (7) and (8) and computed L from formula (1) yield good results. The statistics are shown in Table (1).

Fig. (8) Comparison of estimated M-O scaling length absolute L using local gradient Richardson number, computed values of absolute L with time using Businger's constants

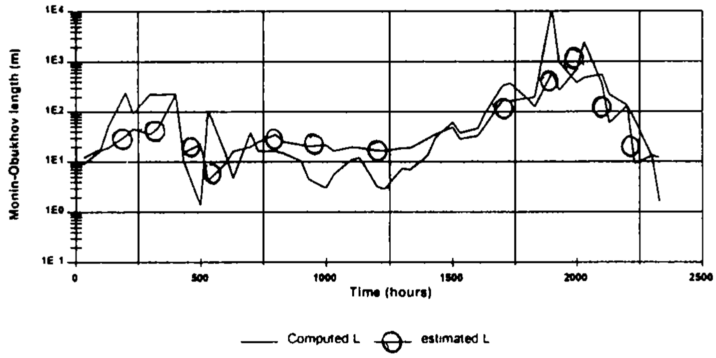


Fig. (9) Comparison of estimated M-O scaling length absolute L using local bulk Richardson number, computed values of absolute L with time using Businger's constant

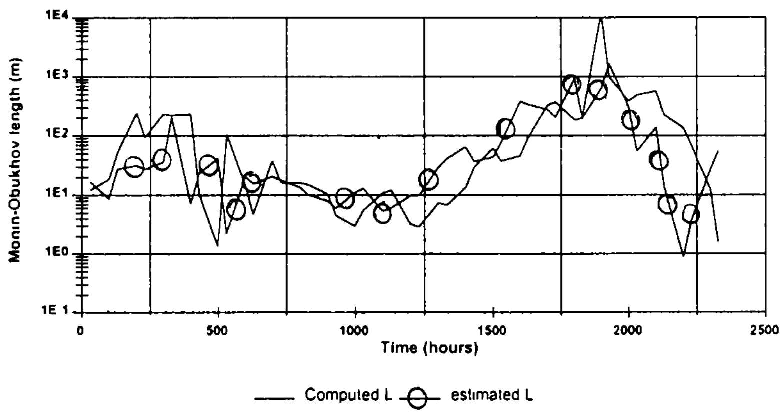


Table 1. Evaluation statistics for the comparison of computed and estimated Monin-Obukhov lengths

	Dyer's constants				Businger's constants			
	Richardson number		bulk Richardson number		Richardson number		bulk Richardson number	
	OBS	PRED	OBS	PRED	OBS	PRED	OBS	PRED
Mean	399 5	141 5	441 6	161 3	431 9	120 5	431 9	120 5
SD	1303 8	298 4	1395 7	485 7	1378 3	328 7	1378 3	328 7
OBS RANG	79-9337 5		79-9337 5		79-9337 5		79-9337 5	
NS	226		220		226		226	
R	0 33		0 49		0 45		0.45	
FB	0 95		0 93		1 13		1 13	
NMSE	28 2		22 3		32 4		32 4	

where the mean is the average of the observed and predicted L, SD is the standard deviation of the two values, NS is the sampling size, R is the correlation coefficient, FB is the friction bias, and NMSE is the normalized mean square error. A good model is indicated by FB value close to zero; a NMSE value of about 1.0 indicated that the typical difference between predictions and observations is approximately equal to the mean. From Table (1), one finds that Dyer's constant for estimating L from the bulk Richardson number is better than estimating L from gradient Richardson number. Also when we use Businger's constants one gets the same result in both cases, and this result agrees with Irwin [11].

Figures (10) and (11) show variation values between computed "L" and estimated "L" from gradient and bulk Richardson number using Dyer's and Businger's constant values.

Fig. (10) Variation values between **computed** L **and** estimated L from gradient and bulk Richardson number using Dyer's constants.

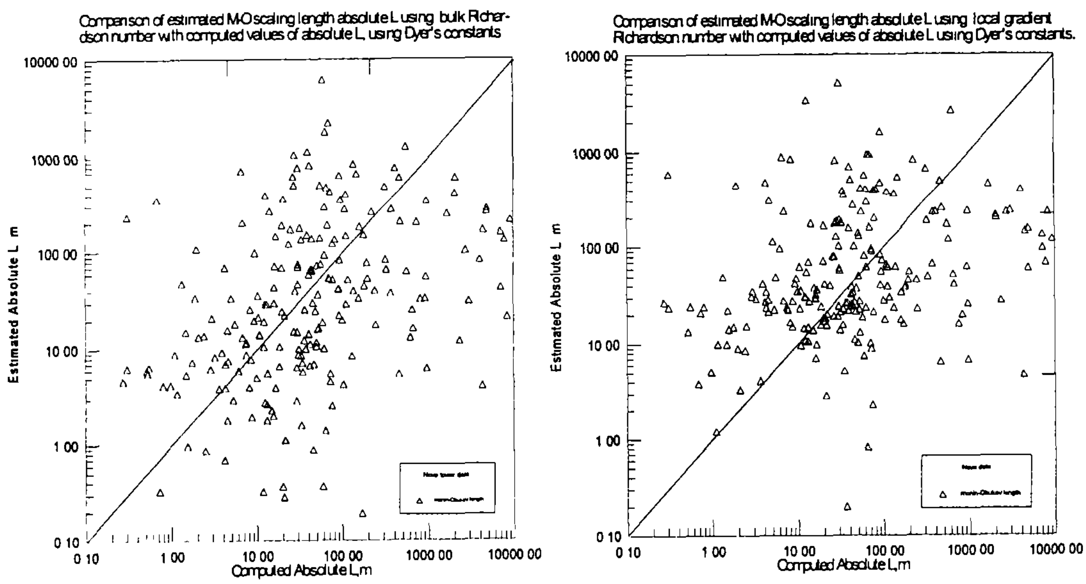


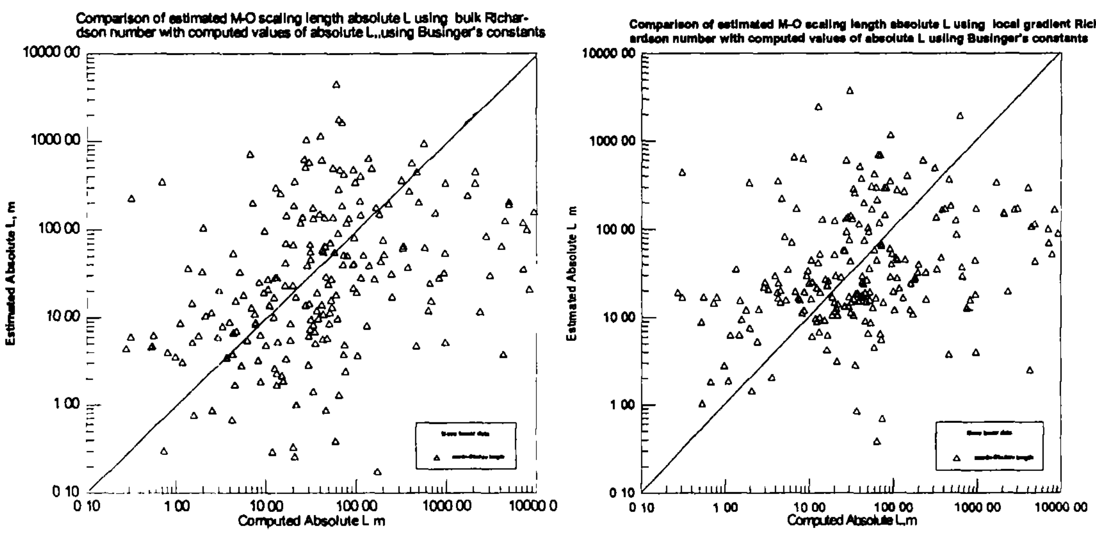
Table 2. Values of Monin-Obukhov length with stability class for two estimation schemes, compared with direct calculation from Eq.1.

	Stability	1/L(Eq.1)	1/L(Eq.7)	1/L(Eq.8)	
	A	38	24	26	
	B	24	75	29	
	C	44	47	39	
	D	49	51	49	
	E	46	25	31	
	F	13	5	10	
	G	22	13	48	

where A is extremely-, B moderately-, and C slightly unstable, D Neutral, E is slightly-, F moderately-, and G extremely stable.

Table 2 gives the relation to the Pasquill stability using the previous three methods which shows that the number of the stability class in the case of the bulk Richardson number (Eq. 8) yields as good an estimate the Monin-Obukhov length (Eq. 1) as the gradient Richardson number (Eq. 7)

Fig. (11) Variation values between computed L and estimated L from gradient and bulk Richardson number using Businger's constants.



CONCLUSIONS

Upon comparing calculated and observed values of friction velocity, which agree very well, it is to be noted that the correlation coefficient is 0.88, the mean of calculated and observed value equal 0.17 and 0.16 respectively, and the standard deviation of calculated and observed value equal 0.09 and 0.10 respectively. The correlation between calculated and observed sensible heat flux at 2 and 10 m, equals 0.86. When predicting Monin-Obukhov lengths to evaluate atmospheric stability, we find that the agreement between the predicted and observed " L" is better in the case where L is estimated from the bulk Richardson number "Ri_b", rather than from the gradient Richardson number "Ri" except at stability G.

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