E2-2006-16

Kh. M. Beshtoev

EXPRESSIONS FOR NEUTRINO WAVE FUNCTIONS AND TRANSITION PROBABILITIES AT THREE-NEUTRINO OSCILLATIONS IN VACUUM AND SOME OF THEIR APPLICATIONS

Бештоев Х. М. E2-2006-16

Выражения для волновых функций и вероятностей переходов при трехнейтринных осцилляциях в вакууме и некоторые их применения

Рассмотрены трехнейтринные смешивания и осцилляции в общем случае и получены выражения для волновых функций в трех случаях: с CP-нарушением, без CP-нарушения и когда переходы $\nu_e \leftrightarrow \nu_\tau$ отсутствуют (в некоторых работах указывается на такую возможность). Проведен анализ с использованием имеющихся экспериментальных данных. Этот анализ определенно указывает на то, что для солнечных нейтрино переходы $\nu_e \leftrightarrow \nu_\tau$ не могут быть закрыты. Показано, что такая же возможность, когда $\beta = 0$, не может быть реализована при использовании механизма резонансного усиления осцилляций нейтрино в веществе (Солнце). Обнаружено, что требование положительной определенности вероятности переходов при $\nu_e \leftrightarrow \nu_e$-осцилляциях выполняется только, если угол смешивания ν_e-, ν_τ-нейтрино $\beta \leqslant 15 \div 17^\circ$.

Работа выполнена в Лаборатории физики частиц ОИЯИ и Научно-исследовательском институте прикладной математики и автоматизации КБНЦ РАН, Нальчик, Россия.

Сообщение Объединенного института ядерных исследований. Дубна, 2006

Beshtoev Kh. M. E2-2006-16

Expressions for Neutrino Wave Functions and Transition Probabilities at Three-Neutrino Oscillations in Vacuum and Some of Their Applications

I have considered three-neutrino vacuum transitions and oscillations in the general case and obtained expressions for neutrino wave functions in three cases: with CP violation, without CP violation and in the case when direct $\nu_e \leftrightarrow \nu_\tau$ transitions are absent $\beta(\theta_{13}) = 0$ (some works indicate this possibility). Then using the existing experimental data some analysis has been fulfilled. This analysis definitely has shown that direct transitions $\nu_e \leftrightarrow \nu_\tau$ cannot be closed for the Solar neutrinos, i. e., $\beta(\theta_{13}) \neq 0$. It is also shown that the possibility that $\beta(\theta_{13}) = 0$ cannot be realized by using the mechanism of resonance enhancement of neutrino oscillations in matter (the Sun). It was found out that the probability of $\nu_e \leftrightarrow \nu_e$ neutrino transitions is positive defined value, if in reality neutrino oscillations take place, only if the angle of ν_e, ν_τ mixing $\beta \leqslant 15 \div 17^\circ$.

The investigation has been performed at the Laboratory of Particle Physics, JINR, and at the Scientific Research Institute of Applied Mathematics and Automation, KBSC of RAS, Nalchik, Russia.

Communication of the Joint Institute for Nuclear Research. Dubna, 2006

1. INTRODUCTION

The suggestion that, in analogy with $K^0, \bar{K}^0$ oscillations, there could be neutrino–antineutrino oscillations ($\nu \to \bar{\nu}$) was considered by Pontecorvo in 1957 [1]. It was subsequently considered by Maki et al. [2] and Pontecorvo [3] that there could be mixings (and oscillations) of neutrinos of different flavors (i. e., $\nu_e \to \nu_\mu$ transitions).

In the general case there can be two schemes (types) of neutrino mixings (oscillations): mass mixing schemes and charge mixing schemes (as it takes place in the vector dominance model or vector boson mixings in the standard model of electroweak interactions) [4].

In the standard theory of neutrino oscillations [5] it is supposed that physically observed neutrino states ν_e, ν_μ, ν_τ have no definite masses and that they are directly produced as mixture of the ν_1-, ν_2-, ν_3-neutrino states. However, the computation has shown that these neutrinos have definite masses and transition widths [4]. Then neutrino mixings are determined by the neutrino mass matrix and neutrino mixing parameters are expressed through elements of the neutrino mass matrix.

In the scheme of charge mixings the oscillation parameters are expressed through weak interaction couple constants (charges) and neutrino masses [4].

In both cases the neutrino mixing matrix V can be given [4] in the following convenient form proposed by Maiani [6] ($\theta = \theta_{12}, \beta = \theta_{13}, \gamma = \theta_{23}$):

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} c_\beta & 0 & s_\beta \exp(-i\delta) \\ 0 & 1 & 0 \\ -s_\beta \exp(i\delta) & 0 & c_\beta \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

$$\begin{aligned} c_{e\mu} &= \cos\theta, \quad s_{e\mu} = \sin\theta, \quad c^2_{e\mu} + s^2_{e\mu} = 1; \\ c_{e\tau} &= \cos\beta, \quad s_{e\tau} = \sin\beta, \quad c^2_{e\tau} + s^2_{e\tau} = 1; \\ c_{\mu\tau} &= \cos\gamma, \quad s_{\mu\tau} = \sin\gamma, \quad c^2_{\mu\tau} + s^2_{\mu\tau} = 1; \\ &\exp(i\delta) = \cos\delta + i\sin\delta. \end{aligned} \tag{2}$$

Now we will come to computation of neutrino wave functions $\Psi_{\nu_e}, \Psi_{\nu_\mu}, \Psi_{\nu_\tau}$ and to a probability of transitions (oscillations) of these neutrinos.

2. GENERAL EXPRESSIONS FOR NEUTRINO WAVE FUNCTIONS AND PROBABILITIES AT THREE-NEUTRINO TRANSITIONS (OSCILLATIONS) IN VACUUM IN DEPENDENCE ON TIME

Using the above matrix V, we can connect the wave functions of physical neutrino states $\Psi_{\nu_e}, \Psi_{\nu_\mu}, \Psi_{\nu_\tau}$ with the wave functions of intermediate neutrino states $\Psi_{\nu_1}, \Psi_{\nu_2}, \Psi_{\nu_3}$ and write down them in a component-wise form [5]:

$$\Psi_{\nu_l} = \sum_{k=1}^{3} V^*_{\nu_l \nu_k} \Psi_{\nu_k},$$

$$\Psi_{\nu_k} = \sum_{k=l}^{3} V_{\nu_k \nu_l} \Psi_{\nu_l}, \qquad l = e, \mu, \tau, \qquad k = 1 \div 3, \tag{3}$$

where Ψ_{ν_k} is a wave function of neutrino with momentum p and mass m_k. We suppose that neutrino mixings (oscillations) are virtual if neutrinos have different masses. If we suppose that these transitions are real, as it is supposed in the standard theory of neutrino oscillations, then it is necessary to accept that expression (3) is based on a supposition that mass differences of ν_k neutrinos is so small that coherent neutrino states are formed in the weak interactions (computation has shown that this condition is not fulfilled, i. e., neutrino as wave packet is unstable and decays).

$$\Psi_{\nu_k}(t) = \mathrm{e}^{-iE_k t} \Psi_{\nu_k}(0). \tag{4}$$

Then

$$\Psi_{\nu_l(t)} = \sum_{k=1}^{3} \mathrm{e}^{-iE_k t} V^*_{\nu_l \nu_k} \Psi_{\nu_k}(0). \tag{5}$$

Using unitarity of matrix V or expression (3) we can rewrite expression (5) in the following form:

$$\Psi_{\nu_l}(t) = \sum_{l'=e,\mu,\tau} \sum_{k=1}^{3} V_{\nu_{l'} \nu_k} \mathrm{e}^{-iE_k t} V^*_{\nu_l \nu_k} \Psi_{\nu_{l'}(0)}, \tag{6}$$

and introducing symbol $b_{\nu_l \nu_{l'}}(t)$

$$b_{\nu_l \nu_{l'}}(t) = \sum_{k=1}^{3} V_{\nu_{l'} \nu_k} \mathrm{e}^{-iE_k t} V^*_{\nu_l \nu_k}, \tag{7}$$

we obtain

$$\Psi_{\nu_l}(t) = \sum_{l'=e,\mu,\tau} b_{\nu_l \nu_{l'}}(t) \Psi_{\nu_{l'}}(0), \tag{8}$$

where $b_{\nu_l\nu_{l'}}(t)$ is the amplitude of transition probability $\Psi_{\nu_l} \to \Psi_{\nu_{l'}}$.
And the corresponding transition probability $\Psi_{\nu_l} \to \Psi_{\nu_{l'}}$ is:

$$P_{\nu_l\nu_{l'}}(t) = \mid \sum_{k=1}^{3} V_{\nu'_l\nu_k} \mathrm{e}^{-iE_k t} V^*_{\nu_l\nu_k} \mid^2 . \tag{9}$$

It is obvious that

$$\sum_{l'=e,\mu,\tau} P_{\nu_{l'}\nu_l}(t) = 1. \tag{10}$$

2.1. Expressions for Neutrino Wave Functions of ν_e, ν_μ, $\nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ Transitions (Oscillations) with CP Violation in Vacuum. The wave functions of $\nu_e, \nu_\mu, \nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ transitions with CP violation have the following form:

1) For the case of $\nu_e \to \nu_e, \nu_\mu, \nu_\tau$ transitions:

$$\begin{gathered}
\Psi_{\nu_e\to\nu_e,\nu_\mu,\nu_\tau}(t) = [\cos^2(\beta)\cos^2(\theta)\exp(-iE_1 t) + \cos^2(\beta)\sin^2(\theta) \\
\exp(-iE_2 t) + \sin^2(\beta)\exp(-iE_3 t)]\Psi_{\nu_e}(0)+ \\
+ [\cos(\beta)\cos(\theta)\exp(-iE_1 t)(-\cos(\gamma)\sin(\theta)- \\
- \sin(\beta)\exp(-i\delta)\sin(\gamma)\cos(\theta))+ \\
+ \cos(\beta)\sin(\theta)\exp(-iE_2 t)(\cos(\gamma)\cos(\theta)- \\
-\sin(\beta)\exp(-i\delta)\sin(\gamma)\sin(\theta))+ \\
+ \sin(\beta)\exp(-i\delta)\exp(-iE_3 t)\sin(\gamma)\cos(\beta)]\Psi_{\nu_\mu}(0)+ \\
+ [\cos(\beta)\cos(\theta)\exp(-iE_1 t)(\sin(\gamma)\sin(\theta)- \\
- \sin(\beta)\exp(-i\delta)\cos(\gamma)\cos(\theta))+ \\
+ \cos(\beta)\sin(\theta)\exp(-iE_2 t)(-\sin(\gamma)\cos(\theta)- \\
- \sin(\beta)\exp(-i\delta)\cos(\gamma)\sin(\theta))+ \\
+\sin(\beta)\exp(-i\delta)\exp(-iE_3 t)\cos(\gamma)\cos(\beta)]\Psi_{\nu_\tau}(0).
\end{gathered} \tag{11}$$

2) For the case of $\nu_\mu \to \nu_e, \nu_\mu, \nu_\tau$ transitions:

$$\begin{gathered}
\Psi_{\nu_\mu\to\nu_e,\nu_\mu,\nu_\tau}(t) = [(-\sin(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)- \\
- \cos(\gamma)\sin(\theta))\exp(-iE_1 t)\cos(\beta)\cos(\theta)+ \\
+ (-\sin(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)+ \\
+ \cos(\gamma)\cos(\theta))\exp(-iE_2 t)\cos(\beta)\sin(\theta)+
\end{gathered}$$

$$+\sin(\gamma)\cos(\beta)\exp(-iE_3t)\sin(\beta)\exp(i\delta)]\Psi_{\nu_e}(0)+$$
$$+[(-\sin(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)-$$
$$-\cos(\gamma)\sin(\theta))\exp(-iE_1t)(-\cos(\gamma)\sin(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\sin(\gamma)\cos(\theta))+$$
$$+(-\sin(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)+$$
$$+\cos(\gamma)\cos(\theta))\exp(-iE_2t)(\cos(\gamma)\cos(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\sin(\gamma)\sin(\theta))+$$
$$+\sin^2(\gamma)\cos^2(\beta)\exp(-iE_3t)]\Psi_{\nu_\mu}(0)+$$
$$+[(-\sin(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)-$$
$$-\cos(\gamma)\sin(\theta))\exp(-iE_1t)(\sin(\gamma)\sin(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\cos(\gamma)\cos(\theta))+$$
$$+(-\sin(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)+$$
$$+\cos(\gamma)\cos(\theta))\exp(-iE_2t)(-\sin(\gamma)\cos(\theta)- \qquad (12)$$
$$-\sin(\beta)\exp(-i\delta)\cos(\gamma)\sin(\theta))+$$
$$+\sin(\gamma)\cos^2(\beta)\exp(-iE_3t)\cos(\gamma)]\Psi_{\nu_\tau}(0).$$

3) For the case of $\nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ transitions:

$$\Psi_{\nu_\tau\to\nu_e,\nu_\mu,\nu_\tau}(t) = [(-\cos(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)+\sin(\gamma)\sin(\theta))$$
$$\exp(-iE_1t)\cos(\beta)\cos(\theta)+(-\cos(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)-$$
$$-\sin(\gamma)\cos(\theta))\exp(-iE_2t)\cos(\beta)\sin(\theta)+$$
$$+\cos(\gamma)\cos(\beta)\exp(-iE_3t)\sin(\beta)\exp(i\delta)]\Psi_{\nu_e}(0)+$$
$$+[(-\cos(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)+$$
$$+\sin(\gamma)\sin(\theta))\exp(-iE_1t)(-\cos(\gamma)\sin(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\sin(\gamma)\cos(\theta))+$$
$$+(-\cos(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)-$$
$$-\sin(\gamma)\cos(\theta))\exp(-iE_2t)(\cos(\gamma)\cos(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\sin(\gamma)\sin(\theta))+$$
$$+\sin(\gamma)\cos^2(\beta)\exp(-iE_3t)\cos(\gamma)]\Psi_{\nu_\mu}(0)+$$
$$+[(-\cos(\gamma)\sin(\beta)\exp(i\delta)\cos(\theta)+$$

$$+\sin(\gamma)\sin(\theta))\exp(-iE_1t)(\sin(\gamma)\sin(\theta)-$$
$$-\sin(\beta)\exp(-i\delta)\cos(\gamma)\cos(\theta))+$$
$$+(-\cos(\gamma)\sin(\beta)\exp(i\delta)\sin(\theta)-$$
$$-\sin(\gamma)\cos(\theta))\exp(-iE_2t)(-\sin(\gamma)\cos(\theta)- \tag{13}$$
$$-\sin(\beta)\exp(-i\delta)\cos(\gamma)\sin(\theta))+$$
$$+\cos^2(\gamma)\cos^2(\beta)\exp(-iE_3t)]\Psi_{\nu_\tau}(0).$$

2.2. Expressions for Neutrino Wave Functions and Probability of ν_e, ν_μ, $\nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ Transitions (Oscillations) without CP Violation in Vacuum. If we do not take CP violation into account, then the expression for the amplitude of neutrino transitions has the following forms:
1) If primary neutrinos are ν_e ones, then for the neutrino wave function for $\nu_e \to \nu_e$, $\nu_e \to \nu_\mu$, and $\nu_e \to \nu_\tau$ transitions we get

$$\Psi_{\nu_e\to\nu_e,\nu_\mu,\nu_\tau}(t) = [\cos^2(\beta)\cos^2(\theta)\exp(-iE_1t)+$$
$$+\cos^2(\beta)\sin^2(\theta)\exp(-iE_2t)+$$
$$+\sin^2(\beta)\exp(-iE_3t)]\Psi_{\nu_e}(0)+ \tag{14}$$
$$+[\cos(\beta)\cos(\theta)\exp(-iE_1t)(-\sin(\gamma)\sin(\beta)\cos(\theta)-$$
$$-\cos(\gamma)\sin(\theta))+\cos(\beta)\sin(\theta)\exp(-iE_2t)(-\sin(\gamma)\sin(\beta)\sin(\theta)+$$
$$+\cos(\gamma)\cos(\theta))+\sin(\beta)\exp(-iE_3t)\sin(\gamma)\cos(\beta)]\Psi_{\nu_\mu}(0)$$
$$+[\cos(\beta)\cos(\theta)\exp(-iE_1t)(-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta))+$$
$$+\cos(\beta)\sin(\theta)\exp(-iE_2t)(-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta))+$$
$$+\sin(\beta)\exp(-iE_3t)\cos(\gamma)\cos(\beta)]\Psi_{\nu_\tau}(0).$$

Expression (14) can be rewritten in the following form:

$$\Psi_{\nu_e\to\nu_e,\nu_\mu,\nu_\tau}(t) = b_{\nu_e\nu_e}\Psi_{\nu_e}(0)+b_{\nu_e\nu_\mu}\Psi_{\nu_\mu}(0)+b_{\nu_e\nu_\tau}\Psi_{\nu_\tau}(0), \tag{14a}$$

where $b_{...}$ are coefficients before neutrino wave functions.

2.2.1. Probability of $\nu_e \to \nu_e$ neutrino transitions obtained from expression (14) is as follows:

$$P_{\nu_e\to\nu_e}(t) = 1-\cos^4(\beta)\sin^2(2\theta)\sin^2(-t(E_1-E_2)/2)$$
$$\cos^2(\theta)\sin^2(2\beta)\sin^2(-t(E_1-E_3)/2)- \tag{15}$$
$$-\sin^2(\theta)\sin^2(2\beta)\sin^2(-t(E_2-E_3)/2).$$

2.2.2. Probability of $\nu_e \to \nu_\mu$ neutrino transitions obtained from expression (14) is as follows:

$$P_{\nu_e\to\nu_\mu}(t) = 4\cos^2(\beta)\cos(\theta)\sin(\theta)[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]$$
$$[\sin(\gamma)\sin(\beta)\cos(\theta) + \cos(\gamma)\sin(\theta)]\sin^2(-t(E_1 - E_2)/2)- \tag{16}$$
$$+ 4\cos^2(\beta)\sin(\beta)\cos(\theta)\sin(\gamma)[\sin(\gamma)\sin(\beta)\cos(\theta) + \cos(\gamma)\sin(\theta)]\times$$
$$\times \sin^2(-t(E_1 - E_3)/2) - 4\cos^2(\beta)\sin(\beta)\sin(\theta)\sin(\gamma)[-\sin(\gamma)\sin(\beta)\sin(\theta)+$$
$$+ \cos(\gamma)\cos(\theta)]\sin^2(-t(E_2 - E_3)/2).$$

2.2.3. Probability of $\nu_e \to \nu_\tau$ neutrino transitions obtained from expression (14) is as follows:

$$P_{\nu_e\to\nu_\tau}(t) = 4\cos^2(\beta)\cos(\theta)\sin(\theta)[-\cos(\gamma)\sin(\beta)\cos(\theta)+ \tag{17}$$
$$+ \sin(\gamma)\sin(\theta)][\cos(\gamma)\sin(\beta)\sin(\theta) + \sin(\gamma)\cos(\theta)]\sin^2(-t(E_1 - E_2)/2)-$$
$$- 4\cos^2(\beta)\cos(\theta)\sin(\beta)\cos(\gamma)[-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]$$
$$\times \sin^2(-t(E_1 - E_3)/2) + 4\cos^2(\beta)\sin(\theta)\sin(\beta)\cos(\gamma)[\cos(\gamma)\sin(\beta)\sin(\theta)+$$
$$+ \sin(\gamma)\cos(\theta)]\sin^2(-t(E_2 - E_3)/2).$$

The control confirmed that $P_{\nu_e\to\nu_e}(t) + P_{\nu_e\to\nu_\mu}(t) + P_{\nu_e\to\nu_\tau}(t) = 1$.

2) For the case of $\nu_\mu \to \nu_e, \nu_\mu, \nu_\tau$ transitions we get

$$\Psi_{\nu_\mu\to\nu_e,\nu_\mu,\nu_\tau}(t) = [\cos(\beta)\cos(\theta)\exp(-iE_1t)$$
$$(-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta))+$$
$$+ \cos(\beta)\sin(\theta)\exp(-iE_2t)(-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta))+$$
$$+ \sin(\beta)\exp(-iE_3t)\sin(\gamma)\cos(\beta)]\Psi_{\nu_e}(0)+$$
$$+ [(-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta))^2\exp(-iE_1t)+$$
$$+ (-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta))^2\exp(-iE_2t)+$$
$$+ \sin^2(\gamma)\cos^2(\beta)\exp(-iE_3t)]\Psi_{\nu_\mu}(0)+$$
$$[(-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta))\exp(-iE_1t)$$
$$(-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta))+$$
$$+ (-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta))\exp(-iE_2t) \tag{18}$$
$$(-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)) + \sin(\gamma)\cos^2(\beta)$$
$$\exp(-iE_3t)\cos(\gamma)]\Psi_{\nu_\tau}(0).$$

Expression (18) can be rewritten in the following form:

$$\Psi_{\nu_\mu\to\nu_e,\nu_\mu,\nu_\tau}(t) = b_{\nu_\mu\nu_e}\Psi_{\nu_e}(0) + b_{\nu_\mu\nu_\mu}\Psi_{\nu_\mu}(0) + b_{\nu_\mu\nu_\tau}\Psi_{\nu_\tau}(0), \tag{18a}$$

where $b_{\ldots}$ are coefficients before neutrino wave functions.

2.2.4. Probability of $\nu_\mu \to \nu_\mu$ neutrino transitions obtained from expression (18) is as follows:

$$\begin{gathered} P_{\nu_\mu\to\nu_\mu}(t) = 1 - 4[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]^2 \\ [-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]^2\sin^2(-t(E_1 - E_2)/2) \\ - 4[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]^2\sin^2(\gamma)\cos^2(\beta) \\ \sin^2(-t(E_1 - E_3)/2) \\ - 4[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]^2\sin^2(\gamma) \\ \cos^2(\beta)\sin^2(-t(E_2 - E_3)/2). \end{gathered} \tag{19}$$

2.2.5. Probability of $\nu_\mu \to \nu_e$ neutrino transitions obtained from expression (18) is as follows:

$$\begin{gathered} P_{\nu_\mu\to\nu_e}(t) = -4\,[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]\cos^2(\beta)\cos(\theta) \\ [-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]\sin(\theta)\sin^2(-t(E_1 - E_2)/2) \\ - 4\,[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]\cos^2(\beta) \\ \cos(\theta)\sin(\gamma)\sin(\beta)\sin^2(-t(E_1 - E_3)/2) \\ - 4\,[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]\cos^2(\beta)\sin(\theta)\sin(\gamma)\sin(\beta) \\ \sin^2(-t(E_2 - E_3)/2). \end{gathered} \tag{20}$$

2.2.6. Probability of $\nu_\mu \to \nu_\tau$ neutrino transitions obtained from expression (18) is as follows:

$$\begin{gathered} P_{\nu_\mu\to\nu_\tau}(t) = -4\,[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)] \\ [-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)][-\sin(\gamma)\sin(\beta)\sin(\theta) \\ + \cos(\gamma)\cos(\theta)][-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)]\sin^2(-t(E_1 - E_2)/2) \\ - 4\,[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)] \\ [-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]\sin(\gamma)\cos^2(\beta)\cos(\gamma) \\ \sin^2(-t(E_1 - E_3)/2) \\ - 4\,[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)] \end{gathered} \tag{21}$$

$$[-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta)]\sin(\gamma)\cos^2(\beta)\cos(\gamma)$$
$$\sin^2(-t(E_2-E_3)/2).$$

The control confirmed that $P_{\nu_\mu\to\nu_e}(t)+P_{\nu_\mu\to\nu_\mu}(t)+P_{\nu_\mu\to\nu_\tau}(t)=1$.

3) For the case of $\nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ transitions we get

$$\Psi_{\nu_\tau\to\nu_e,\nu_\mu,\nu_\tau}(t)=[\cos(\beta)\cos(\theta)\exp(-iE_1t)$$
$$(-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta))+\cos(\beta)\sin(\theta)\exp(-iE_2t)$$
$$(-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta))+$$
$$+\sin(\beta)\exp(-iE_3t)\cos(\gamma)\cos(\beta)]\Psi_{\nu_e}(0)+$$
$$+[(-\sin(\gamma)\sin(\beta)\cos(\theta)-\cos(\gamma)\sin(\theta))\exp(-iE_1t)$$
$$(-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta))+(-\sin(\gamma)\sin(\beta)\sin(\theta)+$$
$$+\cos(\gamma)\cos(\theta))\exp(-iE_2t)(-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta))+$$
$$+\sin(\gamma)\cos^2(\beta)\exp(-iE_3t)\cos(\gamma)]\Psi_{\nu_\mu}(0)+ \qquad (22)$$
$$+[(-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta))^2\exp(-iE_1t)+$$
$$+(-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta))^2\exp(-iE_2t)+$$
$$+\cos^2(\gamma)\cos^2(\beta)\exp(-iE_3t)]\Psi_{\nu_\tau}(0).$$

Expression (22) can be rewritten in the following form:

$$\Psi_{\nu_\tau\to\nu_e,\nu_\mu,\nu_\tau}(t)=b_{\nu_\tau\nu_e}\Psi_{\nu_e}(0)+b_{\nu_\tau\nu_\mu}\Psi_{\nu_\mu}(0)+b_{\nu_\tau\nu_\tau}\Psi_{\nu_\tau}(0), \qquad (22a)$$

where $b_{...}$ are coefficients before neutrino wave functions.

2.2.7. Probability of $\nu_\tau \to \nu_\tau$ neutrino transitions obtained from expression (22) is as follows:

$$P_{\nu_\tau\to\nu_\tau}(t)=1-4[-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta)]^2$$
$$[-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta)]^2\sin^2(-t(E_1-E_2)/2)$$
$$-4[-\cos(\gamma)\sin(\beta)\cos(\theta)+\sin(\gamma)\sin(\theta)]^2\cos^2(\gamma)\cos^2(\beta)$$
$$\sin^2(-t(E_1-E_3)/2) \qquad (23)$$
$$-4[-\cos(\gamma)\sin(\beta)\sin(\theta)-\sin(\gamma)\cos(\theta)]^2\cos^2(\gamma)\cos^2(\beta)$$
$$\sin^2(-t(E_2-E_3)/2).$$

2.2.8. Probability of $\nu_\tau \to \nu_e$ neutrino transitions obtained from expression (22) is as follows:

$$P_{\nu_\tau\to\nu_e}(t) = -\,4\,[-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]$$
$$\cos^2(\beta)\cos(\theta)[-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)]\sin(\theta)$$
$$\sin^2(-t(E_1 - E_2)/2)$$
$$-\,4\,[-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]\cos^2(\beta)\cos(\theta)$$
$$\cos(\gamma)\sin(\beta)\sin^2(-t(E_1 - E_3)/2) \tag{24}$$
$$-\,4\,[-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)]\cos^2(\beta)\sin(\theta)\cos(\gamma)\sin(\beta)$$
$$\sin^2(-t(E_2 - E_3)/2).$$

2.2.9. Probability of $\nu_\tau \to \nu_\mu$ neutrino transitions obtained from expression (22) is as follows:

$$P_{\nu_\tau\to\nu_\mu}(t) = -\,4\,[-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]$$
$$[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]$$
$$[-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)] \tag{25}$$
$$[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]\sin^2(-t(E_1 - E_2)/2)$$
$$-\,4\,[-\cos(\gamma)\sin(\beta)\cos(\theta) + \sin(\gamma)\sin(\theta)]$$
$$[-\sin(\gamma)\sin(\beta)\cos(\theta) - \cos(\gamma)\sin(\theta)]\cos(\gamma)\cos^2(\beta)\sin(\gamma)$$
$$\sin^2(-t(E_1 - E_3)/2)$$
$$-\,4\,[-\cos(\gamma)\sin(\beta)\sin(\theta) - \sin(\gamma)\cos(\theta)]$$
$$[-\sin(\gamma)\sin(\beta)\sin(\theta) + \cos(\gamma)\cos(\theta)]$$
$$\cos(\gamma)\cos^2(\beta)\sin(\gamma)\sin^2(-t(E_2 - E_3)/2).$$

The control confirmed that $P_{\nu_\tau\to\nu_e}(t) + P_{\nu_\tau\to\nu_\mu}(t) + P_{\nu_\tau\to\nu_\tau}(t) = 1$.

We can rewrite expressions (14a), (18a), (22a) for three-neutrino wave functions in the following compact form:

$$\begin{pmatrix} \Psi_{\nu_e\to\nu_e,\nu_\mu,\nu_\tau}(t) \\ \Psi_{\nu_\mu\to\nu_e,\nu_\mu,\nu_\tau}(t) \\ \Psi_{\nu_\tau\to\nu_e,\nu_\mu,\nu_\tau}(t) \end{pmatrix} = \begin{pmatrix} b_{\nu_e\nu_e} & b_{\nu_e\nu_\mu} & b_{\nu_e\nu_\tau} \\ b_{\nu_\mu\nu_e} & b_{\nu_\mu\nu_\mu} & b_{\nu_\mu\nu_\tau} \\ b_{\nu_\tau\nu_e} & b_{\nu_\tau\nu_\mu} & b_{\nu_\tau\nu_\tau} \end{pmatrix} \begin{pmatrix} \Psi_{\nu_e}(0) \\ \Psi_{\nu_\mu}(0) \\ \Psi_{\nu_\tau}(0) \end{pmatrix}. \tag{26}$$

We can also introduce matrix $V_{\rm prob}(t)$ probabilities of three-neutrino transitions (oscillations) in dependence on time and write three-neutrino transition probabilities in the following compact form:

$$V_{\rm prob}(t) = \begin{pmatrix} P_{\nu_e\to\nu_e}(t) & P_{\nu_e\to\nu_\mu}(t) & P_{\nu_e\to\nu_\tau}(t) \\ P_{\nu_\mu\to\nu_e}(t) & P_{\nu_\mu\to\nu_\mu}(t) & P_{\nu_\mu\to\nu_\tau}(t) \\ P_{\nu_\tau\to\nu_e}(t) & P_{\nu_\tau\to\nu_\mu}(t) & P_{\nu_\tau\to\nu_\tau}(t) \end{pmatrix}. \quad (27)$$

Now we consider neutrino wave functions and a probability of neutrino transitions at the absence of $\nu_e \to \nu_\tau$ transitions.

2.3. Expressions for Neutrino Wave Functions and Probability of ν_e, ν_μ, $\nu_\tau \to \nu_e, \nu_\mu, \nu_\tau$ Transitions (Oscillations) in Vacuum in the Absence of the Direct $\nu_e \to \nu_\tau$ Transitions. If primary neutrinos are ν_e ones and there are no direct transitions between ν_e and ν_τ neutrinos, i. e., these transitions are closed, then only $\nu_e \to \nu_e$, $\nu_e \to \nu_\mu$, and $\nu_\mu \to \nu_\tau$ after $\nu_e \to \nu_\mu$ transitions can exist. The wave function for these transitions has the following form:

$$\Psi_{\nu_e\to\nu_e,\nu_\mu,\nu_\tau}(t) = [\cos^2(\theta)\exp(-iE_1 t) + \sin^2(\theta)\exp(-iE_2 t)]\Psi_{\nu_e}(0)+ \quad (28)$$

$$+[-\cos(\theta)\sin(\theta)\cos(\gamma)\exp(-iE_1 t) + \cos(\theta)\sin(\theta)\cos(\gamma)\exp(-iE_1 t)]\Psi_{\nu_\mu}(0)+$$

$$\cos(\theta)\sin(\theta)\sin(\gamma)[\exp(-iE_1) - \exp(-iE_2)]\Psi_{\nu_\tau}(0).$$

Probabilities of these neutrino transitions (oscillations) are described by the following expressions (in reality after transitions $\nu_e \to \nu_\mu$ there must be transitions between $\nu_\mu \to \nu_\tau$):

For $\nu_e \to \nu_e$:

$$P(\nu_e \to \nu_e, t) = 1 - \sin^2(2\theta)[\cos^2(2\gamma) + \sin^2(2\gamma)]\sin^2(L/L_{12}). \quad (29)$$

For $\nu_e \to \nu_\mu$:

$$P(\nu_e \to \nu_\mu, t) = \sin^2(2\theta)\cos^2(2\gamma)\sin^2(L/L_{12}). \quad (30)$$

For $\nu_e \to \nu_\tau$:

$$P(\nu_e \to \nu_\tau, t) = \sin^2(2\theta)\sin^2(2\gamma)\sin^2(L/L_{12}), \quad (31)$$

where

$$L_{ik}(m) = 1.27\frac{E_{\nu_e}(\text{MeV})}{|m_i^2 - m_k^2|(\text{eV}^2)} \qquad L = ct, \quad (32)$$

E_{ν_e} is energy of primary neutrino and $E_k = \sqrt{m_k^2 + p_{\nu_e}^2} \simeq p_{\nu_e} + \frac{m_k^2}{p_{\nu_e}}$, $i, k = 1 \div 3$.

If primary neutrinos are ν_μ neutrinos and there are no transitions between ν_e and ν_τ neutrinos, then

$$\Psi_{\nu_\mu \to \nu_e,\nu_\mu,\nu_\tau}(t) = [-\cos(\theta)\exp(-\,itE_1)\cos(\gamma)\sin(\theta)+$$
$$+\sin(\theta)\exp(-\,itE_2)\cos(\gamma)\cos(\theta)]\Psi_{\nu_e}(0) + [\cos^2(\gamma)\sin^2(\theta)\exp(-\,itE_1)+$$
$$+\cos^2(\gamma)\cos^2(\theta)\exp(-\,itE_2) + \sin^2(\gamma)\exp(-itE_3)]\Psi_{\nu_\mu}(0)+ \qquad (33)$$
$$[-\cos(\gamma)\sin^2(\theta)\exp(-\,itE_1)\sin(\gamma) - \cos(\gamma)\cos^2(\theta)\exp(-\,itE_2)\sin(\gamma)+$$
$$\sin(\gamma)\exp(-\,itE_3)\cos(\gamma)]\Psi_{\nu_\tau}(0).$$

If primary neutrinos are ν_τ neutrinos, there are no transitions between ν_e and ν_τ neutrinos

$$\Psi_{\nu_\tau \to \nu_e,\nu_\mu,\nu_\tau}(t) = [\cos(\theta)\exp(-\,itE_1)\sin(\gamma)\sin(\theta)-$$
$$-\sin(\theta)\exp(-\,itE_2)\sin(\gamma)\cos(\theta)]\Psi_{\nu_e}(0)+$$
$$+[-\cos(\gamma)\sin^2(\theta)\exp(-\,itE_1)\sin(\gamma)- \qquad (34)$$
$$-\cos(\gamma)\cos^2(\theta)\exp(-\,itE_2)\sin(\gamma) + \sin(\gamma)\exp(-itE_3)\cos(\gamma)]\Psi_{\nu_\mu}(0)+$$
$$[\sin^2(\gamma)\sin^2(\theta)\exp(-\,itE_1)+$$
$$\sin^2(\gamma)\cos^2(\theta)\exp(-\,itE_2) + \cos^2(\gamma)\exp(-\,itE_3)]\Psi_{\nu_\tau}(0).$$

3. SOME ANALYSIS OF NEUTRINO OSCILLATION POSSIBILITIES

The value of the Solar neutrino flow measured (through elastic scattering) on SNO [7] is in good agreement with the same value measured in Super-Kamiokande [8].

Ratio of ν_e flow measured on SNO (CC) to the same flow computed in the framework of SSM [9] ($E_\nu > 6.0$ MeV) is:

$$\frac{\phi^{CC}_{\rm SNO}}{\phi_{\rm SSM2000}} = 0.306 \pm 0.026({\rm stat.}) \pm 0.024({\rm syst.}). \qquad (35)$$

This value is in good agreement with the same value of ν_e relative neutrino flow measured on Homestake (CC) [10] for energy threshold $E_\nu = 0.814$ MeV.

$$\frac{\Phi^{\exp}}{\Phi^{\rm SSM2000}} = 0.34 \pm 0.03. \qquad (36)$$

From these data we can come to a conclusion that the angle mixing for the Sun ν_e neutrinos does not depend on neutrino energy thresholds (0.8 $\div$ 15 MeV).

Now it is necessary to know the value of this angle mixing $\theta_{\nu_e \nu_\mu}$. Estimation of the value of this angle can be extracted from KamLAND [11] data as follows:

$$\sin^2(2\theta_{\nu_e \nu_\mu}) \cong 1.0, \quad \theta \cong \frac{\pi}{4}, \quad \Delta m^2_{12} = 6.9 \cdot 10^{-5} \mathrm{eV}^2 \tag{37}$$

or

$$\sin^2(2\theta_{\nu_e \nu_\mu}) \cong 0.83, \quad \theta = 32^\circ, \quad \Delta m^2_{12} = 8.3 \cdot 10^{-5} \mathrm{eV}^2.$$

The angle mixing for vacuum $\nu_\mu \to \nu_\tau$ transitions obtained on Super-Kamiokande [12] for atmospheric neutrinos is:

$$\sin^2(2\gamma_{\nu_\mu \nu_\tau}) \cong 1, \quad \gamma \cong \frac{\pi}{4} \quad \Delta m^2_{23} = 2.1 \div 2.5 \cdot 10^{-3} \mathrm{eV}^2. \tag{38}$$

Now we can estimate the third angle mixing for $\nu_e \to \nu_\tau$ transitions by using expression (15). For this aim we average the time dependence of expression (15) taking into account that the Earth is moving over the elliptic orbit and then $(\bar{\sin}^2(-t(E_i - E_j)/2) = 1/2, \quad i, j = 1, 2, 3)$

$$\bar{P}_{\nu_e \to \nu_e} = 1 - \frac{1}{2}[\cos^4(\beta)\sin^2(2\theta) + \cos^2(\theta)\sin^2(2\beta) + \sin^2(\theta)\sin^2(2\beta)] =$$

$$= 1 - \frac{1}{2}[\cos^4(\beta)\sin^2(2\theta) + \sin^2(2\beta)]. \tag{39}$$

By substituting the value of $\sin^2(2\theta)$ in (39) from expression (37) and the value of $\bar{P}_{\nu_e \to \nu_e}$ from expressions (37), (38) we get

$$1.30 \simeq [\cos^4(\beta) + \sin^2(2\beta)]. \tag{40}$$

From the above expression we can come to conclusion that

$$\beta \leqslant \pi/4, \tag{41}$$

i. e., this angle is close to the maximal angle $\pi/4$.

If we suppose that direct $\nu_e \to \nu_\tau$ transitions are closed ($\beta = 0$), then we can use expression (15) to estimate value of $\bar{P}_{\nu_e \to \nu_e}(t)$. For this aim we average the time dependence of expression (15) taking into account that the Earth is moving over the elliptic orbit, then

$$\bar{P}_{\nu_e \to \nu_e}(t) = 1 - \frac{1}{2}\sin^2(2\theta). \tag{42}$$

By substituting the value of $\sin^2(2\theta)$ from expression (37) in (42) we get

$$\bar{P}_{\nu_e \to \nu_e}(t) = 0.585, \tag{43}$$

and then we come to a contradiction with expressions (37), (38), i. e. with experiments, but it was supposed that this contradiction can be removed by using the mechanism of resonance enhancement of neutrino oscillations in matter [13]. However, this is not the case. If we suppose that on the Sun surface all ν_e neutrinos are transformed into ν_μ neutrinos, through the resonance mechanism, then by neutrino vacuum oscillations at neutrinos moving to the Earth we get

$$\bar{P}_{\nu_\mu \to \nu_e}(t) = 1 - \frac{1}{2}\sin^2(2\theta) = 0.415. \tag{44}$$

Again we came to contradiction with expressions (37), (38). It is clear that the supposition $\beta(\theta_{13}) = 0$ is not confirmed.

Besides, it is necessary to remark that the resonance mechanism is not confirmed by the Solar neutrino spectrum (the Solar neutrino spectrum is not distorted) and the Day–Night effect is not observed [8]. Unlikely, it is possible to obtain a flat neutrino energy spectrum without distortion in broad energy region $E = 1 \div 15$ MeV by using this mechanism.

It is also detected that using expression (15) we can obtain limitation on value of angle β. For this purpose we have fulfilled graphical modelling of this function by using the following values for $\theta = 32.45°, \Delta m^2_{12}$ [11], Δm^2_{23} from expressions (37), (38) [12] for the case when $\Delta m^2_{13} = \Delta m^2_{12} + \Delta m^2_{23}$ and $\Delta m^2_{13} = 10^{-5}\text{eV}^2$, $5.7 \cdot 10^{-5}\text{eV}^2$, $8.3 \cdot 10^{-4}\text{eV}^2$ (for examination) for different values of $\beta = 10 \div 45°$ at the average electron neutrino energy $\bar{E}_{\nu_e} = 7$ MeV and established that the value for $P_{\nu_e \to \nu_e}(t)$ become a positive defined value at $\beta = 15 \div 17°$ ($P_{\nu_e \to \nu_e}(t) \simeq 0$ at some values of t). If $\beta \geqslant 15 \div 17°$, then $P_{\nu_e \to \nu_e}(t)$ becomes negative at some values of t. Since the value for the probability of $\nu_e \leftrightarrow \nu_e$ transitions $P_{\nu_e \to \nu_e}(t)$ must be positive defined one then, if in reality neutrino oscillations take place, the value for β must be $\beta \leqslant 15 \div 17°$.

4. CONCLUSION

I have considered three-neutrino vacuum transitions and oscillations in the general case and obtained expressions for neutrino wave functions in three cases: with CP violation, without CP violations and when direct $\nu_e \leftrightarrow \nu_\tau$ transitions are absent $\beta(\theta_{13}) = 0$ (some works indicate this possibility). Then using the existing experimental data some analysis has been fulfilled. This analysis definitely has shown that direct transitions $\nu_e \leftrightarrow \nu_\tau$ cannot be closed for the Solar neutrinos, i. e., $\beta(\theta_{13}) \neq 0$. It is also shown that the possibility that $\beta(\theta_{13}) = 0$ cannot be realized by using the mechanism of resonance enhancement of neutrino oscillations in matter (the Sun). It was found out that the probability of $\nu_e \leftrightarrow \nu_e$ neutrino transitions is positive defined value, if in reality neutrino oscillations take place, only if the angle of ν_e, ν_τ mixing $\beta \leqslant 15 \div 17°$.

REFERENCES

1. *Pontecorvo B. M.* // JETP. 1957. V. 33. P. 549; JETP. 1958. V. 34. P. 247.

2. *Maki Z. et al.* // Prog. Theor. Phys. 1962. V. 28. P. 870.

3. *Pontecorvo B. M.* // JETP. 1967. V. 53. P. 1717.

4. *Beshtoev Kh. M.* JINR Commun. E2-2004-58. Dubna, 2004; hep-ph/0406124; JINR Commun. E2-2005-123. Dubna, 2005, hep-ph/0506248;

5. *Bilenky S. M., Pontecorvo B. M.* // Phys. Rep. C. 1978. V. 41. P. 225;
   Boehm F., Vogel P. // Physics of Massive Neutrinos. Cambridge: Cambridge Univ. Press, 1987. P. 27, 121;
   Bilenky S. M., Petcov S. P. // Rev. Mod. Phys. 1977. V. 59. P. 631;
   Gribov V., Pontecorvo B. M. // Phys. Lett. B. 1969. V. 28. P. 493.

6. *Maiani L.* // Proc. of the Intern. Symp. on Lepton–Photon Interaction. Hamburg: DESY, 1977. P. 867.

7. *Ahmad Q. R. et al.* Internet Pub. nucl-ex/0106015, June 2001;
   Ahmad Q. R. et al. // Phys. Rev. Lett. 2002. V. 89. P. 011301-1; Phys. Rev. Lett. 2004. V. 92. P. 181301.

8. *Fukuda S. et al.* // Phys. Rev. Lett. 2001. V. 86. P. 5651; Phys. Lett. B. 2002. V. 539. P. 179;
   Koshio Y. (Super-Kamiokande Collab.) // Proc. of 28th Intern. Cosmic Ray Conf., Japan, 2003. V. 1. P. 1225.

9. *Bahcall D. et al.* // Astrophys. J. 2001. V. 555. P. 990.

10. *Davis R.* // Prog. Part. Nucl. Phys. 1994. V. 32. P. 13.

11. *Eguchi K. et al.* // Phys. Rev. Let. 2003. V. 90. P. 021802;
    Mitsui T. // Proc. of the 28th Intern. Cosmic Ray Conf., Japan, 2003. V. 1. P. 1221.

12. *Habig A.* // Proc. of the Intern. Cosmic Ray Conf., Japan, 2003. V. 1. P. 1255;
    Kearns Ed. (Super-Kamiokande Collab.). Report at Intern. Conf. «Neutrino 2004», Paris, 2004.

13. *Mikheyev S. P., Smirnov A. Yu.* // Yad. Fiz. 1986. V. 42. P. 1441; JETP. 1986. V. 91. P. 7;
    Mikheyev S. P., Smirnov A. Yu. // Nuovo Cimento. C. 1986. V. 9. P. 17;
    Goswami S. Report at Intern. Conf. «Neutrino 2004», Paris, 2004.

Received on February 6, 2006.

Корректор *Т. Е. Попеко*

Подписано в печать 29.03.2006.
Формат $60 \times 90/16$. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 1,67. Тираж 415 экз. Заказ № 55287.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/