Progress Towards 3D Black Hole Merger Simulations

Edward Seidel*

Max-Planck-Institut für Gravitationsphysik, Golm, Germany

Lecture given at the
Conference on Gravitational Waves:
A Challenge to Theoretical Astrophysics
Trieste, 5-9 June 2000

LNS013008

*eseidel@aei.mpg.de

Abstract

I review recent progress in 3D numerical relativity, focused on simulations involving black holes evolved with singularity avoiding slicings, but also touching on recent results in advanced techniques like black hole excision. After a long series of axisymmetric and perturbative studies of distorted black holes and black hole collisions, similar studies were carried out with full 3D codes. The results showed that such black hole simulations can be carried out extremely accurately, although instabilities plague the simulation at uncomfortably early times. However, new formulations of Einstein's equations allow much more stable 3D evolutions than ever before, enabling the first studies of 3D gravitational collapse to a black hole. With these new formulations, for example, it has been possible to perform the first detailed simulations of 3D grazing collisions of black holes with unequal mass, spin, and with orbital angular momentum. I discuss the 3D black hole physics that can now be studied, and prospects for the future, which look increasingly bright due to recent progress in formulations, black hole excision, new gauge conditions, and larger computers. Simulations may soon be able to provide information about the final plunge of two black holes, of relevance for gravitational wave astronomy.

1 Introduction

In the last few years, a major focus of the "traditional" 3D numerical relativity community has been mainly in four areas: (1) the development of improved formulations of the equations, (2) evolutions of black holes, especially the development of black hole excision techniques, and evolutions of full 3D black hole systems, from distorted black holes to collisions with increasingly general initial conditions, (3) the evolution of pure gravitational waves, and (4) the evolution of full 3D general relativistic (GR) hydro, applied primarily to neutron star binaries. By "traditional" 3D numerical relativity, I mean 3D evolutions of the full Einstein equations carried out using 3+1 evolution of Cauchy initial data on spacelike slices, with finite difference techniques. There are a number of important alternatives that have made very significant progress recently, such as the characteristic approach to black hole evolution [1], the conformal field equation approach [2, 3, 4, 5, 6, 7] and others that I will not be able to discuss here. Further, I will restrict my discussion to vacuum spacetimes. Generally speaking, in spite of the perception that progress is rather slow in this field, I want to share some of the excitement that many of us in the numerical community are feeling with progress that *has* been made.

Serious problems remain, especially in the area of long term evolution of 3D black hole spacetimes. But as I argue below, in the last few years the community has had many achievements in 3D numerical relativity: it has developed far more stable evolution schemes than ever before; it has been able to show that highly distorted 3D black holes and limited black hole collisions can be very accurately performed, with very accurate waveform extraction even for very low energy waves ($10^{-7}M$) [8, 9]; that the collapse of pure waves can form black holes [10]; and that 3D grazing collisions of spinning black holes can be simulated, as long as the evolution times are fairly short (less than 50M) [11, 12, 13], although very recently much longer times now seem possible. It has also made major progress in developing black hole excision techniques that will likely be the key to extending these evolutions to much longer time periods [14, 15]. Finally, an innovative and exciting project called "Lazarus", that uses a hybrid numerical-perturbative approach to rescue the results of an ailing (but not dead) numerical simulation has made major progress in pushing 3D black hole simulations farther than ever before [16, 17]. I see this as significant and exciting progress, even though the ultimate goal of evolving binary black hole mergers for a number of orbits may not be achieved for some years.

This article is based on an earlier review, published in [13], but substantially updated with newer material and references.

2 Why are Black Holes so Difficult? A brief history

Numerical evolutions of Einstein's equations in 3D are extremely difficult, as evidenced by the relatively slow progress over the years in spite of the efforts of a great many people. Even in vacuum 3D spacetimes without black holes, where there are no complications due to evolution of hydrodynamics, equations of state, and so on, the evolution of pure gravitational waves has been very difficult [18, 19, 20]. When black holes are considered, the presence of singularities makes evolution even much more problematic. One must simultaneously deal with singularities inside them, follow the highly nonlinear regime near the horizons, and also calculate the linear regime in the radiation zone where the waves represent a very small perturbation on the background spacetime metric. In axisymmetry this has been achieved, for example, for stellar collapse [21], rotating collisionless matter [22], distorted vacuum black holes with rotation [23] and without [24], and for equal mass colliding black holes [25, 26], but with difficulty. These 2D evolutions can be carried out to roughly $t = 100 - 150M$, where M is the ADM mass of the spacetime, but beyond this point large gradients related to singularity avoiding slicings usually cause the codes to become very inaccurate and crash. This is one of the fundamental problems associated with black hole evolutions: if one uses the gauge freedom in the Einstein equations to bend time slices up and around the singularities, one ends up with pathological behavior in metric functions describing the warped slices that eventually leads to numerical instabilities.

In 3D the problems are even more severe with this traditional, singularity avoiding time slicing approach. To simulate the coalescence of two black holes in 3D, evolutions of time scales $t \approx 10^2 - 10^3 M$ will be required. Traditional slicing approaches, coupled with the standard ADM-like formulations of the Einstein equations, can presently carry evolutions only to about $t = 50M$ or less, just for Schwarzschild! (But see below for major improvements enabled by recent variations in the formulations.) However, in spite of these difficulties, great progress is being made on several fronts. Alternative approaches to standard numerical evolution of black holes, such as apparent horizon boundary conditions (also known as black hole excision), characteristic evolution, and possibly evolution on hyperboloidal slices [2, 3, 4, 5, 6, 7], promise much longer evolutions. Apparent horizon conditions cut away the causally disconnected region interior to the black hole horizon, allowing better behaved slicings. These have been well developed in 1D, spherically symmetric studies [27, 28, 29, 30] and to a lesser extent, in full 3D evolutions [31, 32]. Recently, [14] reported success in evolving the merger of two black holes, with unequal masses and momenta on each hole, through the point where a common horizon appears, and excision work at AEI [15] has been able to evolve Schwarzschild spacetimes indefinitely, and distorted black holes, with very accurate waveform extraction, for well over $t = 100M$. (The waveforms are extracted according to a gauge-invariant proceedure developed originally by Abrahams, and applied and developed in various ways in the 3D case by many, including, for example [33, 34, 35, 8, 9, 33, 36, 37, 11].) Characteristic evolution with ingoing null slices has very recently been successful in evolving 3D single black holes for essentially unlimited times, even with distortions away from spherical or axisymmetry [38]. These alternate approaches look promising, but will take time to develop into general approaches to the two black hole coalescence problem.

I will focus most of the rest of this paper on recent results obtained using singularity avoiding slicings to evolve black hole and gravitational wave spacetimes. Although these techniques are limited, they have so far enabled many investigations of the physics of highly distorted black holes, rotating holes, and black hole collisions, in axisymmetry and full 3D. The emphasis here is on what physics one can do now. This work will provide a foundation for future studies with advanced excision, characteristic, or other techniques, once they are perfected.

2.1 Axisymmetric Black Hole Simulations

In axisymmetry, even with singularity avoiding slicings, it is possible to perform accurate and long term ($t \approx 150M$) simulations of several classes of black hole systems that teach us much about the full problem that we ultimately wish to solve. A class of highly distorted black holes was developed and studied numerically [34, 39, 35, 24, 40, 41], showing that even highly nonlinear black hole evolutions can be very cleanly studied. The initial data typically have a three–metric with the form $d\ell^2 = \tilde{\psi}^4 \left(e^{2q}\left(d\eta^2 + d\theta^2\right) + \sin^2\theta d\phi^2\right)$, where η is a radial coordinate related to Cartesian coordinates by $e^\eta = \sqrt{x^2+y^2+z^2}$ [34]. Given a choice for the "Brill wave" function q, the Hamiltonian constraint leads to an elliptic equation for the conformal factor $\tilde{\psi}$. The function q represents the wave surrounding the black hole (BH), and is chosen to be $q\left(\eta,\theta,\phi\right) = a\sin^n\theta\left(e^{-\left(\frac{\eta+b}{w}\right)^2} + e^{-\left(\frac{\eta-b}{w}\right)^2}\right)\left(1 + c\cos^2\phi\right)$. (Here I have given the full 3D generalization of the original axisymmetric data; axisymmetric data sets are recovered for $c = 0$.) If the amplitude a vanishes, the undistorted Schwarzschild solution results; small values of a correspond to a perturbed BH. Depending on the choice of Brill wave parameters and extrinsic curvature, these black holes can represent highly distorted rotating black holes mimicking those that are formed during the spiralling merger of two spinning black holes. These studies proved very rich in black hole physics, for example allowing black hole apparent and event horizon dynamics to be studied [42], and waveforms to be extracted, from the numerical evolutions.

These simulations were extended to axisymmetric black hole collisions, including equal mass BH's [25, 26], boosted BH's, and unequal mass BH's. The black hole collision work led to a revival of black hole perturbation theory, which turns out to be an essential tool in both interpreting and confirming numerical simulation in the right regimes. First, if the two holes are so close together initially that they have actually already merged into one, they might be considered as a single perturbed Schwarzschild hole (the so-called "close limit"). Price, Pullin, and others [43, 44, 45, 46, 47] used this technique to produce waveforms for colliding black holes in the Misner [48] and Brill and Lindquist [49] black hole initial data. Second, when the holes are very far apart, one can consider one black hole as a test particle falling into the other. Then one rescales the answer obtained by formally allowing the "test particle" to be a black hole with the same mass as the one it is falling into [25, 26, 44, 50].

The details of this success of the synergistic perturbative/numerical program has provided insights into the nature of collisions of holes, and should also apply to many systems of dynamical black holes. The waveforms and energies agree remarkably well with numerical simulations. Moreover, second order perturbation theory [47] spectacularly improved the agreement between the close limit and full numerical results for even larger distances between the holes, although ultimately beyond a certain limit the approximation is simply inappropriate and breaks down.

Together this large body of work provides a detailed and very well understood picture of axisymmetric black hole interactions, which also provides an excellent testbed for studies in 3D cartesian coordinates of the very same system. If one is unable to reproduce the results of this large body of axisymmetric spacetime studies with general 3D codes, one will not be able to go on the general problem of orbiting, spinning, and coalescing black holes.

Most of this perturbative work has been based on the gauge-invariant perturbation formalism developed by Moncrief [51], but it is now being extended to the more general case of the curvature based on the Teukolsky approach, which naturally handles rotating black holes. A recent application of this approach to distorted black holes with both even- and odd-parity black hole distortions has been very successful [52]. We will see below how this perturbative treatment can be a powerful aid in conducting, and extending, full 3D nonlinear simulations.

2.2 3D Testbeds

Armed with robust and well understood axisymmetric black hole codes, we now consider the 3D evolution of axisymmetric distorted black hole initial data. I first discuss results obtained with standard 3+1 ADM formulations, and later move to more recent formulations. These same axisymmetric initial data sets can be ported into a 3D code in cartesian coordinates, evolved in 3D, and the results can be compared to those obtained with the 2D, axisymmetric code discussed above, with excellent results showing rather precise agreement in waveforms [39, 34].

We can also take fully 3D distorted black hole data sets, for which there is no axisymmetric testbed, and compute the evolution in 3D cartesian coordinates. In this case, if the amplitude of the Brill wave is low enough, we may also compute the evolution perturbatively as shown in Ref. [8, 9], and compare with the full 3D evolution in cartesian coordinates. The non-axisymmetric initial data set ($a = -0.1, b = 0, c = 0.5, w = 1, n = 4$) was evolved in both full 3D numerical relativity, with cartesian coordinates, maximal slicing, and zero shift, waveforms were extracted, and compared to perturbative evolution. In Figs. 1 and 2 we show a sample of results for a large part of the BH spectrum of modes excited. The waveforms for the linear and nonlinear evolutions are each plotted on the same graphs, extracted at $r = 12.6M$. The total energy radiated for these modes runs from $E \sim 3 \times 10^{-4} M$ for the $\ell = 2$, $m = 0$ mode, to $E \sim 3 \times 10^{-7} M$ for the very low energy, nonaxisymmetric $\ell = 4$, $m = 2$ mode. The agreement between these two completely independent treatments is remarkable, giving complete confidence in the reliability of these waveforms. Not only do the perturbative results confirm those of full 3D numerical relativity, but the 3D results confirm the perturbative treatment. These results show that now it is possible in full 3D numerical relativity, to study the evolution and waveforms emitted from highly distorted black holes, even when the final waves leaving the system carry a very small amount of energy, in cartesian coordinates.

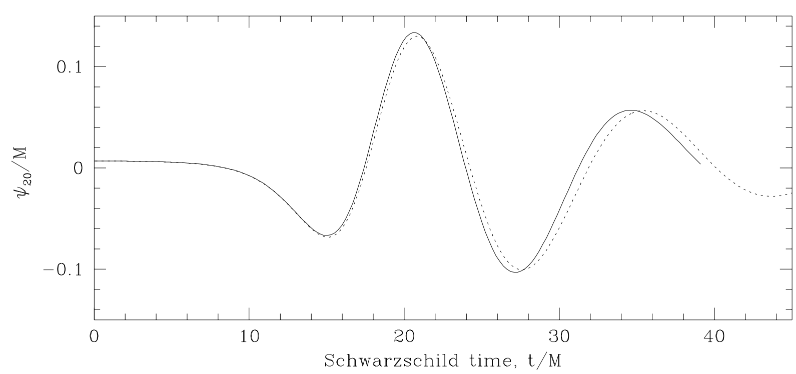

Figure 1: We show the waveform for the $\ell = 2, m = 0$ mode, extracted from the linear and nonlinear evolution codes. The dotted (solid) line shows the linear (nonlinear) evolution.

Similar comparisons were carried out with full 3D evolutions of axisymmetric colliding black holes, studied as described above with fully nonlinear axisymmetric codes and perturbation theory. The first 3D simulations of colliding black holes were carried out by the NCSA/WashU collaboration in 1994-1995 [53]. Following the success of axisymmetric calculations of Misner data by the same group, and by the beautiful perturbative results pioneered by Price, Pullin, and collaborators,

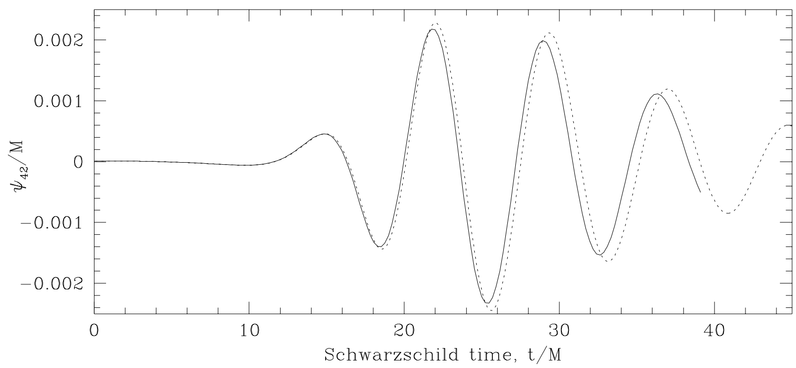

Figure 2: Waveforms are shown for the $\ell = 4, m = 2$ mode, extracted from the linear and nonlinear evolution codes. The dotted (solid) line shows the linear (nonlinear) evolution.

Misner data provides a useful testbed for the two black hole problem in full 3D. Results of the 3D simulations agree very well with those carried out in 2D and through perturbation theory in the appropriate regimes, including comparisons of wave energies and event horizon studies. Together, these studies of distorted and colliding black holes in 3D, with the perturbative and nonlinear axisymmetric comparisons, form a very solid basis for believing that full 3D numerical codes are able to produce very accurate, if limited, 3D back hole physics results. However, as I emphasized, these 3D simulations, all obtained with the standard 3+1 ADM formalism, generally develop numerical instabilities before $t = 40M$ and crash. We will return to this point below, where new formulations will prove their worth in stabilizing these simulations. But first I discuss a new community infrastructure that I hope will help accelerate progress of the community, by allowing it to easily explore different techniques, and to share each other's work, to help break through these barriers and to speed progress.

3 Cactus Computational Toolkit and its use in Numerical Relativity and Astrophysics

The computational and collaborative needs of numerical relativity are clearly immense. To develop a basic 3D code with all the different modules, including parallelling layers, adaptive mesh refinement, elliptic solvers, initial value solvers, gauge conditions, black hole excision modules, analysis tools, wave extraction, hydrodynamics modules, visualization tools, etc., require dozens of person years of effort from many different disciplines (in fact, such a feat has still not been done by the entire community!). Different groups often needlessly repeat each other's effort, further slowing the progress of the field. The NSF Black Hole Grand Challenge was a first attempt to address this problem, and an outgrowth of that effort led to the development of the "Cactus" Computational Toolkit (CCTK), developed by the Potsdam group, in collaboration first with NCSA and Washington University, and now with a growing number of international collaborators in various disciplines. Originally designed to solve Einstein's equations, the CCTK has grown into a general purpose parallel environment for solving complex PDE's [54, 55, 56, 57, 58] that is being picked up by various communities in computational science. Here I focus on its application to Einstein's equations.

Cactus is designed to minimize barriers to the community development and use of the code, including the complexity associated with both the code itself and the networked supercomputer environments in which simulations and data analysis are performed. This complexity is particularly noticeable in large multidisciplinary simulations such as ours, because of the range of disciplines that must contribute to code development (relativity, hydrodynamics, astrophysics, numerics, and computer science) and because of the geographical distribution of the people and computer resources involved in simulation and data analysis.

The collaborative technologies that we are developing within Cactus include:

- *A modular code structure and associated code development tools.* Cactus defines coding rules that allow one, with only a working knowledge of Fortran or C, to write new code modules that are easily plugged in as "thorns" to the main Cactus code (the "flesh"). The "flesh" contains basic computational infrastructure needed to develop and connect parallel modules into a working code. It allows one to plug in not only different physics modules, such as the basic Einstein solver, other formulations of the equations, analysis routines, etc., but also different parallel domain decomposition modules, I/O modules, utilities, elliptic solvers, and so forth. A thorn may be any code that the user wants, in order to provide different initial data, different matter fields, different gauge conditions, visualization modules, etc. Thorns need not have anything to do with relativity: the flesh could be used as basic infrastructure for any set of PDE's, from Newtonian hydrodynamics equations to Yang Mills equations, that are coded as thorns. The user inserts the hook to their thorn into the flesh code in a way that the thorn will not be compiled unless it is designated to be active. We have developed a makefile and perl-based thorn management system that, through the use of preprocessor macros that are appropriately expanded to the arguments of the flesh and additional arguments defined by each thorn, is able to create a code which can configure itself to have a variety of different modules. This ensures that *only* those variables needed for a particular simulation are actually used, and that no conflicts can be created in subroutine argument calling lists.
- *A consistency test suite library.* An increased number of thorns makes the code

more attractive to its community but also increases the risk of incompatibilities. Hence, we provide technology that allows each developer to create a test/validation suite for their own thorn. These tests are run prior to any check in of code to the repository, ensuring that new code reproduces results consistent with previous ones, and hence cannot compromise the work of other developers relying on a given thorn.

• *Grid Computing Technologies, Remote Steering, Visualization, and Distributed Computing.* With such a large computational problem, simulations often run for many hours or days on even the largest supercomputers. A composite code developed by a group of collaborators means that many people, at different sites, may be responsible for a simulation. Hence, it is important for a number of people to be able to monitor the simulation in real time, wherever they are, to make sure things proceed properly and that the results make sense. If something is wrong, it is often possible to adjust it on the fly, and save a simulation, avoiding the need to start over again (which may take days or weeks to get a slot on a crowded supercomputer!). We have developed many such technologies, which are available to all users with any thorn set.

A detailed user guide is available with the code (see www.cactuscode.org), but in a nutshell, one specifies which physics modules, and which computational/parallelism modules, are desired in a configuration file, and makes the code on the desired architecture, which can be any one of a number of machines from SGI/Cray Origin or T3E, Dec Alpha, Linux workstations or clusters, NT clusters, and others. The make system automatically detects the architecture and configures the code appropriately. Control of run parameters is then provided through an input file. Additional modules can be selected from a large community-developed library, or new modules may be written and used in conjunction with the pre-developed modules.

Cactus seems to be a very effective collaborative code infrastructure that is enabling groups to work together better, and it relieves much of the computational science burden on the physicist, allowing more time to focus on physics. This flexible, open code architecture allows, for example, a relativity expert to contribute to the code without knowing the details of, say, the computational layers (e.g., message passing or AMR libraries) or other components (e.g., hydrodynamics). We encourage users from throughout the relativity and astrophysics communities to make use of this freely downloadable code infrastructure and physics modules, either for their own use, or as a collaborative tool to work with other groups in the community.

4 New Formulations: Towards a Stable Evolution System

With this collaborative Cactus infrastructure, it is easy to insert alternate variations on many things, e.g., evolution equations. In what follows I report on new results obtained with Cactus, using the "BSSN" formulations, in the last year.

As discussed above, the 3D evolution of Einstein's equations has proved very difficult, with instabilities developing on rather short time scales, even in cases of weakly gravitating, vacuum systems, such as low amplitude gravitational waves, as summarized in an important paper by Baumgarte and Shapiro [18]. In this work, it was shown how one can achieve highly improved stability by making a few key changes to the formulation of the ADM equations, most notably through a conformal decomposition and by rewriting certain terms in the 3D Ricci tensor to eliminate terms that spoil its elliptic nature. These same tricks were already noticed a few years earlier by Shibata and Nakamura [20]. Hence we refer to these formulations collectively as "BSSN" after the four authors. These subtle changes to the standard ADM formalism have a very powerful stabilizing effect on the evolutions, as discussed extensively in [59, 60]. Evolutions of weak waves that would develop instabilities and crash with the standard ADM formulation run much longer with the new system, and as shown in Alcubierre et al. [10], the new system and variations allow for the first time the successful evolution of highly nonlinear gravitational waves to form a black hole in 3D while the standard ADM treatment would fail well in advance of black hole formation. Further work by the Palma group, showed the deep connection between the BSSN formulations and the Bona-Massó family of formulations [61], leading to the possibility of a fully hyperbolic, very stable formulation that shares advantages from many sides.

4.1 Evolving Pure Gravitational Waves

With these new formulations, we are now able to study the nonlinear dynamics of pure gravitational waves with much more stability than ever before. This allows us to use numerical relativity to probe general relativity in the highly nonlinear regime. Can one form a black hole in full 3D from pure gravitational waves? Does one see critical phenomena in full 3D? These inherently nonlinear phenomena have been investigated in 1D and 2D studies, but little is known about generic 3D behavior.

In our investigations, we take as initial data a pure Brill [62] type gravitational wave, later studied by Eppley [63, 64] and others [65]. The metric takes the form

$$ds^2 = \Psi^4 \left[e^{2q} \left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right] = \Psi^4 \hat{ds}^2, \qquad (1)$$

where q is a free function subject to certain boundary conditions. Following [9, 39, 66], we choose q of the form

$$q = a\,\rho^2\,e^{-r^2}\left[1 + c\,\frac{\rho^2}{(1+\rho^2)}\;\cos^2(n\phi)\right], \tag{2}$$

where a, c are constants, $r^2 = \rho^2 + z^2$ and n is an integer. For $c = 0$, these data sets reduce to the Holz [65] axisymmetric form, recently studied in full 3D Cartesian coordinates [67]. Taking this form for q, we impose the condition of time-symmetry, and solve the Hamiltonian constraint numerically in Cartesian coordinates. An initial data set is thus characterized only by the parameters (a, c, n). For the case $(a, 0, 0)$, we found in [67] that no apparent horizon (AH) exists in initial data for $a < 11.8$, and we also studied the appearance of an AH for other values of c and n.

We have surveyed a large range of this parameter space, but here I discuss two cases of interest: (i) a subcritical (but highly nonlinear) case where after a violent collapse of the self-gravitating waves, there is a subsequent rebound and after a few oscillations the waves all disperse, and (ii) a supercritical case where the waves collapse in on themselves and immediately form a black hole.

The subcritical case studied in Ref. [10] has parameters (a=4, c=0, n=0) in the notation above. It is a rather strong axisymmetric Brill wave (BW). The evolution of this data set shows that part of the wave propagates outward while part implodes, re-expanding after passing through the origin. However, due to the nonlinear self-gravity, not all of it immediately disperses out to infinity; again part re-collapses and bounces again. After a few collapses and bounces the wave completely disperses out to infinity. At late times, the lapse returns to unity, and the Riemann invariant J settles on a constant value that converges rapidly to zero as we refine the grid. With these results, and direct verification that the metric functions become stationary at late times, we conclude that spacetime returns to flat (in non-trivial spatial coordinates; the metric is decidedly non-flat in appearance!). The same simulation carried out with the standard ADM systems crashes far earlier than in the present case with the BSSN systems, which essentially run forever.

With this experience, we next try the case of an even stronger amplitude wave, which in this case will actually collapse on itself and form a black hole. In Fig. 3, we study the development of the 3D data set (a=6, c=0.2, n=1), and watch it collapse to form a black hole (the first such 3D simulation). The figure also compares this black hole formation to results obtained with an axisymmetric data set. The system clearly collapses on itself and rapidly forms a black hole. The waveform extraction shows that the newly formed hole then rings at its quasinormal mode frequency. High quality images and movies of these simulations can be found at http://jean-luc.aei-potsdam.mpg.de .

These results are exciting examples of how numerical relativity can act as a laboratory to probe the nonlinear aspects of Einstein's equations. Pure gravitational waves are clearly a rich and exciting research area that allows one to study Ein-

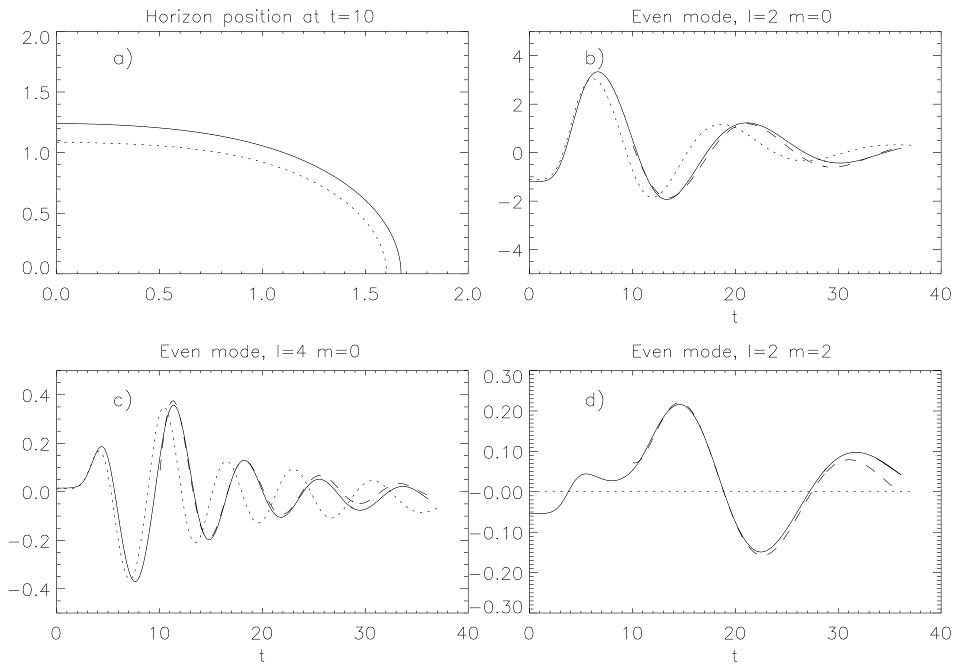

Figure 3: a) The solid (dotted) line is the AH for the full 3D data set $(6, 0.2, 1)$ $((6, 0, 0))$ at t=10 on the x-z plane. b) The $\{l$=2,m=0$\}$ waveform for the 3D case at $r = 4$ (solid line) is compared to an axisymmetric case (dotted line). The dot-dashed line shows the fit of the 3D case to the two lowest lying QNM's (quasi-normal modes) for a BH of mass 0.99. c) Same comparison for the $\{l$=4,m=0$\}$ waveform. d) Same comparison for the non-axisymmetric $\{l$=2,m=2$\}$ waveform.

stein's equations as a nonlinear theory of physics. With these new capabilities of accurate 3D evolution that can follow the implosion of waves to a black hole, there is much more physics to study, including the structure of horizons, full 3D studies of critical phenomena, and much more. Further, this study of pure vacuum waves has helped us understand the importance of developing and testing new formulations of Einstein's equations for numerical purposes. Without the new formulations, these results simply could not have been obtained. Further, we have run literally hundreds of simulations like these in order to determine which variation on the "BSSN" families of formulations performs best. With this new knowledge, we move back to the problem of 3D black holes.

5 Black Holes

Having tested these new formulations of Einstein's equations on the problem of pure gravitational waves, we now apply what we have learned to the considerably more complex problem of black hole evolutions. We first applied these new formulations to black hole spacetimes that have been very carefully tested in axisymmetry and with 3D nonlinear numerical codes, but with standard 3+1 formulations, as well as with perturbative methods. As discussed above and shown in many papers [8, 9, 34, 35], the evolutions can be actually very accurate with the ADM system—allowing the extraction of very delicate waveforms from a large spectrum of modes—and remain so until the code crashes. With the new formulations, we find that we can break far through the former crash barrier, but as features become underresolved the results naturally become less accurate. For a fuller discussion of these results, and mathematical analysis giving insight into the improved behavior of the new formulations, please see Ref. [60].

5.1 True 3D Grazing Black Hole Collision

Having allowed us to break through the barriers seen in evolving pure nonlinear gravitational waves, and having shown that these BSSN formulations can extend very accurate simulations of highly distorted black holes, we now apply them to full 3D grazing collisions of black holes. These results can found in more detail in [11].

The initial data sets we use here for binary BH systems were developed originally by Brandt and Brügmann [68]. They are very convenient, since no isometry is needed and hence the elliptic solver can be applied on standard cartesian grid without the need to apply boundary conditions on strangely shaped (e.g., non-planar!) surfaces. A few of these data sets were first evolved by Brügmann [69] using the standard ADM formulation. This was a first pioneering attempt to go beyond the highly symmetric black hole collisions that had been studied previously, combining for the first time unequal mass, spinning black holes with linear and orbital angular momentum. Brügmann was able to use nested grids to provide reasonable resolution near the holes, while putting the boundary reasonably far away. The result was that for selected data sets he was able to carry out the evolution far enough to observe a merger, where two separate apparent horizons (AHs) were replaced by an outer one. However, the difficulties of the ADM formulation, discussed above, coupled with poor resolution achievable at that time limited these evolutions to about $t = 7M$, and it was not possible to extract detailed physics, such as horizon masses, waveforms, energies, spins, etc.

We compute BH initial data corresponding to two BHs in orbit about each other, with unequal masses, linear momentum, and individual spins on each BH. The construction of such data sets, which involves solving the nonlinear elliptic Hamiltonian constraint equation numerically, is described in [68]. A detailed survey of a sequence of such data sets including various physical properties is discussed in [70]. We chose punctures for each BH on the y-axis at $\pm 1.5m$, masses $m_1 = 1.5m$ and $m_2 = m$, linear momenta $P_{1,2} = (\pm 2, 0, 0)m$, and spins $S_1 = (-1/2, 0, -1/2)m^2$ and $S_2 = (0, 1, -1)m^2$. The linear momentum is perpendicular to the line connecting the BHs, equal but opposite for a vanishing net linear momentum, and that the spins are somewhat arbitrarily chosen to obtain a general configuration.

For this case, after solving the Hamiltonian constraint, an asymptotic estimate for the initial ADM mass is $M = 3.22m$. The angular momentum for puncture data is given by (independent of the solution to the Hamiltonian constraint) $\vec{J} = 2\vec{d}_1 \times \vec{P}_1 + \vec{S}_1 + \vec{S}_2$, where $\vec{d}_1$ is the vector from the origin to the first puncture. The total angular momentum is therefore $J = 7.58m^2$, which corresponds to an angular momentum parameter of $a/M = J/M^2 = 0.73$. In this configuration the individual spins increase the total angular momentum, so we call it the "high-J" case. We also consider other cases where the individual spins vanish (medium-J, $M = 3.00m$, $J = 6.00m^2$, $a/M = 0.67$) or where $S_1 \to -S_1$ and $S_2 \to -S_2$ (low-J, $M = 3.07m$, $J = 4.64m^2$, $a/M = 0.49$).

I begin with qualitative measurements of the physics we extract. In Fig. 4 I show a sequence of visualizations of simulations near the time just before, during, and after the merger of the two holes. The coordinate locations of the AHs are shown as colored surfaces. The grayscale represents the local gaussian curvature of the surface, computed from the induced 2–metric on the horizon. As the holes approach each other and merge, a global AH develops. Meanwhile, a burst of gravitational waves, indicated by the wisps emanating from the BH system develops and propagates away. The Newman-Penrose quantity Ψ_4, computed fully nonlinearly, is used to indicate the gravitational waves. As this system has no symmetries, and includes rotation, all $\ell - m-$ modes and both even- and odd-parity polarizations of the waves are present, leading to a much more complex structure in the wave patterns than one is used to seeing in such simulations. But this is now moving much closer to what one expects to see in Nature, and that, too, will be rather complicated!

We can also extract quantitative waveforms, using the gauge invariant waveform extraction technique, developed originally by Abrahams [71] and applied to the 3D case in [9], to extract gravitational wave modes of arbitrary ℓ, m. As shown in [9, 52], this technique can be used on numerically evolved 3D distorted BH spacetimes to produce very accurate waveforms away from the BH, even if errors are rather large near the horizon. Here, for the "high-J" binary black hole initial data set, we extract for example the nonaxisymmetric $\ell = m = 2$ mode, expected to be one of the most important modes in binary BH coalescence [72]. Fig. 5 shows for three resolutions the Zerilli function $\psi_{22}^{even}(t)$ extracted at $R = 7.8M$. Up to a time $t \approx 30M$ the dependence on resolution is rather small, which suggests that the resolution reaches the convergent regime.

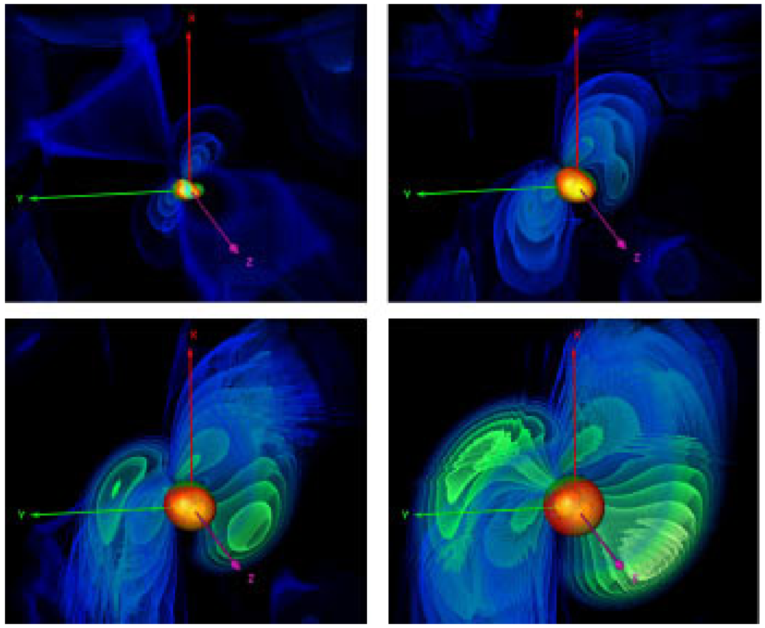

Figure 4: We show a sequence of visualizations of the merger of two black holes with unequal mass and spin. The apparent horizons are shown as the surfaces at the center of the image, and the graytones represent the gaussian curvature. The waves, shown emanating from the merger, are visualizations of the Newman-Penrose quantity Ψ_4. The top left panel shows the system just at the time of the merger, while the bottom right shows the system much later.

In Fig. 7, we show a sequence of extracted waves at different radii, obtained by integration over the corresponding coordinate spheres, as a function of time. The outermost detectors show late time problems due to spurious signals propagating in from the outer boundary, while the inner detectors are affected by the closeness to the strong field region. Note that these methods assume a Schwarzschild background, but they can be applied on a rotating BH, the primary effect being an offset depending on the rotation parameter a [23]. In Fig. 6, we show the $l = m = 2$ even parity wave for the detector at $R = 7.8M$ and a match to the corresponding lowest quasi-normal mode plus the first overtone. The values for M and a determine the quasi-normal frequency, while the amplitude and the offset in time are fitted. The observed period is $13M$, which is consistent with a final distorted Kerr BH with $a/M = 0.73$. Gravitational waves carry away energy and momentum from the BHs. By integrating the $l = 2, 3, 4$ modes up to $t = 35M$, we find the total energy radiated $\Delta E \approx 1\% M$.

In Fig. 8 we show waveforms from evolutions of three different initial data sets having the same masses and linear momenta on the initial black holes, but quite different spins. The waveforms all show the quasinormal ringing at late times, but

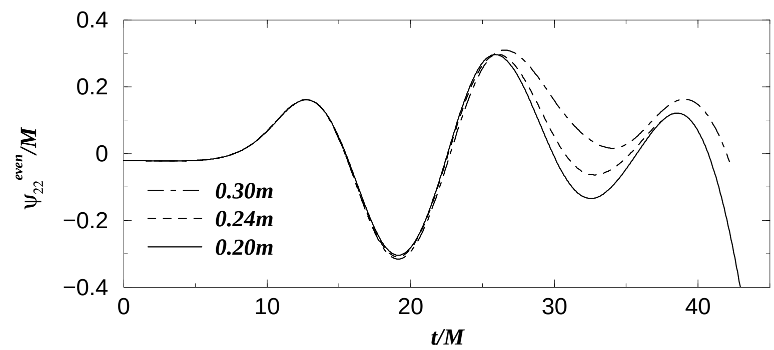

Figure 5: We show the extracted waveform (Zerilli function) at resolutions $0.30m$, $0.24m$, $0.20m$.

they clearly distinguish the different initial data cases from each other. Hence, we are now for the first time able to carry out full 3D black hole binary merger simulations and use the waveforms to distinguish between different starting conditions having different physics parameters. This is exactly the capability that will be needed when gravitational wave detectors begin seeing black hole collision events, which could be very soon!

These results indicate that for the first time we are indeed now able to simulate the late merger stages of two black holes colliding, with rather general spin, mass, and momenta, and that we can now begin to study the fine details of the physics. Without more advanced techniques, such as black hole excision, these simulations will be limited to the final merger phase of black hole coalescence. But while that is under development, we can take advantage of our capabilities and explore this phase of the inspiral now. Our goal is several fold: (a) to explore new black hole physics of the "final plunge" phase of the binary BH merger, (b) to try to determine some useful information relevant for gravitational wave astronomy, and (c) to provide a strong foundation of knowledge for this process that will be useful when more advanced techniques, such a black hole excision, are fully developed. When these techniques are used to extend the ability of the community to handle the earlier orbital phase, it will be important have an understanding of details of this most violent phase in advance, both as a testbed to ensure that results are correct, and because the understanding we gain may be useful in devising the appropriate techniques for longer term evolution.

5.2 Lazarus

As the simulations above are still limited in the time they can evolve a full 3D BH merger, we can use any help we can get to extend them. A very ambitious project

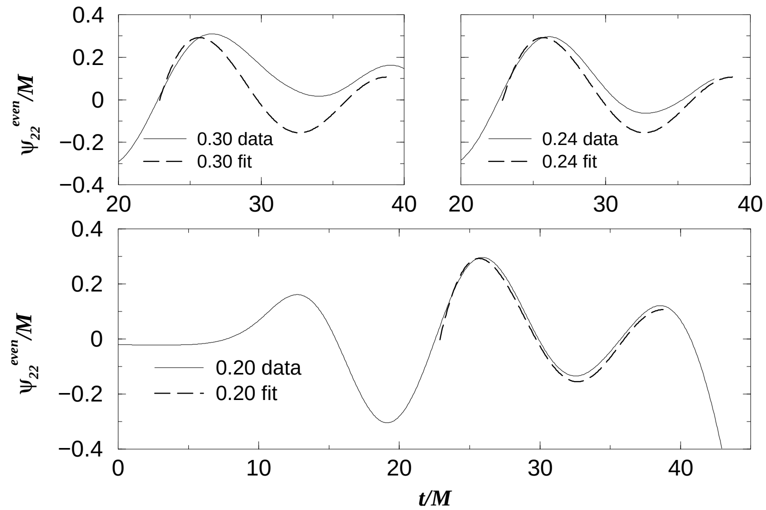

Figure 6: For the $l = m = 2$ even-parity mode, extracted at $R = 7.8M$, we show a fit to the lowest quasi-normal mode plus the first overtone modes, determined by M and a. The result shows good agreement in the frequency and decay rate at late times for a resolution of $0.2m$.

ongoing in Potsdam (the so-called "Lazarus" project) aims to rescue an ailing evolution of a binary merger [16, 17]. This exciting project goes far beyond previous work in the area of synergistic numerical-perturbative work to date. Lazarus is a *hybrid* nonlinear-linear technique for black hole evolution. It aims to take an initial binary black hole system, evolve it with the *full nonlinear* Einstein equations until it reaches a stage where it can be considered perturbative, and then *continue* it forward using perturbative techniques of the type described above, using the Teukolsky equation. Hence, it makes the best of both worlds, using full numerical relativity where it is needed and perturbative evolution where it is best. A demonstration of this technique was shown to work remarkably well in treatment of the Misner black holes, where results described above could be reproduced, even in the case where the initial data are clearly not in the perturbative regime. Lazarus is now being applied to more general 3D cases where initial black holes are in orbit around each other, and preliminary results are very encouraging. See Baker's contribution to this volume for more details.

6 Conclusion

Numerical relativity in 3D continues to move forward, as evidenced by new and exciting results in pure gravitational waves, fully relativistic neutron star mergers, grazing collisions of unequal mass, spinning black holes, and the development of

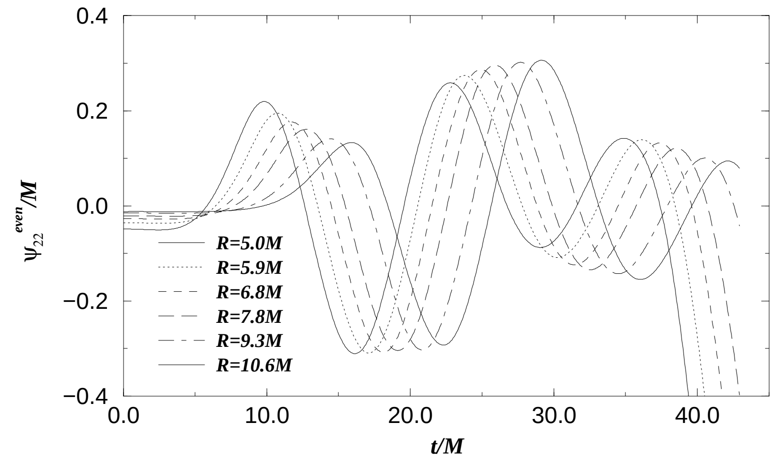

Figure 7: Mode $l = m = 2$ of the even Zerilli function extracted for different radii as a function of time. A wave that develops after the BHs collide is propagating out.

more stable formulations of Einstein's equations. I have focused here on recent progress in vacuum black hole evolutions with traditional 3+1 evolutions with singularity avoiding slicings, because at present these are the most mature techniques that are already yielding physics in full 3D simulations. In particular, the collapse of 3D gravitational waves to form black holes offers a laboratory for studying nonlinear aspects of Einstein's theory, including generic 3D critical phenomena, without complications due to hydrodynamics or singularities present from the beginning; the grazing merger of two black holes with unequal mass and spin can also be accomplished now, although for limited evolution times (up to about $t = 50M$), but this is enough time for us to study important physical processes of this final plunge that simply cannot be obtained without simulations. The Lazarus project can extend such evolutions, allowing the full scale nonlinear code to be focused just where it is needed, handing the result off to a perturbative treatment at the appropriate time. As alternate techniques are developed that promise to extend the evolutions, such as black hole excision, and the conformal hyperbolic approach, they may provide much more powerful approaches in the future. But the physics simulations we are able to do on a shorter time scale will be important testbeds for these future approaches, and we hope they will be able to provide some urgently needed information about gravitational wave astronomy.

Many images and movies of the simulations reported here are available at http://jean-luc.aei-potsdam.mpg.de or http://jean-luc.ncsa.uiuc.edu .

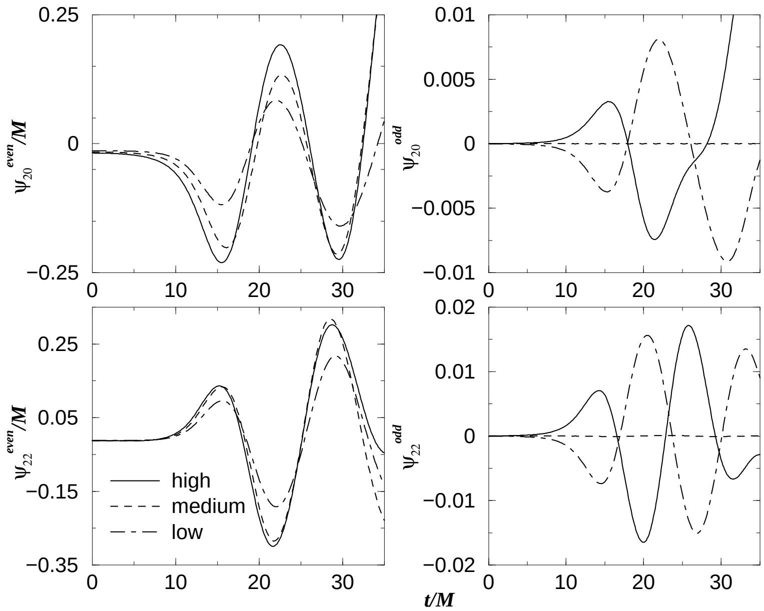

Figure 8: Even and odd wave parts showing differences depending on high, medium, and low-J data.

Acknowledgements

The work presented in this paper has been the result of a collaborative effort from many talented people with whom I am privileged to work. In particular, I thank Miguel Alcubierre, Gabrielle Allen, Pete Anninos, Toni Arbona, John Baker, Werner Benger, Carles Bona, Steve Brandt, Bernd Brügmann, Karen Camarda, Manuela Campanelli, Changxue Deng, Ed Evans, Tom Goodale, Christian Hege, Daniel Holz, Gerd Lanfermann, Carlos Lousto, Joan Massó, Andre Merzky, Mark Miller, Lars Nerger, Thomas Radke, John Shalf, Joan Stela, Wai-Mo Suen, Ryoji Takahashi, and Malcolm Tobias, and especially Ryoji, Michael Koppitz, and Mihai Bondarescu for carefully reading the manuscript.

References

[1] J. Winicour, Living Reviews in Relativity **1**, (1998).

[2] H. Friedrich, Class. Quant. Grav. **13**, 1451 (1996).

[3] P. Hübner, Phys. Rev. D **53**, 701 (1996).

[4] P. Hübner, gr-qc/9804065 (1998).

[5] P. Hübner, Class. Quant. Grav. **16**, 2823 (1999).

[6] J. Frauendiener, Phys. Rev. D **58**, 064002 (1998).

[7] J. Frauendiener, Phys. Rev. D **58**, 064003 (1998).

[8] G. Allen, K. Camarda, and E. Seidel, (1998), gr-qc/9806014, submitted to Phys. Rev. D.

[9] G. Allen, K. Camarda, and E. Seidel, (1998), gr-qc/9806036, submitted to Phys. Rev. D.

[10] M. Alcubierre *et al.*, Phys. Rev. D **61**, 041501 (2000), gr-qc/9904013.

[11] M. Alcubierre *et al.*, (2000), gr-qc/0012079.

[12] B. Brügmann, Ann. Phys. (Leipzig) **9**, 227 (2000), gr-qc/9912009.

[13] E. Seidel, Prog. Theor. Phys. Suppl. **136**, 87 (2000).

[14] S. Brandt *et al.*, , submitted, gr-qc/0009047.

[15] M. Alcubierre and B. Brügmann, (2000), gr-qc/0008067.

[16] J. Baker, B. Brügmann, M. Campanelli, and C. O. Lousto, Class. Quant. Grav. **17**, L149 (2000).

[17] J. Baker, M. Campanelli, and C. O. Lousto, in prepapartion (unpublished).

[18] T. W. Baumgarte and S. L. Shapiro, Physical Review D **59**, 024007 (1999).

[19] P. Anninos *et al.*, Phys. Rev. D **56**, 842 (1997).

[20] M. Shibata and T. Nakamura, Phys. Rev. D **52**, 5428 (1995).

[21] R. F. Stark and T. Piran, Phys. Rev. Lett. **55**, 891 (1985).

[22] A. M. Abrahams, G. B. Cook, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. D **49**, 5153 (1994).

[23] S. Brandt and E. Seidel, Phys. Rev. D **52**, 870 (1995).

[24] A. Abrahams *et al.*, Phys. Rev. D **45**, 3544 (1992).

[25] P. Anninos *et al.*, Phys. Rev. Lett. **71**, 2851 (1993).

[26] P. Anninos *et al.*, Phys. Rev. D **52**, 2044 (1995).

[27] E. Seidel and W.-M. Suen, Phys. Rev. Lett. **69**, 1845 (1992).

[28] P. Anninos *et al.*, Phys. Rev. D **51**, 5562 (1995).

[29] M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. D **51**, 4208 (1995).

[30] R. Marsa and M. Choptuik, Phys Rev D **54**, 4929 (1996).

[31] P. Anninos *et al.*, Phys. Rev. D **52**, 2059 (1995).

[32] G. B. Cook *et al.*, Phys. Rev. Lett **80**, 2512 (1998).

[33] A. M. Abrahams *et al.*, Physical Review Letters **80**, 1812 (1998), gr-qc/9709082.

[34] K. Camarda and E. Seidel, Phys. Rev. D **57**, R3204 (1998), gr-qc/9709075.

[35] K. Camarda and E. Seidel, Phys. Rev. D **59**, 064026 (1999), gr-qc/9805099.

[36] M. E. Rupright, A. M. Abrahams, and L. Rezzolla, Phys. Rev. D **58**, 044005 (1998).

[37] L. Rezzolla *et al.*, Phys. Rev. D **59**, 064001 (1999).

[38] R. Gomez, L. Lehner, R. Marsa, and J. Winicour, Phys. Rev. D **57**, 4778 (1998), gr-qc/9710138.

[39] K. Camarda, Ph.D. thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, 1998.

[40] D. Bernstein, D. Hobill, E. Seidel, and L. Smarr, Phys. Rev. D **50**, 3760 (1994).

[41] S. Brandt and E. Seidel, Phys. Rev. D **54**, 1403 (1996).

[42] P. Anninos *et al.*, Phys. Rev. Lett. **74**, 630 (1995).

[43] R. H. Price and J. Pullin, Phys. Rev. Lett. **72**, 3297 (1994).

[44] P. Anninos *et al.*, Phys. Rev. D **52**, 4462 (1995).

[45] A. Abrahams and R. Price, Phys. Rev. D **53**, 1972 (1996).

[46] J. Baker *et al.*, Phys. Rev. D **55**, 829 (1997).

[47] R. J. Gleiser, C. O. Nicasio, R. H. Price, and J. Pullin, Phys. Rev. Lett. **77**, 4483 (1996).

[48] C. Misner, Phys. Rev. **118**, 1110 (1960).

[49] D. Brill and R. Lindquist, Phys. Rev. **131**, 471 (1963).

[50] C. O. Lousto and R. H. Price, Phys. Rev. **D55**, 2124 (1997).

[51] V. Moncrief, Annals of Physics **88**, 323 (1974).

[52] J. Baker *et al.*, Phys. Rev. D **62**, 127701 (2000), gr-qc/9911017.

[53] P. Anninos, J. Massó, E. Seidel, and W.-M. Suen, Physics World **9**, 43 (1996).

[54] G. Allen, T. Goodale, and E. Seidel, in *7th Symposium on the Frontiers of Massively Parallel Computation-Frontiers 99* (IEEE, New York, 1999).

[55] G. Allen *et al.*, IEEE Computer **32**, (1999).

[56] G. Allen, T. Goodale, J. Massó, and E. Seidel, in *Proceedings of Eighth IEEE International Symposium on High Performance Distributed Computing, HPDC-8, Redondo Beach, 1999.*

[57] G. Allen *et al.*, in *Proceedings of Ninth IEEE International Symposium on High Performance Distributed Computing, HPDC-9, Pittsburgh* (2000).

[58] G. Allen *et al.*, in *IEEE International Symposium on Cluster Computing and the Grid* (2001), submitted.

[59] M. Alcubierre *et al.*, Phys. Rev. D **62**, 044034 (2000), gr-qc/0003071.

[60] M. Alcubierre *et al.*, Phys. Rev. D **62**, 124011 (2000), gr-qc/9908079.

[61] A. Arbona, C. Bona, J. Massó, and J. Stela, Phys. Rev. D **60**, 104014 (1999), gr-qc/9902053.

[62] D. S. Brill, Ann. Phys. **7**, 466 (1959).

[63] K. Eppley, Phys. Rev. D **16**, 1609 (1977).

[64] K. Eppley, in *Sources of Gravitational Radiation*, edited by L. Smarr (Cambridge University Press, Cambridge, England, 1979), p. 275.

[65] D. Holz, W. Miller, M. Wakano, and J. Wheeler, in *Directions in General Relativity: Proceedings of the 1993 International Symposium, Maryland; Papers in honor of Dieter Brill*, edited by B. Hu and T. Jacobson (Cambridge University Press, Cambridge, England, 1993).

[66] S. Brandt, K. Camarda, and E. Seidel, in *Proceedings of the 8th Marcel Grossmann Meeting on General Relativity*, edited by T. Piran (World Scientific, Singapore, 1999), pp. 741–743.

[67] M. Alcubierre *et al.*, Class. Quant. Grav. **17**, 2159 (2000), gr-qc/9809004.

[68] S. Brandt and B. Brügmann, Phys. Rev. Lett. **78**, 3606 (1997).

[69] B. Brügmann, Int. J. Mod. Phys. D **8**, 85 (1999).

[70] L. Nerger, Master's thesis, Universität Bremen, 2000.

[71] A. Abrahams, Ph.D. thesis, University of Illinois, Urbana, Illinois, 1988.

[72] Éanna É. Flanagan and S. A. Hughes, Phys. Rev. D **57**, 4535 (1998), gr-qc/9701039.