Neutron Stars, Magnetic Fields, and Gravitational Waves

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Abstract

The r-modes of rapidly spinning young neutron stars have recently attracted attention as a promising source of detectable gravitational radiation. These neutron stars are expected to have magnetic fields $\sim 10^{12}$ G.

The r-mode velocity perturbation causes differential motion of the fluid in the star; this is a *kinematic* effect. In addition, the radiation-reaction associated with emission of gravitational radiation by r-waves drives additional differential fluid motions; this is a *dynamic* effect. These differential fluid motions distort the magnetic fields of neutron stars and may therefore play an important role in determining the structure of neutron star magnetic fields. If the stellar field is $\sim 10^{16}\,(\Omega/\Omega_B)$ G or stronger, the usual r-modes are no longer normal modes of the star; here Ω and Ω_B are the angular velocities of the star and at which mass shedding occurs. Much weaker magnetic fields can prevent gravitational radiation from amplifying the r-modes or damp existing r-mode oscillations on a relatively short timescale by extracting energy from the modes faster than gravitational wave emission can pump energy into them.

The onset of proton superconductivity in the cores of newly formed magnetic neutron stars typically increases the effect on the r-modes of the magnetic field in the core by many orders of magnitude. Once the core has become superconducting, magnetic fields of the order of 10^{12} G or greater are usually sufficient to damp r-modes that have been excited by emission of gravitational radiation and to suppress any further emission. A rapid drop in the strength of r-mode gravitational radiation from young neutron stars may therefore signal the onset of superconductivity in the core and provide a lower bound on the strength of the magnetic field there. Hence, measurements of r-mode gravitational waves from newly formed neutron stars may provide valuable diagnostic information about magnetic field strengths, cooling processes, and the transition to superconductivity in neutron stars.

If the neutrons and protons in the cores of the neutron stars in low-mass X-ray binary systems are superfluid and superconducting, respectively, the resultant strong coupling between different regions of the core and between the core and the solid crust appears likely to prevent gravitational radiation by r-wave fluid motions from amplifying them. If so, gravitational radiation by r-waves would not play a significant role in determining the spin rates of these neutron stars, in accordance with the standard picture in which their spins are determined by magnetic spin evolution.

1 Introduction

Three years ago, Andersson [4] and Friedman & Morsink [18] demonstrated that the gravitational radiation produced by mass currents of r-mode (r-wave) character in rotating fluid stars causes these modes to grow. The discovery of this instability is an exciting development in relativistic astrophysics. It raises the possibility that observations of gravitational radiation by rapidly spinning neutron stars may eventually provide data on the validity of general relativity and the properties of ultradense matter, as well as on neutron star spins and magnetic fields (see, e.g., [30, 39, 7, 15, 29, 46]). For recent reviews, see [16, 6] and the lectures in this volume by Lindblom and Andersson.

In this lecture I focus on the likely effects on r-waves of magnetic fields, proton superconductivity, and neutron superfluidity. I first summarize what is known about the spin rates and magnetic fields of neutron stars. Next I introduce the linear r-wave velocity perturbation and discuss briefly the fluid motions produced by this perturbation, gravitational and electromagnetic radiation by r-waves, and the effects of nonlinear hydrodynamic and radiation-reaction forces. I then consider the interaction between r-waves and the stellar magnetic field, and the critical magnetic field strengths at which the r-modes are severely distorted and damped. Finally, I discuss the implications for r-waves in newly formed and old neutron stars.

An important conclusion is that interaction of r-waves with the magnetic fields of newly formed neutron stars are likely to suppress them and their gravitational radiation after days to months, depending on the physics of the interior. Detection of this suppression could be a valuable diagnostic of the interior properties of neutron stars. Damping in the fluid cores of the old neutron stars found in low-mass X-ray binary systems (LMXBs) is likely to prevent excitation of r-modes in these stars, in which case gravitational radiation by these modes is not important in determining their spin rates. My discussion of the interaction between r-waves and the stellar magnetic field is based on the work reported in [46, 47, 48, 10]. Further discussion of this interaction may be found in the lecture by Rezzolla in this volume.

2 Neutron Star Spins and Magnetic Fields

Before discussing the interaction of r-waves in neutron stars with the stellar magnetic field, it is useful to summarize the present evidence concerning their spin rates and field strengths.

2.1 Neutron star spins

The initial spin rates of neutron stars are important in determining whether strong gravitational radiation from newly formed neutron stars is likely. Observations of relatively young rotation-powered pulsars and soft γ-ray repeaters (SGRs) show that they have spin frequencies ranging from ~ 0.15 Hz up to ~ 60 Hz (see [58, 34]). Using the dipole braking law or their measured slowing down rates to extrapolate

backward in time to the epoch at which these stars are thought to have formed typically gives apparent initial spin frequencies that are only a factor of 2 or 3 higher than the star's current spin rate. Thus, although spin-down by gravitational radiation from an initial spin rate near the mass-shedding limit may have occurred, in most cases it is not required.

To the extent that accreting neutron stars in LMXBs are the progenitors of the millisecond rotation-powered pulsars, and there is strong evidence that they are (see below), these neutron stars must have magnetic dipole moments that correspond to surface fields $\sim 10^8$–10^{10} G. As discussed below, there is also a large body of evidence, most importantly the observed correlation between the spin rates and dipole magnetic moments of the millisecond rotation-powered pulsars, which supports the standard picture that the spin rates of accreting neutron stars in LMXBs are determined primarily by magnetic spin equilibrium (see, e.g., [2, 56, 20, 42, 14]) and not by gravitational radiation.

2.2   Magnetic fields of neutron stars

Neutron stars could have magnetic fields as strong as $\sim 10^{18}$ G and remain bound. However, the magnetic dipole spin-down timescale of such a strongly magnetic neutron star spinning near breakup (~ 900 Hz) is ~ 1 ms; even at 1 Hz, its spin-down timescale is only ~ 30 yr. Hence stars with such high fields are likely to be directly observable as strong emitters only for a very brief period. Some neutron stars may have magnetic fields less than $\sim 10^6$ G, but such low fields are too weak to be easily detectable. There is currently no compelling evidence that the magnetic fields of isolated neutron stars evolve appreciably with time. There is circumstantial evidence that the magnetic fields of neutron stars which have been accreting for a long time are much smaller than the magnetic fields of most young pulsars. As I now discuss, there is direct or indirect evidence that the surface magnetic fields of neutron stars range from as low as $\sim 10^7$ G to as high as $\sim 10^{15}$ G.

SGRs and young rotation-powered pulsars.—The small spin-down timescales of SGRs and their associations with supernova remnants have been interpreted as indicating that their magnetic fields have dipole components corresponding to surface magnetic fields $\sim 10^{13}$–10^{15} G (see [58]). Their X-ray and soft γ-ray emissions are widely thought to be powered primarily by dissipative processes associated with reconfiguration of their magnetic fields and consequent cracking of their crusts. Application of the dipole formula to measurements of the spin-down rates of most rotation-powered pulsars gives dipole moments that correspond to surface magnetic fields in the range $\sim 10^{11}$–10^{14} G [57]. This evidence, and the absence of evidence for any substantial decrease in the magnetic fields of isolated neutron stars over time, indicate that newly formed neutron stars probably have magnetic dipole moments that correspond to surface magnetic fields $\sim 10^{11}$–10^{15} G.

Recycled pulsars.—There is a class of rotation-powered pulsars with relatively high spin rates (~ 10–600 Hz) but relatively long spin-down timescales (~ 5–10 Gyr; see, e.g., [42, 14]). Application of the magnetic dipole braking law to these pulsars

yields dipole moments that correspond to surface magnetic fields in the range $\sim$ 10^8–10^9 G [42]). These neutron stars are thought to have high spin rates and low magnetic fields because they were (or are) in close binary systems and underwent a phase of mass accretion during which they accreted a substantial amount of angular momentum (see, e.g., [2, 20, 42, 43]) and their magnetic fields were suppressed, at least at the stellar surface (see, e.g., [56, 50]). They are often referred to as "recycled" neutron stars. The progenitors of the recycled rotation-powered pulsars are thought to be the accretion-powered neutron stars found in LMXBs (see, e.g., [2, 42, 59]).

Accreting neutron stars in HMXBs.—All bright accretion-powered neutron stars are accreting gas from their companion star in a close binary system. Of the subset that produce strong periodic oscillations ("X-ray pulsars"), almost all are found in high-mass systems (systems in which the companion to the pulsar has a mass $\gtrsim 10 M_\odot$). The neutron stars in these high-mass X-ray binary systems (HMXBs) can have accreted only a relatively small amount of matter. Their dipole moments can be inferred from their spin-rate behavior (see, e.g., [38]). The dipole moments estimated in this way correspond to surface magnetic fields $\sim 10^{11}$–10^{14} G. Cyclotron scattering features have been detected in the X-ray spectra of about a dozen X-ray pulsars and give values of their surface magnetic fields in the range $\sim 10^{12}$–10^{13} G (see, e.g., [33, 22]). This range is at least partly determined by observational selection, because the cyclotron features produced by magnetic fields outside this range would be at X-ray energies too low or too high to be easily detected.

Accreting neutron stars in LMXBs.—A few accreting neutron stars in low-mass systems produce strong, periodic X-ray oscillations. Here the adjective "low-mass" refers to the companion star, not the neutron star, and indicates that the companion star has a mass $\lesssim 1 M_\odot$. The dipole moments of these pulsars appear similar to the dipole moments of the X-ray pulsars found in HMXBs. However, most neutron stars in LMXBs are thought to have surface magnetic fields $\sim 10^7$–10^{10} G. The arguments supporting this thinking include:

(1) The accreting neutron stars in LMXBs are thought to be the progenitors of the millisecond rotation-powered pulsars, which have magnetic fields $\sim 10^8$–10^{10} G.

(2) Strong, coherent oscillations are observed in the tails of X-ray bursts from these neutron stars [55], indicating the existence of a stable marker in their surface layers. A nonuniform magnetic field is the most plausible marker.

(3) Antipodal hot spots have been detected during some X-ray bursts from 4U 1636−536. Funneling of nuclear fuel onto two antipodal spots by a dipolar surface magnetic field is the most plausible explanation for this observation [36].

(4) The twin kilohertz quasi-periodic oscillations (QPOs) observed from more than 20 neutron stars in LMXBs are most likely the orbital frequency of the accreting matter and the beat between this frequency and the spin frequency of the star, which requires a stable marker on the stellar surface. A magnetic field is the most plausible marker (see [37]).

(5) The lower-frequency ($\sim$ 15–80 Hz) QPOs produced by neutron stars in LMXBs are likely generated by beating of the stellar spin with the orbital mo-

tion of accreting matter further away, which again requires a stable marker on the stellar surface (see [37]).

(6) Surface magnetic fields $\sim 10^8$–10^9 G are required to produce the observed X-ray spectra of some neutron stars in LMXBs without violating the blackbody limit [45].

(7) Magnetic spin-equilibrium naturally explains the spin rates and lower-frequency QPOs of the accreting neutron stars called Z sources, if they have magnetic moments corresponding to surface fields $\sim 10^8$–10^9 G [44].

(8) The amplitudes of the lower-frequency QPOs are positively correlated with the magnetic field strengths inferred from their X-ray spectra, as would be expected if these neutron stars have magnetic fields $\sim 10^7$–10^9 G [45].

(9) The amplitudes of the kilohertz QPOs are negatively correlated with the magnetic field strengths inferred from their spin rates, assuming they are in magnetic spin equilibrium, as would be expected if the stars have magnetic fields $\sim 10^7$–10^9 G [45].

Taken together, this evidence makes a consistent, compelling case that most neutron stars in LMXBs have magnetic fields $\sim 10^7$–10^9 G. These stars are thought to have weaker surface fields than the neutron stars in HMXBs because they have accreted ~ 0.1–$0.5 M_\odot$ [56, 50], although there is as yet no generally accepted physical explanation for this behavior.

3   *r*-Waves and Emission of Radiation

Consider now the velocity perturbation produced by a small-amplitude *r*-wave in a slowly and uniformly rotating, nonmagnetic, inviscid, barytropic Newtonian star with mean mass density $\bar{\rho}$. The structure, magnetic field, and dynamical properties of a real neutron star can distort significantly the *r*-waves discussed here and can affect whether they grow or are damped. Some of these effects will be considered in subsequent sections.

3.1   *r*-wave velocity field

Important parameters include the dimensionless mode amplitude α and the ratio of the angular velocity Ω of the star to $\Omega_B \equiv (2/3)(\pi G \bar{\rho})^{1/2}$; the latter is close to the angular velocity at which mass shedding begins [30]. Solution of the nonrelativistic fluid equations to first order in both α and $(\Omega/\Omega_B)^2$ gives the Eulerian velocity field

$$\delta \vec{v}_1(r, \theta, \phi, t) = \alpha \Omega R \, (r/R)^l \, \vec{Y}_{ll}^B e^{i\sigma t} \, , \tag{1}$$

where r is the radius, $\vec{Y}_{ll}^B$ is the vector spherical harmonic of magnetic type, and σ is the angular frequency of the wave in the inertial frame; to this same order, $\delta v_r = 0$ (see [18, 30]). The velocity field $\delta \vec{v}_1$ becomes exact to arbitrary order in the wave amplitude as $(\Omega/\Omega_B)^2 \to 0$. The *r*-wave velocity field $\delta \vec{v}_1$ causes elements of fluid to gyrate and drift, as do sound and water waves. Over time, it will be modified by nonlinear hydrodynamic effects and the gravitational radiation-reaction force.

3.2   *r*-wave fluid motions

The Eulerian *r*-wave velocity field $\delta\vec{v}_1$ causes fluid elements to drift as well as gyrate, just as sound and water waves cause fluid elements to drift as well as oscillate [46, 47, 48]. Although the Eulerian velocity perturbation (i.e., the velocity perturbation at a *fixed* location in the corotating frame) is periodic to first order in α, the Lagrangian motion of fluid elements produced by this velocity perturbation is not periodic (for an example of this behavior in a similar context, see [25], §64). As Figure 1 shows, a given fluid element makes excursions of comparable size in the ϕ and θ directions during each gyration and, in addition, moves steadily in the azimuthal direction with a drift velocity $\vec{v}_d$ that depends on r and θ and therefore shears the fluid in latitude and radius. Fluid elements in the northern and southern hemispheres equidistant from the rotation equator drift in the same direction at the same rate, although their gyrational motions are different.

The drift velocity $\vec{v}_d$ is a *kinematic* effect, in the sense that it is not produced by any force but is simply a consequence of the presence of the *r*-wave: it is present any time the *r*-wave is present, and its magnitude and properties are determined entirely by the *r*-wave. This kinematic drift is therefore fundamentally different from the differential rotation varying with radius that Spruit [53] conjectured would be produced by the gravitational radiation-reaction force, from the latitude-dependent drift produced by radiation-reaction in the simplified shell model studied analytically by Levin and Ushomirsky [26], and from the differential motion found by Friedman, Lockitch, and Sá [17], which is produced by nonlinear hydrodynamic forces. These *dynamic* drifts are zero initially and then grow in time at rates determined by the magnitudes of the driving forces, whereas the kinematic drift is present from the beginning, with the same size it always has; if there are no driving forces, there will be no dynamic drifts, whereas the kinematic drift will be unaffected.

The *r*-modes of a neutron star are sufficiently complex that exact analytical solutions of the fluid equations valid for arbitrary wave amplitudes α are not available for these waves, even to first order in $(\Omega/\Omega_B)^2$. The fluid equations have, however, been solved exactly for some sound waves, for shallow water waves ($kh_0 \gg 2\pi$, where k is the wavenumber and h_0 is the water depth), and for *r*-waves in a thin shell of fluid. The fluid motions produced by these waves are described correctly to second order in the wave amplitude by integrating the fluid motions given by the solutions $\delta\vec{v}_1$ of the relevant linearized fluid equations, just as was done for the *r* waves in [46, 47, 48]. For example, the displacement with time of a fluid element caused by a sinusoidal shallow water wave is $x_+(x_0, t) = x_0 + c_0[1 + \alpha\cos(kx_0)]t$, where c_0 is the wave speed [48]. Characteristics first cross after the wave has traveled for a time $t_s = 1/3\alpha k c_0$ and a distance $\Delta x \approx \lambda/\alpha$. For $t \ll t_s$, the average of the *exact* fluid velocity over one period gives $v_d = 2\alpha^2 c_0$ to $\mathcal{O}(\alpha^2)$. Using $\delta\vec{v}_1$ to compute the drift velocity also gives $v_d = 2\alpha^2 c_0$ to $\mathcal{O}(\alpha^2)$. The nonlinear terms in the Euler equation vanish for a single *r*-wave, if the radial velocity is constrained to be zero, i.e., if motion is confined to spherical surfaces. Levin and Ushomirsky [26] used this fact to compute the fluid motion generated by *r*-waves in a model in which a uniformly rotating fluid is confined between closely spaced concentric spherical
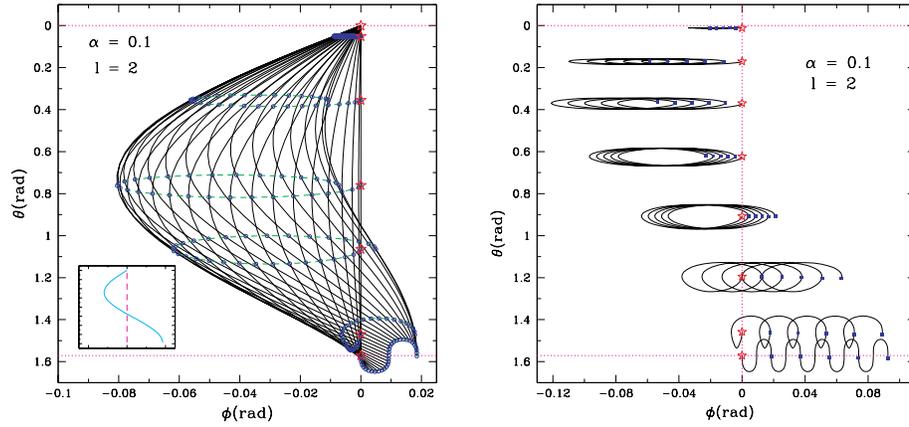
Figure 1: Motions of fiducial fluid elements in the northernhemisphere of a star produced by the velocity perturbation $\delta\vec{v}_1$ for $l = m = 2$ and $\alpha = 0.1$, as seen by a distant corotating observer in the rotation equator of the star; Ω was set equal to Ω_B for clarity. *Left*: Motions over one oscillation period [47]. *Right*: Motions over five oscillation periods [46, 47]. The initial positions of the fluid elements are indicated by the 'star' symbols; their positions after each oscillation period are indicated by the filled squares.

shells. They find exactly the same latitude-dependent azimuthal drift found earlier by Rezzolla et al. [46]. These examples give us confidence that the drift of fluid elements given by the r-wave velocity field $\delta\vec{v}_1$ is qualitatively correct and perhaps exact to order α^2.

Stergioulas and Font [54] have studied numerically the time evolution of an initial perturbation of a rapidly, uniformly rotating relativistic star that approximates the $l = m = 2$ r-mode. They found an apparent kinematic drift with a magnitude that scales as α^2, but lacked the resolution needed to investigate the latitude dependence of the drift. They concluded that simulations with more precise initial data and higher resolution are needed to identify the cause of the drift.

3.3 Gravitational and electromagnetic radiation

The growth time for $l = 2$ r-waves driven by gravitational radiation is [18, 30]

$$\frac{1}{\tau_{\mathrm{GW}}} \approx \frac{c}{R}\left(\frac{GM}{Rc^2}\right)\left(\frac{R\Omega}{c}\right)^6 \approx \frac{1}{37\,\mathrm{s}}\left(\frac{\nu_{\mathrm{s}}}{900\,\mathrm{Hz}}\right)^6 . \tag{2}$$

For comparison, the magnetic dipole radiation timescale is

$$\frac{1}{\tau_{\mathrm{MD}}} \approx \frac{c}{R}\left(\frac{B^2 R^3}{Mc^2}\right)\left(\frac{R\Omega}{c}\right)^2 \approx \frac{(B/10^{14}\,\mathrm{G})^2}{1.5 \times 10^5\,\mathrm{s}}\left(\frac{\nu_{\mathrm{s}}}{900\,\mathrm{Hz}}\right)^4 \tag{3}$$

and hence the ratio of these two time scales is

$$\frac{\tau_{\mathrm{MD}}}{\tau_{\mathrm{GW}}} \approx \left(\frac{GM^2}{B^2 R^4}\right) \left(\frac{R\Omega}{c}\right)^4 \approx \frac{4.2 \times 10^3}{(B/10^{14}\,\mathrm{G})^2} \left(\frac{\nu_{\mathrm{s}}}{900\,\mathrm{Hz}}\right)^4 . \tag{4}$$

Thus, the time required for magnetic dipole radiation to spin the star down is much longer than the growth time of low-order r-waves unless the dipole component of the star's magnetic field is $\gtrsim 10^{16}$ G (see also [23]). Electromagnetic radiation produced by perturbations of the stellar magnetic field by the r-wave fluid motions is much weaker.

3.4   Evolution of r-waves in a fluid star

Initial work on the evolution of r-waves in neutron stars assumed that gravitational radiation-reaction simply increases the amplitude of the r-wave and that nonlinear hydrodynamic effects simply limit the r-wave amplitude α to some saturation value $\alpha_s \sim 1$. Here I report on recent work exploring the validity of these assumptions.

Nonlinear hydrodynamic effects.—In their numerical simulations, Stergioulas and Font [54] found that even for amplitudes $\alpha \lesssim 1$, an initial perturbation of a rapidly rotating relativistic star that approximates its $l = m = 2$ r-mode persists for ~ 20 spin periods without obvious evidence of nonlinear hydrodynamic effects. On the other hand, Friedman, Lockitch, and Sá [17] (see also [52]) find in an analytical treatment that the source terms in Euler's equation that are $\mathcal{O}(\alpha^2)$ drive differential rotation that is stratified on cylinders. These two results could be consistent if the coefficients of the $\mathcal{O}(\alpha^2)$ terms in Euler's equation are small. Nonlinear effects may be reduced if the star is highly stably stratified, because such stratification tends to suppress radial fluid motions.

Effects of the gravitational radiation.—The gravitational radiation-reaction force not only increases the amplitude of the r-wave, but also generates other fluid motions, including differential rotation of the star. In the simplified r-wave model of Levin and Ushomirsky [26], for example, the back-reaction produced by $l = m = 2$ r-wave gravitational radiation not only increases the amplitude of the r-wave, it also generates an $\mathcal{O}(\alpha^2)$ uniform rotation counter to the rotation of the star and an $\mathcal{O}(\alpha^2)$ flow that varies with latitude.

Recently Lindblom, Tohline, and Vallisneri [32] constructed a rapidly rotating Newtonian star and then studied the time evolution of its $l = m = 2$ r-mode in the presence of a simplified post-Newtonian radiation-reaction force that drives the mass current quadrupole associated with this mode. In order to study the growth of the mode with a reasonable amount of computing time, they increased the radiation-reaction force by a factor of 4,500. They found that the r-wave grew to $\alpha \sim 2$, at which point its amplitude plunged. By this time there is significant differential motion within the star. Lindblom et al. attribute the damping of the r-wave to dissipation produced by shocks in the outer layers of the star. They conjecture that once the r-wave is damped, the dissipation will die away, and the r-wave will grow again, repeating the cycle. However, as they themselves note, the artificially

increased radiation-reaction force means that nonlinear hydrodynamic effects will appear much smaller in the simulation than they are in reality. The true r-wave growth time is $\sim 6 \times 10^4$ rather than ~ 100 oscillation periods, so it is possible that in reality nonlinear hydrodynamic effects cause the wave to saturate at a lower amplitude.

These results show that the simplified evolutionary model used by Owen et al. [39] and others, which assumes that the only effect of the gravitational radiation-reaction force is to increases the amplitude of the r-wave, is incomplete. One consequence of the increasing differential rotation produced by the gravitational radiation-reaction force is that after a time the star cannot be treated as uniformly rotating, even if it was uniformly rotating initially; at this point the canonical r-modes are no longer normal modes of the star.

4   Interaction of r-Waves with the Star's Magnetic Field

4.1   Oscillatory distortion of the stellar magnetic field by r-waves

The timescale for magnetic diffusion in the normal cores of newly formed neutron stars is [24] $\tau_{\text{diff}} = 4\pi L^2 \sigma_{E}/c^2 \approx 7 \times 10^7 \, (x/0.1) L_5{}^2 T_{10}{}^{-2} \rho_{14}$ yr, where x is the charged particle fraction in the core, L_5 is the characteristic lengthscale of the field configuration in units of 10^5 cm, T_{10} is the temperature in units of 10^{10} K, and ρ_{14} is the mass density in units of 10^{14} g cm^{-3}. Hence, even at the temperatures $\sim 10^{10}$ K relevant to initial r-wave growth in newly formed neutron stars, τ_{diff} is much longer than the timescales of interest. Thus the distortion of the stellar magnetic field $\vec{B}$ by r-waves can be computed using the ideal MHD approximation.

In the ideal MHD approximation, the evolution of $\vec{B}$ in a normal fluid star is given by the induction equation $d\vec{B}/dt = \nabla \times (\vec{u} \times \vec{B})$, where the operator $d/dt \equiv \partial/\partial t + \vec{\Omega} \times \vec{r} \cdot \nabla - \vec{\Omega} \times$ gives the time rate of change of $\vec{B}$ at a fixed position in the frame corotating with the star and $\vec{u}$ is the fluid velocity in this frame. To lowest order in $(\Omega/\Omega_B)^2$, r-waves do not change the density and we therefore neglect density variations in the following analysis. Then the induction equation can be integrated directly to give [41, 8]

$$B^j(\vec{x}, t) = B^k(\vec{x}_0, t_0)(\partial x^j/\partial x_0^k) , \tag{5}$$

where $\partial x^j/\partial x_0^k$ is the coordinate strain from time t_0 to time t. The rate of change of the magnetic field during an oscillation is $d\vec{B}/dt = \nabla \times (\vec{u} \times \vec{B}) \approx \delta v_1 B/R \approx \alpha \Omega B$, and hence the distortion of the field during one oscillation is $\delta B \approx (dB/dt)(\pi/\Omega) \approx \pi \alpha B$. The change in the magnetic energy during an oscillation is therefore

$$\delta E_m = \int_V (\delta B^2/8\pi) \, dV \approx (\pi^2/6) \, \alpha^2 B^2 R^3 . \tag{6}$$

4.2 Effect of core superconductivity on the magnetic field energy

Once the protons in the neutron star core have become superconducting, the magnetic field there is confined to flux tubes, provided that the field is less than the upper critical field H_{c2} [19]. For proton superconductivity in neutron star cores, H_{c2} is $\sim$ 5 times larger than the thermodynamic critical field $H_c = 4 \times 10^{16}$ G $(x/0.1)^{1/6}(\rho/\rho_0)^{1/6}(\Delta_p/1\,\mathrm{MeV})^{1/6}$ (see [24]). Here $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is the density of bulk nuclear matter and Δ_p is the proton energy gap. To extend a flux tube costs the condensation energy per unit volume, which for fields much less than H_{c2} is very large compared to the magnetic energy per unit volume in the normal fluid. Typically, therefore, the onset of superconductivity in the core increases the energy cost of distorting the magnetic field in the core by orders of magnitude [13]. This increases the effect of the magnetic field on the r-modes by this same factor.

To understand why superconductivity increases the energy required to distort the magnetic field, note that the energy E_f of a single flux tube is the free energy per unit length $\epsilon_f = \phi_0 H_f/(8\pi)$ times the total length ℓ_f of the tube. Here $\phi_0 \equiv hc/2e$ is the quantum of flux and H_f is the field inside a flux tube, which in neutron star cores is expected to be $\sim (0.2\text{--}5) \times H_c$. Estimates of H_f are relatively secure, because unlike H_c, H_f depends only logarithmically on the poorly known gap energy (see [24]). For magnetic field strengths less than H_{c2}, the total energy of a collection of flux tubes is very nearly the sum of their individual energies. Assuming there are no closed-loop flux tubes in the core, the total number of flux tubes there is $\phi_{\mathrm{tot}}/\phi_0 \approx \pi R^2 B^{\hat{p}}/\phi_0$, where ϕ_{tot} is the total flux threading the core and $B^{\hat{p}}$ is the undistorted poloidal field there. Hence the total energy cost of the magnetic flux in the core is $E_f \approx (1/8)H_f B^{\hat{p}} R^2 \ell_f$, which is a factor $H_f/B^{\hat{p}}$ larger than it would be if the core were normal. This result is quite general: when the core becomes superconducting, the magnetic energy density and the stress exerted by the magnetic field become $2H_f/B^{\hat{p}}$ and $H_f/B^{\hat{p}}$ times larger, respectively, than when the core was normal [13].

4.3 Distortion of r-waves by the stellar magnetic field

Consider a thought experiment in which the magnetic field of the star steadily increases. As it does, the back-reaction force created by the distortion of the field by the r-mode increases. Eventually, the back-reaction force significantly distorts the velocity field of the mode. At this point the zero-field r-modes are no longer normal modes of the star; in order to determine the normal modes, the magnetic force must be included.

Computing the low-order normal modes of the star in the presence of a dynamically significant magnetic field requires global solution of the hydromagnetic equations for a rotating fluid star, which is a challenging problem. It is relatively easy, however, to estimate the strength of the magnetic field that severely distorts the r-mode velocity field, by comparing the energy $\delta E_r \approx \int_V (\rho\, \delta v_1^2/8\pi)\, dV$ in the

r-mode with the energy δE_m that it costs to distort the magnetic field in the manner required by the mode [46]. If $\delta E_m > \delta E_r$, the usual r-waves are not normal modes of the star; the corresponding normal modes are instead hydromagnetic r-waves [23, 10].

When the stellar core is normal, $\delta E_m > \delta E_r$ if $B > B^N_{\mathrm{crit},d} \approx 4\times10^{16}\,\mathrm{G}$ ($\nu_{\mathrm{s},900} M_{1.4}^{1/2}$ $R_{12.5}^{-1/2}$), where $\nu_{\mathrm{s},900}$ is the stellar spin rate in units of 900 Hz, $M_{1.4}$ is the stellar mass in units of $1.4 M_\odot$, and $R_{12.5}$ is the stellar radius in units of 12.5 km. If instead the stellar core is superconducting, $\delta E_m > \delta E_r$ if $B > B^{\mathrm{SC}}_{\mathrm{crit},d} \approx 10^{15}\,\mathrm{G}$ ($\nu^2_{\mathrm{s},100} H^{-1}_{c1,16} M^1_{1.4}$ $R_{12.5}^{-1}$), where $\nu_{\mathrm{s},100}$ is the stellar spin rate in units of 100 Hz; for spin rates $\gtrsim 500$ Hz, the critical field $B^{\mathrm{SC}}_{\mathrm{crit},d}$ at which r-like modes become highly distorted is so large that it would probably prevent proton superconductivity in the core. The critical field at which magnetic forces severely distort the zero-field r-modes is only modestly affected by superconductivity in the core because the critical field is close to the thermodynamic critical field H_c.

These estimates show that *magnetic fields in the core of a rapidly spinning neutron star will not significantly distort the r-modes there unless the strength of the field is* $\gtrsim 10^{16}$ *G.*

4.4 Damping of r-waves by the stellar magnetic field

As discussed in §3.2, linear r-waves produce a radius- and latitude-dependent kinematic drift. As discussed in §3.4, the gravitational radiation back-reaction force and nonlinear hydrodynamic forces are expected to generate additional radius- and latitude-dependent dynamic drifts. These differential motions of the conducting fluid in the star will progressively distort the stellar magnetic field. If the stellar core is normal, the electrons, protons, and neutrons in the fluid core behave like an ideal MHD fluid on the times scales of interest and the differential motions of this fluid therefore distort the frozen-in magnetic field. If the protons in the core are superconducting but the neutrons there are normal, the neutron fluid remains tightly coupled to the proton flux tubes, because the neutrons scatter via the strong interaction off the normal protons in the flux-tube cores (see [1]), and the drift of the neutron fluid therefore distorts the flux tubes. Finally, if the protons in the core are superconducting and the neutrons there are superfluid, the protons and electrons are locked together and scattering of electrons and muons by the $\sim 10^{15}\,\mathrm{G}$ magnetic field surrounding each neutron vortex couples the motion of the neutron superfluid to the flux tubes (see [3]).

The secular distortion of the magnetic field of the star costs energy and therefore drains energy from the r-modes. The coupling of r-waves to gravitational radiation is relatively weak and hence even a modest magnetic field can drain energy from the r-waves faster than they can be driven by the radiation-reaction force. I will focus here on the distortion of the magnetic field over a few r-wave oscillation periods (see the lecture by Rezzolla in this volume for a discussion of the long-term distortion of the stellar magnetic field). Such an analysis is sufficient to determine the critical magnetic field strength above which gravitational radiation cannot excite r-waves,

i.e., the critical magnetic field strength above which r-wave perturbations cannot be amplified by gravitational wave emission.

Assuming for simplicity that the magnetic field in the stellar core is initially poloidal, eq. (5) shows that the gyration-averaged azimuthal magnetic field produced by differential rotation of the fluid in the star is [46]

$$\langle B^\phi(\vec{x}, t)\rangle = B_0^\phi + B_0^r \frac{\partial \langle x^\phi(t)\rangle}{\partial r_0} + B_0^\theta \frac{r \partial \langle x^\phi(t)\rangle}{\partial \theta_0} \ . \tag{7}$$

Here $B_0^j \equiv B^j(\vec{x}_0, t_0)$; r_0 and θ_0 are the initial coordinates of the fluid elements.

For simplicity, consider only the kinematic differential fluid motion produced by a linear r-wave with $m = l$ (see §3.2), neglecting the differential motions that will be generated by the gravitational radiation-reaction force and nonlinear hydrodynamic forces (see §3.4). (We expect the latter differential motions to contribute to amplification of the magnetic field.) The gyration-averaged kinematic drift velocity is $\vec{v}_d(r, \theta, t) = \kappa_l(\theta)\alpha^2(t)\omega(t)R(r/R)^2\hat{\phi}$, where $\kappa_2(\theta) \equiv (1/2)^7(5!/\pi)(\sin^2\theta - 2\cos^2\theta)$. Taking the time derivative of eq. (7) and equating the gyration-averaged ϕ-velocity with $\vec{v}_d$ gives an expression for the rate of increase of the azimuthal magnetic field. This expression can then be used to compute the rate dE_m/dt at which the magnetic field energy increases.

Energy must be continually fed into the magnetic field just to sustain an r-wave at a constant amplitude. If $dE_m/dt > dE_{GW}/dt$, where dE_{GW}/dt is the rate at which the mode energy increases due to excitation of the mode by the radiation-reaction force, the energy that must be put into the star's magnetic field in order to sustain the r-wave fluid motion exceeds the energy that is available from gravitational wave emission. Hence, if dE_m/dt exceeds dE_{GW}/dt, the r-wave not only will not grow, it will be damped.

The condition $dE_m/dt > dE_{GW}/dt$ can be used to derive the critical magnetic field strength above which the r-wave is damped, as a function of the stellar spin rate ν, or to derive the critical spin rate below which the r-wave is damped, as a function of the magnetic field strength in the core. Here we give expressions for the critical spin rate. If the stellar core is normal, $dE_m^N/dt \approx v_d B^{\hat{p}} B^{\hat{\phi}} R^2 \approx \alpha^2\Omega(B^{\hat{p}} B^{\hat{\phi}} R^3)$ and $dE_m/dt > dE_{GW}/dt$ if ν is less than

$$\nu_{\text{crit}}^N \equiv 0.3\,\nu_B\,(B^{\hat{p}} B^{\hat{\phi}}/10^{26}\,\text{G}^2)^{1/7} M_{1.4}^{-2/7} R_{12.5}^{-3/7}\ , \tag{8}$$

where $\nu_B \approx 900$ Hz is the circular velocity at which the star begins to shed mass. If the neutron fluid in the core is normal, it is tightly coupled to the flux tubes, because the neutrons scatter via the strong interaction off the normal protons in the flux-tube cores (see §5.2). if instead the neutrons are superfluid, the flux tubes

If instead the stellar core is superconducting, $dE_m^{SC}/dt \approx (\phi_{\text{tot}} H_f/8\pi)(d\ell_f/dt) \approx (\phi_{\text{tot}} H_f/8\pi)\,v_d$ and $dE_m/dt > dE_{GW}/dt$ if ν is less than

$$\nu_{\text{crit}}^{SC} \equiv 0.7\,\nu_B\,(B^{\hat{p}}/10^{12}\,\text{G})^{1/7}(H_f/10^{16}\,\text{G})^{1/7} M_{1.4}^{-2/7} R_{12.5}^{-3/7}\ . \tag{9}$$

These critical spin frequencies depend only weakly on the strength of the magnetic field threading the core.

A key question is whether a stellar magnetic field strong enough to damp an r-wave will distort it in such a way that the drifts which lead to damping are suppressed. As pointed out in [46, 47, 48], the energy density of the $\sim 10^{12}$ G magnetic field needed to prevent amplification of an r-wave by gravitational radiation or to damp an existing r-wave in a star with $\nu_s \sim 0.7\nu_B$ is $\sim 10^{-8}$ of the energy density in the mode. The deviation δ_B of the position of a fluid element in one oscillation period produced by this magnetic field is therefore $\lesssim 10^{-4}\alpha R$ [10], whereas in one period the kinematic drift displaces the fluid element a distance $d = 2\pi\alpha^2 R$. Thus, for magnetic fields $\sim 10^{12}$ G and mode amplitudes $\alpha \gg 10^{-5}$, d is $\gg \delta_B$, i.e., magnetic forces are too weak to suppress the kinematic drift, not to mention the dynamic drifts discussed in §3.4.

5   Implications for Young and Old Neutron Stars

5.1   *r*-waves in young neutron stars

As explained in the previous section, r-waves distort the magnetic fields of neutron stars, which can limit their occurrence. The results presented in §4.3 show that if the magnetic field in the core exceeds $B_{\mathrm{crit,d}} \sim 10^{16}\,(\Omega/\Omega_B)$ G, the zero-field r-modes are no longer normal modes of the star, whether or not the core is superconducting.

If the magnetic field threading the core is weaker than $B_{\mathrm{crit,d}}$, the following scenario is a possibility. The interior of the newly formed neutron star is initially hot and until the central temperature T_c falls below a critical temperature $T_{\mathrm{crit},\eta} \sim 3 \times 10^9$ K, r-waves are damped by the high bulk viscosity [29]. Once T_c falls below $T_{\mathrm{crit},\eta}$, gravitational radiation causes the amplitudes of r-wave velocity perturbations to grow. Eventually the r-wave motions will become distorted by the differential rotation generated by radiation-reaction and nonlinear hydrodynamic forces. This differential rotation could generate a large toroidal magnetic field in the interior of the star. Assuming that time-dependent mass currents continue to be driven by the gravitational radiation-reaction force, potentially detectable gravitational radiation will continue to be emitted and the star will gradually spin down.

Once the temperature in the stellar core falls below the superconducting transition temperature $T_{\mathrm{crit},p} \sim 3 \times 10^9$ K, a region of superconducting protons forms in the core and then spreads through the core as the temperature continues to fall. If neutrino cooling is rapid, e.g., via the direct URCA process, the neutron star's interior temperature will fall below 10^9 K within minutes [40]. Even if neutrino cooling is not rapid, a significant fraction of the core is likely to become superconducting within days to weeks. If the young neutron star has a canonical magnetic field ($\sim 10^{12}$ G) and this field threads its core, r-wave motions will be suppressed in the core once it has become superconducting and the spin rate ν_s of the star has fallen below the critical spin rate $\nu_{\mathrm{crit}}^{\mathrm{SC}}$. At this point, gravitational radiation by r-wave fluid motions in the core will end. For example, the $l = 2$ r-wave will be suppressed

once the core has become superconducting and ν_s has fallen below ~ 600 Hz. When the core temperature reaches $T_{\mathrm{crit},n} \sim 10^9$ K, the neutrons in the core are expected to become superfluid. A solid crust forms when the surface temperature of the star has fallen to $\sim 10^{10}$ K [21, 11].

If this sequence of events is roughly right, detection of gravitational radiation from a newly formed, rapidly spinning neutron star followed by a relatively rapid cessation of the radiation may signal the onset of proton superconductivity in the stellar core, providing the first observational evidence for this transition. Detection of this signal would provide valuable information about the strength of the magnetic field threading the core, the temperature at which the protons become superconducting, and the processes by which the core cools.

5.2 r-waves in old neutron stars

The surfaces of the old neutron stars in LMXBs are expected to have temperatures $\lesssim 10^8$ K and hence to be solid. Their cores are expected to have temperatures $< 10^9$ K and hence the neutrons and protons there are expected to be superfluid and superconducting, respectively. There are a variety of mechanisms that tend to couple motions in the fluid core to the solid crust and to damp differential motions in the core. Whether r-waves can grow in these neutron stars depends on the strength of these damping mechanisms.

If the magnetic field threading the fluid core is $\sim 10^9$ G, the mechanism discussed in §4.4 will suppress r-waves in stars with spin rates $\lesssim 300$ Hz. Stronger magnetic fields will suppress r-waves in stars with higher spin rates. Moreover, several mechanisms may damp r-waves in the core even if the magnetic field there is less than 10^9 G. Assuming the protons in the core form a type I superconductor and the neutrons there are superfluid, the neutrons circulating around each vortex will entrain some protons, creating a proton supercurrent that generates a $\sim 10^{15}$ G magnetic field parallel to the vortex line [1]. This magnetic field strongly scatters electrons. The protons will move with the electrons, because any lag will generate large transient fields that will strongly couple them [12]. Scattering of electrons by the vortices damps motion of the electron-proton plasma relative to the vortices in a few seconds; r-wave motions of the neutron superfluid are therefore damped by the friction between the vortices and the electron fluid and by the electron shear viscosity. Calculations of these damping processes indicate that whether they are strong enough to suppress r waves in the fluid core depends on poorly known properties of the neutron-proton interaction [28]. These calculations neglected coupling of the fluid core to the solid crust.

The neutron fluid in the core may couple to the crust via a viscous or a hydromagnetic boundary layer, depending on the strength of the magnetic field in the core [12, 9, 31, 49, 27, 35]. If the magnetic field in the core is negligible, the boundary layer is viscous and the timescale for the electrons (and hence the electron-proton fluid) to couple to a perfectly rigid crust would be $\tau_{\mathrm{e-cr}}^{\mathrm{visc}} \approx (R^2/\nu_e\Omega)^{1/2} \sim 2\,\nu_{\mathrm{s},300}^{-1/2}\,T_8$ s [12, 1, 24]; here R is the radius of the star, ν_e is the kinematic vis-

cosity of the electron fluid, Ω is the angular velocity of the star, $\nu_{s,300}$ is the spin frequency of the star in units of $300\,\mathrm{Hz}$, and T_8 is the core temperature in units of $10^8\,\mathrm{K}$. Scattering of the electrons by the $\sim 10^{15}\,\mathrm{G}$ magnetic field surrounding the neutron vortices then couples the electron-proton fluid to the neutron superfluid in a time [3] $\tau_{\mathrm{sc}} \approx 10\,(300\,\mathrm{Hz}/\nu_s)\,\mathrm{s}$.

The crust is not perfectly rigid, however. Indeed, the crust is so weak that it is easily deformed by the viscous shear stress exerted on it by the differential motion of the fluid in the core [27]. The resulting deformation of the crust lengthens the timescale on which core-crust coupling damps r-modes in the core by up to a factor ~ 10, provided that the resulting strain angles $\phi \sim \alpha$ do not exceed the critical strain angle ϕ_{crit} at which the crust fractures; ϕ_{crit} is expected to be $\sim 10^{-4}$–10^{-3}. Even so, the timescale $\tau_{\mathrm{GW}} \approx 5 \times 10^4\,(300\,\mathrm{Hz}/\nu_s)^6\,\mathrm{s}$ for gravitational radiation to excite the $l = 2$ r-wave is $\sim 10^3$ times longer. For wave amplitudes $\alpha \gtrsim 10^{-3}$, the r-wave fluid motions in the core are likely to break the crust, causing increased dissipation and damping the r-waves more rapidly.

If the protons in the stellar core are normal, a core magnetic field stronger than $10^9\,\mathrm{G}$ will change significantly the structure of the boundary layer, increasing the damping of r-waves, and core magnetic fields $\gtrsim 10^{12}\,\mathrm{G}$ will completely suppress them [35]. If instead the protons in these old, cold neutron stars are, as expected, superconducting, it is likely that even weaker magnetic fields in the core will completely suppress r-waves, by increasing the magnetic stress in the boundary layer and creating rigid flux tubes that interfere with the r-wave motions of the neutron vortices [51].

Taken together, these and other damping mechanisms seem likely to prevent excitation of r-waves in the accreting neutron stars in LMXBs. If so, gravitational radiation by r-waves plays no role in determining the spin rates of these neutron stars. This is consistent with the evidence supporting the standard picture of the formation of millisecond rotation-powered pulsars, in which accretion of matter and angular momentum spins up these magnetic neutron stars in LMXBs until they reach magnetic spin-equilibrium (see §2.2). This standard picture predicted the subsequently observed correlation between the magnetic dipole moments and spin frequencies of the millisecond pulsars, a correlation that is so far inexplicable if their spin rates are instead determined by gravitational radiation.

6   Concluding Remarks

Excitation of r-waves in rapidly spinning young neutron stars is a promising source of gravitational radiation. Interaction of these waves with the stellar magnetic field is likely to affect both the waves and the field. Assuming that the stellar core is superconducting, magnetic fields $\gtrsim 10^{12}\,\mathrm{G}$ are sufficient to prevent excitation of r-waves by emission of gravitational radiation or to damp r-modes that have previously been excited. Strong gravitational radiation from a newly formed, rapidly spinning neutron star followed by a relatively rapid cessation of the radiation may signal the onset of superconductivity in the stellar core. Detection of this effect

would provide valuable information about the strength of the core magnetic field, the superconducting transition temperature, and the processes by which the core cools.

A variety of damping mechanisms are likely to prevent excitation of r-waves in the old neutron stars in LMXBs. If so, the spin rates of these neutron stars are not determined by r-wave gravitational radiation but instead by some other mechanism, most likely magnetic spin-equilibrium.

Acknowledgments

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