Gravitational Waves from Freely Precessing Neutron Stars

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Abstract

The purpose of this study is to assess the likely detectability of gravitational waves from freely precessing neutron stars. We begin by presenting a neutron star model of sufficient complexity to take into account both the elasticity and fluidity of a realistic neutron star. We then examine the affect of internal dissipation (i.e. heat generation within the star) and gravitational radiation reaction on the wobble. This is followed by an examination of various astrophysical scenarios where some mechanism might pump the precessional motion. We estimate the gravitational wave amplitude in these situations. Finally, we conclude that gravitational radiation from freely precessing neutron stars is almost certainly limited to a level undetectable by a LIGO II detector by internal dissipation.

1 Introduction

The problem of gravitational wave generation from freely precessing neutron stars can be divided into three parts. First it is necessary to describe a model of the free precession simple enough to be able to use in subsequent calculations, but of sufficient complexity to capture the essential features of a real precessing star. Then it is necessary to describe the dissipative energy losses that sap energy from such a precessing star, by converting its mechanical energy into heat. Finally it is necessary to examine particular astrophysical scenarios in which external torques tend to increase the amplitude of precession, and decide whether they are capable of balancing the dissipative energy losses. Only then can gravitational wave field strengths in realistic scenarios be estimated.

2 Modelling free precession

Fortunately, the problem of describing the free precession of a neutron star, is, in many regards, very similar to describing the free precession of the Earth. See table 1 for a summary of this comparison. Both neutron stars and the Earth consist of elastic shells containing a fluid core. In fact, the Earth also contains a very small solid core within the fluid core, so if neutron stars do indeed contain solid cores as some equation of state theorists have suggested, the analogy with the Earth can be extended even further. However, we will consider stars containing a fluid core only. Also, about 10^{-3} of the Earth's moment of inertia is made up by the oceans. Neutron stars will have oceans also, but even in rapidly accreting LMXB systems they make up a fraction of order 10^{-6} of the total moment of inertia, so we will not include them in our model. However, there is one phenomenon that needs to be considered for all but the hottest of neutron stars that does not occur in the Earth at all: Namely superfluidity. We will therefore describe the free precession of an elastic shell with a (pinned) superfluid, containing a liquid core. The strategy that has been employed to explore this free precession is to start with a rigid body, and introduce only one complicating factor at a time. We will follow this approach. However, before looking at this dynamic problem we will summarise how crustal strains can distort the star away from the shape it would have if it were an unstressed rotating fluid, as without such a deformation free precession would be impossible.

2.1 Deformations of neutron stars

The two main candidates for distorting neutron stars are strong internal magnetic fields and crustal strains. The former are discussed in these proceedings by Bonazzola, so we will look only at the latter.

We imagine the crust to have solidified from a hot liquid state in the geologically distant past, leaving it with a 'preferred' or zero-strain oblateness ϵ_0. This parameter will then change only via crust-cracking or a gradual plastic creep. It follows that if the actual oblateness ϵ differs from ϵ_0 the crust will store a strain energy of

$$E_{strain} = B(\epsilon - \epsilon_0)^2. \tag{1}$$

B is a constant to be determined from the equation of state, and will be of order of the total electrostatic binding energy of the ionic crustal lattice.

Following [1] we will proceed by writing down the total energy of the star. Let I_0 denote a moment of inertia characteristic of the spherical star, so that the moment of inertia about the rotation axis is $I_0(1+\epsilon)$. The star's energy is a function of ϵ according to:

$$E = E_0 + \frac{J^2}{2I_0(1+\epsilon)} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2. \tag{2}$$

Here E_0 labels the energy of the spherical star, the second term is the kinetic energy, the third the increase in gravitational potential energy due to the star's shape no longer being spherical, and the fourth the elastic strain energy. The constant A depends upon the stellar equation of state, but will be of the order of the gravitational energy of the star. The equilibrium configuration can be found by minimising the energy at fixed angular momentum:

$$\left.\frac{\partial E}{\partial \epsilon}\right|_J = 0. \tag{3}$$

For an entirely fluid star we would put $B = 0$, giving an oblateness of order of the ratio of kinetic and gravitational energies per unit mass:

$$\epsilon_{\text{fluid}} \approx \frac{\Omega^2 R^3}{GM} \approx 2 \times 10^{-3} \left(\frac{f}{100\,\text{Hz}}\right)^2. \tag{4}$$

We have taken the mass M as $1.4M_\odot$, the radius R as 10 km, and $\Omega = 2\pi f$ is the angular frequency.

When the strain term is included we find

$$\epsilon = \frac{I_0\Omega^2}{4(A+B)} + \frac{B}{A+B}\epsilon_0 \equiv \epsilon_\Omega + b\epsilon_0. \tag{5}$$

Here $b = B/(A+B)$ is the *rigidity parameter*, equal to zero for a fluid star ($B = 0$) and unity for a perfectly rigid one ($B/A \to \infty$). The oblateness is made up of two parts. The first, ϵ_Ω, scales as Ω^2 and is due to centrifugal forces. We will refer to this as the *centrifugal* deformation. The second term, $b\epsilon_0$, is due entirely to the stresses of the crystalline solid, and will be referred to as the *Coulomb* bulge. Realistic neutron star equations of state show that (in the absence of a solid core) $B \ll A$, with $b \approx 10^{-5}$. This is simply the ratio of the crustal electrostatic binding energy to the total stellar gravitational binding energy.

Thus we see that the affect of elastic stresses in the crust is to change the shape only slightly from that of the corresponding fluid body. Physically, the smallness of this distortion is due to the Coulomb forces being much smaller than the gravitational and centrifugal ones. This is the origin of the description of neutron stars as 'jelly-like' in table 1. For the Earth with its comparatively thick crust and low surface gravity, the rigidity parameter is much larger, having a value of 0.7, so that to a crude first approximation the Earth could be considered rigid.

As b is small we have $\epsilon \approx \epsilon_\Omega \approx \epsilon_{\rm fluid}$. Also ϵ and ϵ_0 can differ at most by the breaking strain $u_{\rm break}$ of the crust. As is discussed below $u_{\rm break} \ll 1$. Thus we have $\epsilon \approx \epsilon_\Omega \approx \epsilon_0$. Thus, in general, the crustal elasticity produces an oblateness—and therefore a gravitational quadrupole moment—of order $\epsilon_{\rm Coulomb}$ times the total stellar moment of inertia, where $\epsilon_{\rm Coulomb}$ is given by

$$\epsilon_{\rm Coulomb} \approx b\epsilon_0 \approx b\epsilon_{\rm fluid} \approx 2 \times 10^{-8} \left(\frac{b}{10^-5}\right) \left(\frac{f}{100\,{\rm Hz}}\right)^2 . \tag{6}$$

Property	Earth	Neutron star
Moment of inertia: Solid crust	90%	$< 5\%$
Moment of inertia: Liquid core	10%	$> 95\%$
Moment of inertia: Solid core	1%	?
Moment of inertia: Ocean	0.1%	$< 10^{-4}\%$
Rigidity parameter	0.7	10^{-5}; jelly
Magnetic field	Unimportant	Maybe
Free precession observed?	Yes, 14 month 'Chandler wobble'	Handful of candidates

Table 1: Comparison of properties of Earth and a 'typical' neutron star.

2.2 Free precession of a rigid body

The moment of inertia tensor of a rigid body can be written as

$$I_{ab} = I_0 + \Delta I_{\rm rigid}(n_a n_b - \delta_{ab}/3), \tag{7}$$

where the unit vector $\mathbf{n}$ points along the body's symmetry axis. Then the principal moments are $I_1 = I_2 = I_0 - \Delta I_{\rm rigid}/3$, $I_3 = I_0 + 2\Delta I_{\rm d}/3$, so that $I_3 - I_1 = \Delta I_{\rm d}$. It is then easy to form the angular momentum $\mathbf{J} = \mathbf{I\Omega}$ and derive all the usual relations that describe the motion (see e.g. [2]. In particular, the angular frequency $\dot\phi$ describes the rotation of the symmetry axis $\mathbf{n}$ about $\mathbf{J}$ in a cone of half-angle θ. We will refer to θ as the *wobble angle*. The angular velocity $\dot\psi$ describes an additional rotation about the symmetry axis $\mathbf{n}$. This is usually referred to as the *body frame precessional frequency.* The two frequencies are related by:

$$\dot{\psi} = -\frac{\Delta I_{\rm rigid}}{I_3}\dot{\phi}. \tag{8}$$

For a nearly spherical body equation $|\dot{\psi}| \ll \dot{\phi}$.

2.3 Free precession of an elastic shell

We will write the moment of inertia tensor of a rotating elastic body as the sum of a spherical and two quadrupolar parts (see e.g. [3, 4]):

$$\mathbf{I} = I_0\delta + \Delta I_d(\mathbf{n_d n_d} - \delta/3) + \Delta I_\Omega(\mathbf{n_\Omega n_\Omega} - \delta/3). \tag{9}$$

The first term on the right-hand side is the moment of inertia of the non-rotating undeformed spherical star. The second term is the change due to crustal Coulomb forces, and has the unit vector $\mathbf{n_d}$, fixed in the crust, as its symmetry axis. The third term is the change due to centrifugal forces, and has $\mathbf{n_\Omega}$, the unit vector along $\mathbf{\Omega}$, as its symmetry axis.

Proceeding exactly as in the rigid body case, it is easy to prove that the elastic body undergoes a free precessional motion like that of a rigid *axisymmetric* one with what we will call an *effective oblateness* $\Delta I_{\rm d}$, i.e. equation (8) applies, with the replacement $\Delta I_{\rm rigid} \to \Delta I_{\rm d}$.

2.4 Free precession of a shell containing non-spherical fluid

Now consider the free precession of a shell containing a spherical fluid cavity. There will be a reaction force between the shell and the fluid, due to the fluid tending toward a configuration symmetric about its rotational axis, and therefore 'pushing' on the shell. This pushing is known as *inertial coupling.* In the case of a rigid shell and homogeneous incompressible fluid, the problem of the combined motion of fluid and shell was solved by Poincarè, as described in [5], who found that the fluid tended to increase the body frame free precession frequency, so that equation (8) applies, with $\Delta I_{\rm d}$ being the strain-induced distortion of the whole star, not just the crust.

2.5 Free precession of a shell with pinned superfluid

According to many neutron star models which attempt to explain post-glitch behaviour, the neutron vortices which coexist with the inner crust become pinned to the crustal lattice. This means that the angular momentum of the pinned superfluid is fixed with respect to the crust. The free precession of such a body was solved by [6]. In the simple case, where all the pinned superfluid points along the crust's deformation axis, so that $\mathbf{J}_{\rm SF} = J_{\rm SF}\mathbf{n}$, this simply serves to increase the free precession frequency. An *effective oblateness parameter* can then be defined:

$$\epsilon_{\rm eff} = \frac{\Delta I_{\rm d}}{I_0} + \frac{I_{\rm SF}}{I_0}. \tag{10}$$

consisting of both crustal distortion and pinned superfluid parts. Here I_0 denotes the crust's moment of inertia, $\Delta I_{\rm d}$ is the strain-induced distortion of the whole star, and $I_{\rm SF}$ is the moment of inertia of the pinned crustal superfluid. Then

$$\dot{\psi} = -\epsilon_{\rm eff}\dot{\phi}. \tag{11}$$

2.6 Maximum wobble angle and gravitational wave amplitudes

During the course of one free precession period, the centrifugal bulge, of size ϵ_Ω, moves in a cone of half-angle θ about $\mathbf{n}$. This will produce a wobble-induced strain of order $\theta\epsilon_\Omega$. This must be less than the crustal breaking stress $u_{\rm break}$. This means:

$$\theta_{\rm max} \approx \frac{u_{\rm break}}{\epsilon_\Omega} \approx 0.45 \left(\frac{100\,{\rm Hz}}{f}\right)^2 \left(\frac{u_{\rm break}}{10^{-3}}\right). \tag{12}$$

This bound is most severe for rapidly rotating stars. Qualitatively, we can say that faster spinning neutron stars have larger bulges to re-orientate and therefore can sustain smaller wobble angles prior to fracture.

This upper bound on θ becomes an upper bound on the wave amplitude. For a wobble angle θ the gravitational wave amplitude is given by:

$$h = \frac{G}{c^4}\Omega^2 \Delta I_{\rm d} \theta \frac{1}{r}. \tag{13}$$

Setting $\theta = \theta_{\rm max}$ as given by equation (12) then gives an upper bound on the gravitational wave amplitude for a star at a given frequency with a given deformation ΔI. Some example plots of $h_{\rm max}$ against f are given in figure 1. Three curves are drawn, corresponding to crustal rigidity parameters of $b = 10^{-3}, 10^{-4}$ and 10^{-5}, with $\Delta I_{\rm d}$ set equal to $b\epsilon_{\rm fluid} I_{\rm star}$. We have also put $r = 1$ kpc and $u = 10^{-3}$. As can be seen at once, for the value 10^{-5} predicted by most equation of states, this *maximum* signal is detectable only by LIGO II, and only for spin frequencies in excess of 100 Hz. For detection by a LIGO I the rigidity parameter would have to be an order of magnitude larger.

3 Dissipation mechanisms

In the last section we described a model of free precession for a shell containing a liquid core. As the shell was supposed perfectly elastic and the fluid inviscid there

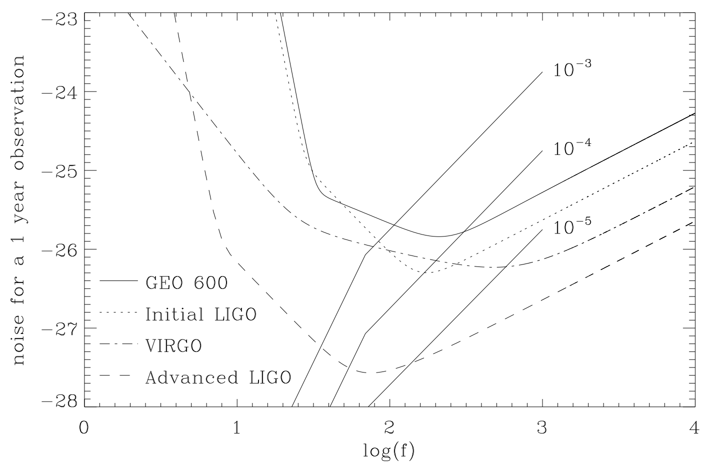

Figure 1: The maximum gravitational wave amplitude for Coulomb deformations. The star is at a distance of 1 kpc with $u_{\rm break} = 10^{-3}$ and rigidity parameter $= 10^{-3}, 10^{-4}, 10^{-5}$. The matched filter has been assumed to accumulate signal for an interval of one year. The noise curves have been taken from [7].

were no dissipative energy losses—such a star set into free precession will continue to precess with the same angular velocity and wobble angle forever. However, for any real star there most certainly will be a whole host of dissipative mechanisms acting. We briefly describe those that are most likely to be of importance for neutron stars.

3.1 Internal dissipation

By the term 'internal dissipation' we mean any process that converts mechanical energy of a freely precessing star into heat energy within it. Such mechanisms act in the Earth. Indeed, the Earth's own free precession is known to be damped on a timescale of the order of tens of decades, or equivalently on a timescale of order tens of free precession periods [3]. Both the cause of this dissipation, and the mechanism responsible for the corresponding excitation, are unknown. Nevertheless, previous workers have applied the most popular models of terrestrial dissipation to neutron stars. In particular, there will exist a viscous boundary layer at the crust-core interface, as the wobbling shell slips past the non-precessing fluid within. There will be dissipation within the crust also, due to its continually changing shape.

However, the dominant dissipation mechanism is probably due to the electromagnetic coupling between the crust and core. The interaction is due to the scattering of electrons (which are very nearly in rigid rotation with the crust) off those protons which circulate around the neutron vortices of the fluid core. Alpar and

Sauls [8] showed that, for a rigid star at least, such an interaction would damp free precession in 400 to 10^4 free precession periods. This result still applies in the more elaborate non-rigid model described in the last section (Jones [9]).

3.2 Gravitational radiation reaction

The gravitational waves emitted by a freely precessing star will carry energy and angular momentum away to infinity. These losses must be subtracted from the energy and angular momentum of the star. This radiation reaction problem was first addressed by [10], using what we will term a *local formulation*. Explicity, they made use of the Burke-Thorne radiation reaction potential:

$$\Phi_{\rm BT} = \frac{G}{c^5} x^a x^b \frac{d^5 \mathcal{I}_{ab}}{dt^5}. \tag{14}$$

The radiation reaction force at a given point in space is then simply $-\nabla\Phi_{\rm BT}$. In the rigid-body case the analysis is straightforward. The above force can be integrated to give a torque $\mathbf{T_{BT}}$ on the body. This then appears in the standard Newtonian equations of motion:

$$\frac{d}{dt}[\mathbf{I}_{\rm N}\mathbf{\Omega}] = \boldsymbol{T_{BT}}. \tag{15}$$

Bertotti & Anile repeated this calculation for an elastic star. However, as described in [11], their calculation contained a subtle flaw. In this elastic case the potential $\Phi_{\rm BT}$ doesn't just exert a torque on the body, it changes its shape too. This shape change must appear as a perturbation of the moment of inertia tensor in the Newtonian equations of motion:

$$\frac{d}{dt}[(\mathbf{I}_{\rm N} + \delta\mathbf{I}_{\rm BT})\mathbf{\Omega}] = \boldsymbol{T_{BT}}. \tag{16}$$

Crucially, the $\delta I_{\rm BT}$ term on the left-hand side cancels a large part of the torque that appears on the right [11]. It is then found gravitational radiation reaction damps wobble in about 10^{10} free precession periods for a canonical neutron star spinning at 1 kHz. This shows that gravitational radiation reaction is not important—it's always much weaker than mutual friction.

The above result has been verified using a 'flux-at-infinity' method too, i.e. by differentiation of $E = E(J, \theta)$, and inserting the values of the energy and angular momentum fluxes $\dot{E}$ and $\dot{J}$ as measured by integration over a surface at infinity [11].

4 Pumping mechanisms

We now consider freely precessing neutron stars in a variety of environments, where a torque is exerted on the star. If the torque were to have a component which tended to excite the wobble a 'tug-of-war' would follow between the pumping torque (tending to increase θ) and the internal dissipation mechanisms of the last section (tending to decrease θ). We wish to see if the corresponding gravitational wave amplitudes are detectable by the laser interferometers.

4.1 Accretion torques

Accretion torques are an obvious place to start when looking for mechanisms to pump precession. Not only are they capable of exciting wobble, but they can also maintain the spin frequency of the system, leading to the possibility of long-lived constant wave amplitude sources.

It can be shown that steady accretion does not lead to a secular change in wobble angle [9]. Here 'steady' means that the accretion torque is fixed in both direction and magnitude, as viewed from the inertial frame. Such a torque might act on an unmagnetised star. It follows that if we wish to find accretion torques capable of pumping precession we need to identify situations in which the torque itself would be non-constant. Fortunately, we would expect this to be the case when magnetospheric affects are taken into account. As pointed out by [12], the accretion rate and therefore also the torque depend upon the balance of gravitational, centrifugal and magnetic forces. The torque is therefore a function of the plasma angular velocity, stellar angular velocity and stellar magnetic moment vectors. However, if the star is itself precessing, the relative orientation of these vectors will be modulated. This in turn must lead to a modulation of the accretion torque locked in phase with the precession. It is precisely this sort of modulation that we would expect to lead to a secular evolution in the wobble angle. Lamb et al. demonstrated that such a modulation can be effective in exciting large amplitude precession of a rigid body, with an excitation timescale of the order of the spin-up timescale.

The evolution of the wobble angle can then be written as

$$\dot{\theta} = \frac{\theta}{\tau_{\rm a}} - \frac{\theta}{\tau_{\rm d}}. \tag{17}$$

The first term describes the accretion torque tending to increase θ on a timescale $\tau_{\rm a}$ connected with the accretion spin-up timescale. The second term describes the internal dissipation, tending to damp the wobble, on a timescale $\tau_{\rm d}$. The criteria for excitation is $\tau_{\rm a} < \tau_{\rm d}$. If this bound is satisfied we would expect the wobble angle to increase to a value close to $\theta_{\rm max}$ given by equation (12). Standard magnetospheric accretion theory can be used to estimate the minimum value of $\tau_{\rm a}$ for a star spinning at a given frequency at a given accretion rate. The inequality $\tau_{\rm a} < \tau_{\rm d}$ can then be used to give an upper bound on the steady gravitational wave amplitude for a star at a given spin frequency, accreting at a given rate, with a given crustal breaking strain [9]. Unfortunately, for all sensible parameters the signal is too weak, by several orders of magnitude, to be detectable by even a LIGO II.

4.2 Electromagnetic torques

Goldreich [13] examined the affect of electromagnetic torques on precessing bodies. He showed that for a particular model of the magnetic field surrounding a neutron star, when the free precession period was less than the electromagnetic spin-down timescale, the electromagnetic torque serves to either pump or damp any free precession the body may be undergoing, depending upon the value of the angle between the dipole moment and the body's deformation axis.

For an isolated spinning down star this mechanism is of limited use—while the electromagnetic torque acts to increase θ it also decreases the frequency f, upon which the gravitational wave amplitude depends quadratically (for a fixed deformation $\Delta I_{\rm d}$). In view of this we will make the (difficult to justify) assumption that the wobble-pumping component of the torque remains active even in an accretion environment, where its spin-down part is suppressed. Then an equation describing the evolution of the wobble angle can be written down which is of the same form as equation (17), with the replacement $\tau_{\rm a} \rightarrow \tau_{\rm e}$, where $\tau_{\rm e}$ is a timescale parameterising the strength of the electromagnetic torque. Then the inequality $\tau_{\rm e} < \tau_{\rm d}$ can be transformed into an upper bound on the steady state gravitational wave amplitude [9]. Unfortunately, we again find that for all sensible parameters the signal is too weak, by several orders of magnitude, to be detectable by even a LIGO II.

4.3 Collisions in dense environments

The possibility of detecting gravitational waves from neutron stars set into free precession following collisions with other stars has recently been discussed in the literature. In particular, de Araujo et al. [14] have suggested that the high stellar densities of globular clusters could lead to a high rate of collision between neutron stars and other stars. Given that globular clusters contain an excess of millisecond pulsars, the latter authors argued that if such collisions were effective in exciting precession, interesting levels of gravitational wave emission would occur.

We begin by considering the collision of a neutron star with a non-compact star. Such collisions have been modelled extensively, e.g. Davies, Benz and Hills [15] who model neutron star-main sequence star collisions, and also neutron star-red giant collisions. They find that for sufficiently small periastrons neutron star-accretion disk and common envelope systems are formed. Despite the violent effect such near-body encounters have on the non-compact star it is difficult to see how the neutron star would be set into free precession by such a collision. The gravitational tidal torque on the neutron star due to the non-compact star is negligible (see next section). This leaves only the material torque on the neutron star, which will be determined by accretion flow onto its surface. This will be described by the standard theory, regardless of the unusual source of the accreting material. The accretion rate will be limited to the Eddington value in the usual manner, so the torque will not be impulsive. A study of the event rate also suggests that such collisions occur too infrequently to give a reasonable probability of detection [9].

Now consider encounters between two neutron stars. Clearly, if a direct collision were to occur, free precession would be the last gravitational wave mechanism that we would wish to consider. We will therefore model a *near* collision, where both gravitational and magnetic affects will come into play, but there is no direct mechanical contact between the stars. We will model the gravitational interaction in a simple Newtonian way: each star exerts a torque on the centrifugal bulge of the other, proportional to the inverse cube of their separation. As the two stars approach one another this torque will grow. If the stars pass very close to one another the steep r^{-3} factor will give rise to an almost impulsive torque, acting when the stars are at and close to periastron. It is easy to show that a wobble angle of order 10^{-3} is produced for a periastron of 100 km. Such a precessing star would be detectable.

Having established that near collisions could excite free precession we must now consider an event rate for such close passages. In fact it is straightforward to show that no such near-collisions will occur over a Hubble time, using a simple model. Suppose there are N neutron stars in a globular cluster of size R_{gc}. Let v_∞ denote their average velocity when far apart. Then in a unit time this population sweeps out an effective volume of order NAv_∞, where A is a collision cross-section. Then the probability of a *given* neutron star colliding with another in this interval is of the order of this volume divided by the globular cluster volume, i.e. of order NAv_∞/R_{gc}^3. As there are N such stars the probability of *any one of them* colliding with any other is then N times this giving a collision rate N^2Av_∞/R_{gc}^3. As there are approximately 200 globular clusters in the Galaxy we obtain a Galactic collision rate $\nu_{\text{collision}}$:

$$\nu_{\text{collision}} \approx 200\frac{N^2Av_\infty}{R_{gc}^3}. \tag{18}$$

If gravitational attractions were neglected, the collision cross-section of interest would be of order $d^2 \sim (100\,\text{km})^2$. However, gravitational focusing will increase the effective cross section as described in Verbunt & Hut [16]

$$A \approx d^2\left[1 + \frac{2GM_{\text{total}}}{v_\infty^2 d}\right] \tag{19}$$

where M_{total} denotes the sum of the masses of the two stars. The second term on the right-hand side describes the affects of gravitational focusing. For the case of interest it is the dominant factor. Parameterising we find an event rate

$$\nu_{\text{collision}} \sim 10^{-11}\,\text{yr}^{-1}\left(\frac{N}{10^3}\right)^2\left(\frac{M_{\text{total}}}{2.8\,M_\odot}\right)\left(\frac{d}{100\,\text{km}}\right)\left(\frac{10\,\text{km}}{v_\infty}\right)\left(\frac{1\,\text{pc}}{R_{gc}}\right)^3. \tag{20}$$

Such an event rate as this makes further comment unnecessary.

4.4 Natal precession

Given their violent birth in supernovae it is tempting to examine the possibility that neutron stars are set into precession when born. Of course, precession cannot occur until the crust has solidified, so we are effectively assuming that a significant perturbation remains from birth long enough for the newly solidified crust to be set into free precession. In other words, we wish to investigate the gravitational wave background due to a population of young spinning-down neutron stars that were set into precession soon after birth. We will describe this as *natal precession*. We will not include any pumping mechanisms in our analysis, so that the wobble angle simply decays under the influence of internal dissipation.

The problem of the gravitational signals due to a population of young non-precessing *triaxial* neutron stars spinning down due to gravitational wave emission has been considered by Blandford as reported in [17]. We wish to make the analogous argument for a population of young isolated *precessing* neutron stars. We will follow in outline the Blandford argument.

We will model the Galaxy as having a radius R and thickness D, giving a volume of order R^2D. This contains $\tau/\Delta t_{\rm sn}$ stars of age τ or younger, where $\Delta t_{\rm sn}$ denotes the Galactic supernova rate. The average separation of this population of young stars is then $(\Delta t_{\rm sn} R^2 D/\tau)^{1/3}$. Explicitly

$$\Delta r = 1.4\,\mathrm{kpc}\left(\frac{\Delta t_{\rm sn}}{30\,\mathrm{years}}\right)^{1/3}\left(\frac{10^3\,\mathrm{years}}{\tau}\right)^{1/3} \tag{21}$$

where we have put $R = 10$ kpc and $D = 1$ kpc. Of course, a more accurate model would take into account the rate of star formation as a function of Galactic position, with different rates applying in the central bulge and spiral arms, for instance. Nevertheless, equation (21) represents a useful first approximation.

We can then take equation (13) for the gravitational wave amplitude and set the source distance r equal to Δr as given in equation (21). The wave amplitude thus obtained will, subject to statistical variation, be the field at Earth due to the closest source of age $\tau_{\rm d}$ or less. If we assume that internal dissipation eliminates the natal wobble in 10^4 free precession periods and neglect pinning, the gravitational wave amplitude at Earth is found to be too small to be detected by several orders of magnitude. If pinning is included the wave amplitude is even smaller [9].

5 Conclusions

Gravitational radiation from freely precessing neutron stars is almost certainly limited to a level undetectable by a LIGO II detector by internal dissipation. Quite simply, it was not possible to find any plausible astrophysical scenarios where the wobble was pumped as strongly as dissipative processes within the star damped the motion.

Nevertheless, a number of open questions (escape routes?) might apply. For instance, what if neutron stars contain large solid cores? Might there exist wobble pumping processes that, rather than exerting a torque on the star, tend to deform it plastically on some short timescale, competitive with the dissipation timescale? Such issues as these are currently under investigation and will be presented elsewhere.

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