Statistical Nuclear Reactions

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Abstract

A review of the statistical model of nuclear reactions is presented. The main relations are described, together with the ingredients necessary to perform practical calculations. In addition, a substantial overview of the width fluctuation correction factor is given.

1 Introduction

Nuclear reactions are described by several models which are linked together to calculate nuclear cross sections. In this process, the basic model is the optical model, which enables us to separate the total cross section into various components. One mainly distinguishes between two kinds of processes according to their reaction times. On the one hand, one has the fast processes, with reaction times comparable to the time needed by the projectile to cross the nucleus without interacting with it, and on the other hand, rather slow processes which may take much longer to occur. The statistical (or compound nucleus) model of nuclear reactions belongs to the second kind of processes as well as the pre-equilibrium models even if they cover an intermediate region in time scale (between fast and slow processes). However, we will not deal with pre-equilibrium models in what follows. In the statistical model, it is assumed that a compound nucleus is formed with an excitation energy high enough so that many states may be excited by the interaction of the incident particle with the target nucleus. Moreover, the incident energy is shared between the individual components of the nucleus which fully equilibrates before decay takes place. This picture implies an independance hypothesis, meaning that the compound nucleus decays without "remembering" the way it has been formed. In other words, in the statistical model picture, there exists no correlation between the formation and decay of the compound nucleus. Apart from the fundamental equations which can be derived within the Statistical model approach, we will describe all the ingredients that may be employed to calculate interaction cross sections. A particular emphasis will be put on the width fluctuation correction factor which is particularly important for achieving a proper description of the compound nucleus decay for low incident particle energies.

2 Main equations of the Statistical model

2.1 Reaction channel definition

The statistical model of nuclear reaction is devoted to the treatment of binary reactions of the type

$$a + A \longrightarrow CN^* \longrightarrow b + B \ ,$$

where a is the projectile, A the target, CN^* the intermediate excited compound nucleus, b the ejectile and B the residual nucleus. The Schrödinger equation for such a complicated interaction is impossible to solve by taking into account all the interacting particles. One rather consider that the interaction between a and A is described by an effective potential (the optical potential) which allows us solve numerically the problem by performing a partial wave expansion in the center of mass system of coordinates. With this mathematical development, incident channels are defined by specifying the projectile orbital angular momentum ℓ_a and the

way it couples with the projectile intrinsic spin s_a and the target spin I_A. Such a coupling can be achieved by two different ways. Historically, the first nuclear reactions were performed with low energy incident projectiles, so that only s-waves (i.e. partial waves with $\ell = 0$) were contributing significantly, implying that the natural approach was then to couple $\vec{s}_a$ with $\vec{I}_A$. However, it is currently more appropriate (we will show why in Section 2.2) to couple first $\vec{s}_a$ and $\vec{\ell}_a$ to form the projectile total angular momentum $\vec{j}_a$, and then couple $\vec{j}_a$ and $\vec{I}_A$ to get the compound nucleus spin J_C. The other channel characteristics are the projectile energy E_a and parity π_a as well as the target excitation energy E_A and parity π_A.

Because the conservation laws,

$$\begin{aligned}
E_a + E_A = E_C = E_b + E_B &\quad \text{Total energy conservation,} \\
\vec{p}_a + \vec{p}_A = \vec{p}_B + \vec{p}_b &\quad \text{Total momentum conservation,} \\
\vec{\ell}_a + \vec{s}_a + \vec{I}_A = \vec{J}_C = \vec{\ell}_b + \vec{s}_b + \vec{I}_B &\quad \text{Total angular momentum conservation,} \\
(-1)^{\ell_a}\pi_a\pi_A = \pi_C = (-1)^{\ell_b}\pi_b\pi_B &\quad \text{Total parity conservation,} \qquad (1)
\end{aligned}$$

where E_C and π_C are the excitation energy and parity of the compound nucleus, are identical for the (projectile+target) and for the (ejectile+residual nucleus) systems, outgoing channels can be defined in the same way incident channels have already been defined.

2.2 Hauser Feshbach expression

In the simplest case of spinless particles [1], the diffusion theory provides us with the reaction cross section for a process leading from an initial channel α to a final channel β as

$$\sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2} \left\langle |\delta_{\alpha\beta} - S_{\alpha\beta}|^2 \right\rangle \ ,$$

where $S_{\alpha\beta}$ is a scattering matrix element, and k_α the wave number of relative motion. The optical model potential yields values for $S_{\alpha\beta}$ as well as transmission coefficients defined by

$$T_\alpha = 1 - |S_{\alpha\alpha}|^2 \ . \qquad (2)$$

Therefore, the compound nucleus formation cross section simply reads

$$\sigma_\alpha^{CN} = \frac{\pi}{k_\alpha^2} T_\alpha \ .$$

The independence hypothesis yields the reaction cross sections,

$$\sigma_{\alpha\beta} = \sigma_\alpha^{CN} P_\beta \ ,$$

where P_β is the probability that the compound nucleus decays into the channel β. This probability can be calculated from the reciprocity theorem and one obtains

$$P_\beta = \frac{T_\beta}{\sum_c T_c} \ , \tag{3}$$

so that the final cross section $\sigma_{\alpha\beta}$ reads

$$\sigma_{\alpha\beta} = \frac{\pi}{k_\alpha^2} \frac{T_\alpha T_\beta}{\sum_c T_c} \ ,$$

which is the Hauser-Feshbach formula for spinless particles.

However, in most experimental situations, many incident and outgoing channels have to be considered together, which considerably complicates the previous expression since summations have to be performed over the various channels contributing to a given process. If we consider a reaction with an incident projectile a leading to an ejectile b, the symbol a and b implicitly containing all the characteristics of the reaction (excitation energies, spins and parities of the interacting particles), one can write [2]

$$\sigma_{ab} = \frac{\pi}{k_a^2} \sum_{J_C=|I_A-s_a|}^{\ell_a^{max}+I_A+s_a} \sum_{\Pi_C=+,-} \frac{2J_C+1}{(2I_A+1)(2s_a+1)} \sum_{j_a=|J_C-I_A|}^{J_C+I_A} \sum_{\ell_a=|j_a-s_a|}^{j_\alpha+s_a} \sum_{j_b=|J_C-I_B|}^{J_C+I_B} \sum_{\ell_b=|j_b-s_b|}^{j_b+s_b} \delta(a,\Pi_C)\delta(b,\Pi_C) \frac{T^J_{\alpha\ell_a j_a} T^{J_C}_{\beta\ell_b j_b}}{\sum_\gamma T_\gamma} \ .$$

In this equation, the sum over J_C and Π_C accounts for the fact that the final cross section results from the contribution of all the partial cross sections tied with compound levels $J_C^{\pi_C}$. Theoretically, all the possible values of J_C should be considered, but in pratice the summation is restricted up to $I_A + s_a + \ell_a^{max}$ because it is known that there is a maximum value for partial waves beyond which the contribution is negligible. Concerning the δ functions, these force compliance with the parity conservation rules given in Eq.(1). Finally, the symbol α implicitly contains (as explained in Section 2.1) all the characteristics defining the channels of the initial system (projectile+target nucleus) and β the channels of the final system (ejectile+residual nucleus). The previous equation enables us to understand that we have adopted the coupling scheme described in Section 2.1 because the transmission coefficient are determined as function of ℓ and j, since in optical model calculations, the $\vec{l}\,\vec{s}$ coupling is generally adopted.

Within the same model, angular distributions can also be calculated in the center of mass as functions of the scattering angle Θ, and read

$$\sigma_{ab}(\Theta) = \sum_{\text{even } L} C_L P_L(cos\Theta) \ ,$$

where the C_L's coefficients are given by

$$\begin{aligned} C_L \ = \ & \frac{\pi}{k_a^2} \sum_{J_C,\pi_C} \frac{2J_C+1}{(2I_A+1)(2s_a+1)} \sum_{j_a,\ell_a,j_b,\ell_b} \\ & \delta(a,\Pi_C)\delta(b,\Pi_C) \frac{T^J_{\alpha\ell_a j_a} T^{J_C}_{\beta\ell_b j_b}}{\sum_\gamma T_\gamma} A^{J_C}_{I_A,\ell_a,j_a,I_B,\ell_b,j_b,L} \ , \end{aligned}$$

with

$$\begin{aligned} A^{J_C}_{I_A,\ell_a,j_a,I_B,\ell_b,j_b,L} = \frac{(-1)^{I_B-s_b-I_A+s_a}}{4\pi}(2J+1) \quad & Z(I_A,j_a,\ell_a,J_C,L) \\ \times \ & Z(I_B,j_b,\ell_b,J_C,L), \end{aligned}$$

and

$$Z(I,j,\ell,J,L) = (2j+1)(2l+1)C(ll00:L0)W(jjll:Ls)W(JjJj:L0),$$

the symbols C and W being respectively the Clebsch-Gordan and Racah coefficients. However, it is important to make the point that the formal expression for angular distributions depends also on the adopted coupling scheme. For the other coupling scheme (i.e. first couple $\vec{s}$ and $\vec{I}$ and then $\vec{l}$) equations are given by Hodgson [3].

2.3 Width Fluctuation concept

The Hauser-Feshbach expression described above is too simple because the independence hypothesis is a too crude an approximation, particularly for low projectile energies. The width fluctuation correction factor is introduced to compensate for this approximation. The main underlying idea is that a cross section $\sigma_{\alpha\beta}(J^\Pi)$ (component of σ_{ab} obtained with an entrance channel α and an outgoing channel β through a compound nucleus with definite spin J and parity π) is obtained by averaging over the many compound nucleus states with spin J and parity π contained in an energy interval whose width is the projectile beam energy spreading. This averaging process can be carried out using the standard Breit-Wigner expression for an isolated resonance [4], and yields

$$\sigma_{\alpha\beta}(J^\pi) = \frac{\pi}{k_\alpha^2} \frac{2\pi}{D(J^\pi)} \left\langle \frac{\Gamma_\alpha^{J^\pi} \Gamma_\beta^{J^\pi}}{\Gamma^{J^\pi}} \right\rangle \ , \tag{4}$$

where the brackets indicate the energy average, $D(J^\pi)$ is the mean spacing of the compound nucleus levels J^π, Γ_α and Γ_β the widths associated with the channels α

and β and Γ the total width (sum of all the channel widths). Since it can be shown that the transmission coefficients are related to the widths by

$$T_\alpha^{J^\pi} = \frac{2\pi \left\langle \Gamma_\alpha^{J^\pi} \right\rangle}{D(J^\pi)} \ , \tag{5}$$

Eq.(4) can trivially be written as

$$\sigma_{\alpha\beta}(J^\pi) = \frac{\pi}{k_\alpha^2} \frac{T_\alpha^{J^\pi} T_\beta^{J^\pi}}{\sum_\gamma T_\gamma^{J^\pi}} W_{\alpha\beta}^{J^\pi} \ ,$$

provided that $W_{\alpha\beta}$, the width fluctuation correction factor (**WFCF**), is defined by

$$W_{ab} = \left\langle \frac{\Gamma_a \Gamma_b}{\Gamma} \right\rangle \frac{\langle \Gamma \rangle}{\langle \Gamma_a \rangle \langle \Gamma_b \rangle} \ , \tag{6}$$

We will see in Section 4 how the WFCF's can be determined using the ingredients of the statistical model.

3 The Statistical model ingredients

Two main kinds of ingredients appear in the statistical model equations, namely the transmission coefficients and the level densities. The level density problem will be developped in an other lecture, so we will only deal here with the transmission coefficients. One may distinguish between several types of transmission coefficients, depending on the type of reaction under consideration. We can separate these in three classes and, accordingly, separate the transmission coefficients in three kinds: the particle transmission coefficients (for outgoing or incoming light particles), the capture transmission coefficients (for outgoing gamma rays) and the fission transmission coefficients (for fissile nuclei).

3.1 Particle transmission coefficients

As we have already mentionned, particle transmission coefficients are given by the optical model using Eq.(2). However, such a relation is only valid for a single channel, and since the compound nucleus may decay to residual nuclei with high excitation energy, it is impossible, in such situations to deal explicitly with every possible decay channels. The, one rather introduces the notion of effective transmission coefficient $< T >$ by summing over all the concerned decay channels that can be found in an arbitrary energy bin. One has then

$$\left\langle T_{b,\ell_b,j_b}^{J^\pi} \right\rangle = \int_{bin} \rho(E, I_B, \pi_B) \delta(b, \Pi_C) T_{b,\ell_b,j_b}^{J^\pi} dE \ , \tag{7}$$

where $\rho(E, I_B, \pi_B)$ is the density of residual nucleus levels J^π with excitation energy E. This effective transmission coefficient replaces all the channels and implicitly contains an approximation which stipulates that all decay channels contribute equally to the decay process, which is of course not too crude an approximation only if the bins are not too wide. Provided this condition is fullfilled, it is then trivial to associate an effective number of channel $< R >$ defined by Eq.(7) in which $T^{J^\pi}_{b,\ell_b,j_b}$ is set to 1. We will see later on how this effective number of channels is used to calculate the WFCF's.

3.2 Gamma transmission coefficients

These transmission coefficents are calculated in pratice from what is called the strength function $f(x, \lambda)$, and read

$$T_{x,\lambda}(\varepsilon_\gamma) = 2\pi f(x, \lambda)\varepsilon_\gamma^{2\lambda+1}, \tag{8}$$

where ε_γ is the emited gamma energy, x gives the type of transition ($x = E$ for electric and M for magnetic transitions), λ is the transition multipolarity, and J^π is the spin of the compound nucleus level that decays. The strength function may have different forms according to various model [5, 6], which we will not describe here. These contain free parameters which must be ajusted as we will see later. Here again, the number of final states is so important that an individual description of each gamma channel is not possible or meaningfull. One therefore performs a summation over all accessible levels, and define a global gamma transmission coefficient $T_\gamma^{J^\pi}(E)$, expressed as

$$T_\gamma^{J^\pi}(E) = \sum_{x,\lambda} \sum_{J_f=|J-\lambda|}^{J+\lambda} \sum_{\pi_f} \int_0^E T_{x,\lambda}(\varepsilon_\gamma)\rho(E - \varepsilon_\gamma, J_f, \pi_f) f(x, \lambda, \pi, \pi_f) \ , \tag{9}$$

where $\rho(E - \varepsilon_\gamma, J_f, \pi_f)$ is the density of final levels $J_f^{\pi_f}$ (of the compound nucleus) to which the initial level J^π decays [1], and $f(x, \lambda, \pi, \pi_f)$ is 0 or 1 to force compliance with the parity selection rules that govern gamma decay. This means that $f = 0$ unless

$$\pi_f = (-1)^\lambda \pi \text{ for electric transitions, or}$$

$$\pi_f = (-1)^{\lambda+1} \pi \text{ for magnetic transitions.}$$

Even if systematics exist for the parameters which specify the strength functions, it is in pratice often necessary to renormalise the gamma transmission coefficients by requiring that the theoretical average gamma width is equal to the experimental value $< \Gamma_\gamma >_{exp}$ (when available) for excitation energies close to B_n. From

[1] It is worth mentioning here that we have chosen to integrate from 0 up to E for simplicity. However, for low excitation energies, the integral should be replaced by a summation over the accessible discrete levels with correct spins and parities.

this condition, the renormalisation factor can be determined as the ratio between $< \Gamma_\gamma >_{exp}$ and the sum

$$\frac{D_0}{2\pi} \sum_{J,\pi} T_\gamma^{J^\pi}(B_n) \ ,$$

where D_0 is the theoretical mean spacing of s-wave resonances at B_n in the compound nucleus (the mean spacing of the compound nucleus levels that can be formed with thermal incident neutrons) and $T_\gamma^{J^\pi}(B_n)$ is given by Eq.(9).

3.3 Fission transmission coefficients

Compared with the two previously described processes, the fission process, is very different because one is not directly concerned with the residual nuclei produced after the compound nucleus undergoes fission, but rather interested in the total fission cross section. Therefore, for a given compound nucleus level J^π, there is only one fission transmission coefficient.

3.3.1 Single humped barrier

This transmission coefficient is usually calculated using the well known Hill-Wheeler model [7] coupled to transition states as introduced by Bohr [8]. The Hill-Wheeler expression describes the quantum penetrability through a fission barrier described by an inverted parabola, which reads

$$T_{HW}(E) = \left[1 + \exp\left(-2\pi \frac{E - B_f}{\hbar\omega}\right)\right]^{-1}$$

where B_f is the barrier height (relative to the nucleus ground state) and $\hbar\omega$ is the barrier curvature. The transition states model, illustrated in Fig. 1, stipulates that the fission process takes place throught intermediate states of the compound nucleus. As shown on Fig. 1, the excitation energy of the nucleus ground state represented as a function of the deformation exhibits a hump, which is the ground state barrier. On the top of this barrier, several compound nucleus levels may be present and to each of these so-called transition states is associated a barrier. In first-order approximation, the transition states barrier is simply that of the ground state translated by the transition states energy relative to the top of the ground state barrier. Starting from a compound nucleus level J^π, all the barriers associated with transition states with spin and parity J^π will contribute to the fission process. If each barrier is described by an inverted parabolic curve, the fission transmission coefficient reads

$$T_{fis}^{J^\pi}(E) = \int_0^{+\infty} \rho(\varepsilon, J, \pi) T_{HW}(E - \varepsilon) dE \qquad (10)$$

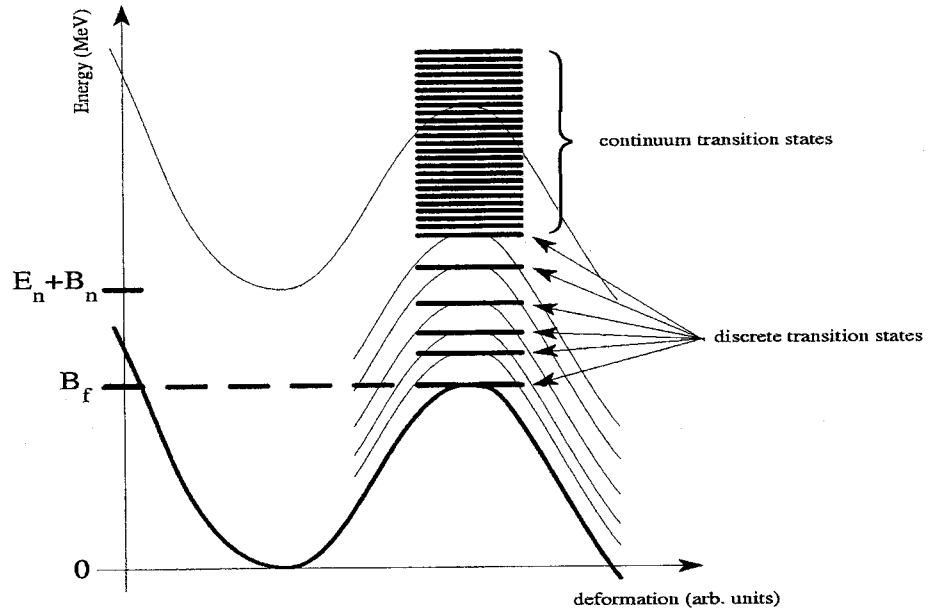

Figure 1: Potential energy surface as function of deformation for a fissile nucleus. Fission barriers for transition states are simply obtained by shifting up the ground state barrier with the transition state energy relative to the top of the ground state barrier. The highest potential energy surface shown correspond to a transition state with energy $E_n + B_n$ (E_n is the incident neutron energy and B_n the neutron binding energy in the compound nucleus) and since a simple translation has been applied, the first well is located at $E_n + B_n$. According to this picture, transition states with higher excitation energies should not contribute to fission.

where $\rho(\varepsilon, J, \pi)$ is the density of transition states located on top of the fission barrier with excitation energy ε relative to the top of the barrier [2]. In Eq.(10), the integration is extended over all possible transition states (infinite upper limit) because the inverted parabolic shape that describes the barrier allows for the tunneling through barriers associated with transition states with excitation energies much higher than the compound nucleus excitation energy. We will see later that an upper limit should be given for proper width fluctuation calculation.

[2]Here again, the integration should be replaced by a sum over discrete transition states for low ε values.

3.3.2 Double humped barrier

All the expressions given above are valid only when one barrier is present. In many cases (for instance in the actinide mass region), a second barrier must be crossed before fission occurs. Even if such a situation is very much more complicated to treat than the single humped barrier problem, a simple approximation is generally good enough to solve the double humped barrier problem.

One can first calculate for both the first (barrier A) and the second (barrier B) barrier, fission transmission coefficients $T_A^{J^\pi}(E)$ and $T_B^{J^\pi}(E)$. Then, we assume that the tunneling through the two barriers can be separated (as the formation and decay of the compound nucleus) into two steps. One first calculate the probability to cross the first barrier (T_A), and then multiply it by the probability to fission. Once the first barrier has been crossed, two solutions are possible : crossing back barrier A with probability T_A or fissioning through barrier B with probability T_B). Therefore, by analogy with the decay probability, Eq.(3), yielding the spinless Hauser-Feshbach expression, it is trivial to deduce that the fission probability once the barrier A is crossed, is

$$P_{fission} = \frac{T_B}{T_A + T_B}.$$

Consequently, the global fission transmission coefficient for double humped barrier reads

$$T_{fis}^{J^\pi}(E) = \frac{T_A^{J^\pi}(E)\, T_B^{J^\pi}(E)}{T_A^{J^\pi}(E) + T_B^{J^\pi}(E)}. \tag{11}$$

3.3.3 Class II states effect

In the previous section, we have implicitly decoupled the two barriers. However, it has been experimentally observed that for compound nucleus with excitation energy below the top of the barrier, the fission probability may display sharp peaks, which have been interpreted as resonances induced by compound nucleus states located in the second well of the potential energy surface (see Fig. 2). Complicated approaches have been developed to study the theoretical effect of these so-called class II states on the global fission transmission coefficient [9]. It has been shown that, the mean value of the fission coefficient was given by Eq.(11) and that the maximum enhancement due to these states is of the order of $4/(T_A + T_B)$ provided that class II states have of course correct spins and parities and that excitation energies are confined in a bin centered on the class II states energies with a width given by $D(T_A + T_B)/2\pi$, by analogy with Eq.(5). These particular states may have a significant effect, both for nuclei that spontaneously undergo fission, and for second, third and other order chance fission.

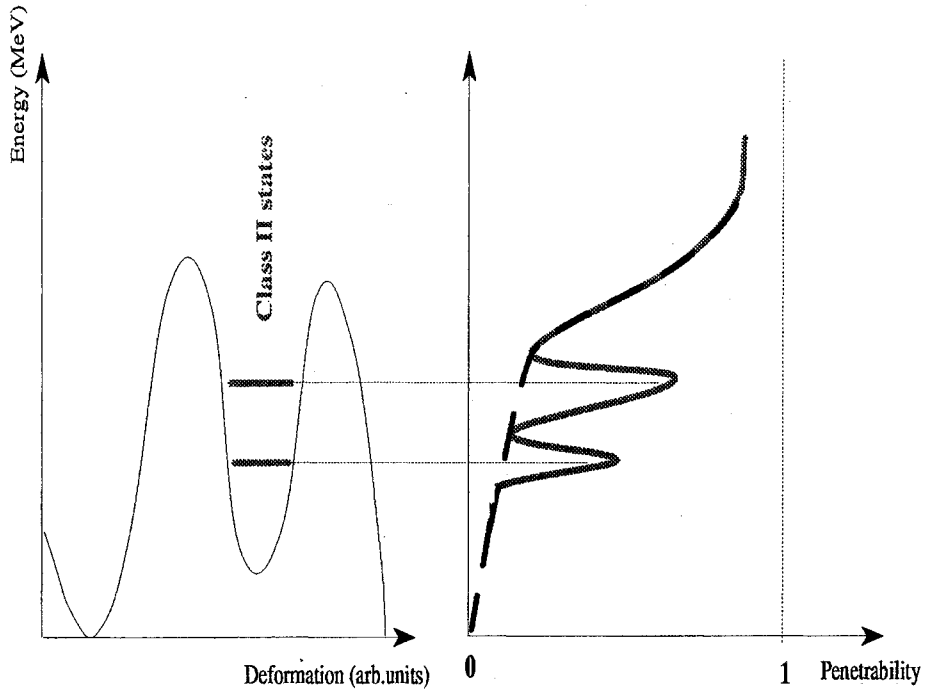

Figure 2: Illustration of the effect of class II states on the fission transmission coefficient through a double humped barrier. The dotted line shows the transmission coefficient without class II states, and the full line (with peaks) shows how class II states enhance locally the transmission.

4 Width fluctuation correction factor expressions

All the previously described ingredients govern the main behaviour of the compound nucleus, but as it has been already mentionned, a width fluctuation correction factor must be introduced to improve the description of the cross sections for low projectile energies. This WFCF has been widely studied with more or less fundamental approaches and from a qualitative point of view, the WFCF implies an enhancement of the elastic cross section, and, generally, a weakening of other cross sections.

4.1 The various methods

We here mainly distinguish three approaches, dedicated to the treatment of the WFCF, by increasing order of complexity : the HRTW method, the Moldauer integral and the most rigourous approach, refered to as the **G**aussian **O**rthogonal **E**nsemble (**GOE**) method. We will not insist on the mathematical expressions obtained within each approach, which can be found in several references [10, 11, 12,

13], but we will briefly describe their physical contents.

In the HRTW method, it is assumed that the cross section can be factorised and calculated by a simple iteration procedure for each channel considered in the calculation. A numerical parametrisation, based on random S matrix analysis, is used to determine the WFCF expression, yielding an analytical expression function of all the transmission coefficients as well as of their second moments. After several iterations, the procedure converges and it is possible to obtain, for every channel, a modified cross section compared to that obtained with the simple Hauser-Feshbach formula.

The second approach, initiated by Dresner [14] and afterwards generalised by Moldauer, is based on a more basic treatment since it concerns the channel widths rather than the final cross section. More precisely, a χ^2 distribution with ν degrees of freedom is assumed for the reduced widths that define formally the WFCF, as given in Eq.(6). Such an assumption enables us to express the WFCF as a simple integral, which depends again on all the channel transmission coefficients. Although $\nu = 1$ for "Dresner", a study performed by Moldauer (again based on random S matrix calculations) provides us with an analytical expression for the number of degrees of freedom which is more appropriate.

Finally, the most fundamental approach is the GOE method, developped by the Heidelberg group [13], where statistical assumptions are made on the Hamiltonian matrix elements, since it is assumed that they form a Gaussian Orthogonal Ensemble. This is the only approximation underlying the whole theory, and after rather complicated algebraic transformations, a triple integral is obtained for the WFCF expression. Even if this integral is very complicated to compute, it is very interesting since it is supposed to give the best results. Therefore, it can be used as a reference to test the reliability of the two other methods which are both easier to implement in a pratical nuclear reaction code.

Both for the Moldauer and for the GOE approaches, the WFCF calculation involves a product over all the channels, which reads

$$\prod_c \left(1 + \frac{2T_c}{\nu_c \sum_i T_i} x\right)^{-0.5\nu_c} , \tag{12}$$

for Moldauer simple integral (x is the variable of integration and ν_c the number of degrees of freedom for each channel c), and

$$\prod_c \frac{1 - T_c\lambda}{\sqrt{(1 + T_c\lambda_1)(1 + T_c\lambda_2)}} , \tag{13}$$

for the GOE triple integral (λ, λ_1 and λ_2 are the three integration variables). As we will see in the next section, these products have to be carefully calculated.

4.2 Approximation for particular reactions

Whatever the method adopted to calculate the WFCF is, approximations have to be made to treat properly several specific channels. The main reason is that the higher the excitation energy, the more the number of channels one has to deal with is important. As for transmission coefficients calculations, three main singular cases have to be considered, namely for capture reactions, for reactions in the continuum and for fission reactions [3]. In each case, the problems comes from the huge number of individual channels which makes a channel per channel treament impossible to perform.

For capture reactions, the method consists in treating all possible gamma channels as a global channel with a transmission coefficient T_γ^{eff} defined as in Eq.(9). However, the calculation of the products (12) and (13) is performed assuming an infinite number of γ-channels with a negligible transmission coefficient, so that Eqs.(12) and (13) are approximated respectively by

$$\exp\left(-\frac{T_\gamma^{eff}}{\sum_i T_i}x\right) \text{ and } \exp\left[-(2\lambda+\lambda_1+\lambda_2)\frac{T_\gamma^{eff}}{2}\right].$$

For continuum reactions, the approximation explained in section 3.1 is used, and the WFCF is calculated as if each continuum bin were equivalent to an average number of channel R_{ave} with identical transmission coefficients T_{ave}. The products over the channels in a given bin is then replaced by the contribution of one channel with transmission coefficient T_{ave} raised to the power R_{ave}. For HRTW, the iterations are performed for one channel and after convergence has been reached, the result is multiplied by R_{ave}.

For fission reactions, the main problem is to define an average number of fission channels in order to use the same approach as for continuum channels. As already mentionned in section 3.3.1, the infinite upper limit in integral (10) does not allow us to define that effective number. For this purpose, one can set the upper limit to the compound nucleus excitation energy. Indeed, as illustrated in Fig 1, above this excitation energy, one can consider that no more barrier can be crossed since all transition states barriers are simply deduced by a shift. With this actual upper limit, it is possible to define, as for continuum transmission coefficients, an effective number of fission channels, and then apply the same method as for continuum channels.

Comparisons have shown that all the previously mentioned methods gave quite identical results. However, for computational time considerations, either Moldauer or HRTW approaches should be employed in applications.

[3] For the HRTW method, this problem only concerns the continuum and fission reactions.

5 Conclusions

The compound nucleus model is currently used in many nuclear reaction codes. However, it is clear that regarding the current needs for nuclear data up to several tens or hundreds of MeV, the compound nucleus model shows severe limitations. For low excitation energies, compound nucleus process is essentially the only relevant reactions mechanism. For excitation energies around 20 MeV, it is still responsible for 60 to 70 % of the cross sections, but its importance decreases with excitation energy until it becomes irrelevant. In fact, even above 10 MeV, a pre-equilibrium component starts to be important, and the higher the excitation energy, the more the pre-equilibrium mechanisms are important to consider.

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