Problems in the Neutron Dynamics of Source-Driven Systems

P. Ravetto*

Politecnico di Torino, Dipartimento di Energetica, Torino, Italy

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*ravetto@polito.it

Abstract

The present paper presents some neutronic features of source-driven neutron multiplying systems, with special regards to dynamics, discussing the validity and limitations of classical methods, developed for systems in the vicinity of criticality. Specific characteristics, such as source dominance and the role of delayed neutron emissions are illustrated. Some dynamic peculiarities of innovative concepts proposed for accelerator-driven systems, such as fluid-fuel, are also discussed. The second portion of the work formulates the quasi-static methods for source-driven systems, evidencing its novel features and presenting some numerical results.

1 Introduction

Subcritical source-injected multiplying systems are today proposed as a viable and acceptable nuclear means to produce energy and to transmute actinide and fission products produced by conventional reactors. [1] A large effort is devoted worldwide to the assessment of accelerator-driven systems (ADS), where source neutrons are produced by spallation reactions caused by high energy protons impinging on a heavy-nuclide target. This concept poses new and interesting problems to reactor physicists, [2] since the methods that were developed for conventional reactors are to be extended and adapted to account for several new aspects in the physics of ADS. [3] That is particularly true for innovative concepts, such as fluid fuel configurations, [4] in which the multiplying material in the form of a molten salt is circulated through a circuit outside the core of the subcritical reactor.

The present paper presents recent developments in the neutron physics and dynamics of source-driven systems. After the presentation of some basic features, the quasi-static method which was developed for standard reactors and proved to be a very efficient algorithm to time-integrate the neutron balance equations is taken into consideration and the novelty connected to its extension to source-driven systems is illustrated. Some numerical results are then presented and discussed.

2 Basic Problems of the Neutronics of Source-Driven Systems

The proposal of subcritical systems for energy production dates back many years. [5] It is obvious that a subcritical reactor behaves more safely than a critical one when responding to a reactivity accident. However, it requires the presence of an external neutron source, whose intensity, for a given power level, increases as its multiplicativity decreases. Therefore, for a rather long time this concept was not considered an interesting means to produce energy, because no neutron sources with the required intensity were available. More recently, the technology of spallation sources driven by accelerators has developed and the proposal has been taken again into consideration (Accelerator-Driven Systems, ADS), also in the perspective of programs for the transmutation of long-lived radioactive products. [6]

Some peculiar physical features of these systems are illustrated in the following. In particular it is evidenced that classic techniques developed for standard critical or close-to-critical reactors need to be extended and adapted in order to be applied for the simulation of subcritical reactors.

2.1 Multiplication Constant and Neutron Amplification

Conventional multiplying systems are usually characterized by their effective multiplication constant, or by some suitable eigenvalue, whose associated eigenstate char-

acterizes the stationary configuration. However, the usual multiplication eigenvalue is not a suitable parameter to describe source-driven subcritical multiplying systems far away from criticality, since the stationary neutron distribution is strongly dominated by the presence of the source and it can be very different from the fundamental critical eigenstate. A more useful parameter is the *neutron amplification*, which can be used to characterize the intrinsic properties of the system, including material, geometry and source.

The neutron amplification may be introduced starting from the steady-state neutron transport equation including fissions and external source: [7]

$$\nabla \cdot (\mathbf{\Omega}\Phi) + \Sigma\Phi = \hat{\Theta}_s[\Phi] + \hat{\Theta}_f[\Phi] + S, \tag{1}$$

where scattering and fission operators $\hat{\Theta}_s$ and $\hat{\Theta}_f$, respectively, are introduced to simplify notation. The angular neutron flux Φ must satisfy proper vacuum boundary conditions

$$\Phi(\mathbf{r}_B, E, \Omega_{in}) = 0, \tag{2}$$

for all points $\mathbf{r}_B$ on the boundary of the non-reentrant domain and for all incoming directions Ω_{in}.

The eigenvalue of the transport equation is introduced for the homogeneous version of Eq. (1). Different formulations of the eigenvalue are possible; however, in nuclear engineering the multiplication eigenvalue is mostly used; such a parameter is defined through the following homogeneous eigen-equation:

$$\nabla \cdot (\mathbf{\Omega}\Phi_0) + \Sigma\Phi_0 = \hat{\Theta}_s[\Phi_0] + \frac{1}{k}\hat{\Theta}_f[\Phi_0]. \tag{3}$$

The only physically significant (fundamental) eigenvalue is the $k = 1$ value associated to a neutron distribution that does not change sign within the domain. In this case, the system is said to be critical, since no source is needed to maintain a steady-state, which presents a phase-space distribution coincident with the eigenstate itself and any arbitrary level, being the solution of the homogeneous Eq. (3). It can be shown that in the absence of an external source k is the ratio between the neutron populations belonging to successive generations at any phase space point, asymptotically, e.g., when the distribution has finally reached the fundamental shape.

In source-injected systems far-away from criticality, characterized by k-values by far smaller than unity, and in the presence of a source, the neutron distribution may be always largely different from the critical one. Therefore, the eigenvalue retains its significance of informing on the distance of the system from criticality, but it loses its capability of characterizing the actual behavior of the reactor. In this case, the nuclear engineer is interested in an integral parameter characterizing the full system, including the source, yielding information on its capability to produce secondaries from fission reactions and thus contributing to maintain the level of the neutron population.

The amplification factor α_S of the source-driven system may be defined as the total number of fission neutrons produced per source neutron. This number can be determined by direct computation as

$$\alpha_S = \frac{\langle \nu\Sigma_f \, | \Phi \rangle}{\langle S \rangle}, \tag{4}$$

where the brackets indicate integration over the whole phase space. Alternatively, it is also interesting to utilize the solution of a suitable adjoint problem, which plays the role of importance of source neutrons, to which reference is made in the following sections when deriving kinetic models. In a way similar to what was proposed by Kobayashi and Nishihara, [8] for the present purposes the neutron importance $\Phi^\dagger$ is introduced as the total number of fission neutrons produced in the system by a source neutron injected at a point in phase space. Consequently, with the usual definition of the scalar product through integration on the full phase space, the transport equation for $\Phi^\dagger$ is written down as: [9]

$$-\nabla \cdot (\mathbf{\Omega}\Phi^\dagger) + \Sigma\Phi^\dagger = \hat{\Theta}_s^\dagger[\Phi^\dagger] + \hat{\Theta}_f^\dagger[\Phi^\dagger] + \nu\Sigma_f, \tag{5}$$

where $\nu\Sigma_f$ plays the role of adjoint source. Hence, the neutron amplification can also be written in a form symmetric to Eq. (4), namely: [10]

$$\alpha_S = \frac{\langle \Phi^\dagger \, | S \rangle}{\langle S \rangle}. \tag{6}$$

The following observations can be made on the amplification factor:

- it characterizes the system including both the material-geometric properties and the source;
- in an accelerator-driven system it can be connected directly to the accelerator current;
- it can be easily computed by Monte Carlo statistical procedures;
- it is directly connected to the stationary power-level of the system.

Consequently, it is believed that the amplification parameter is much more informative to characterize the performance of the source-driven system and can be much more useful in engineering evaluations.

A few results can clarify some aspects of the amplification factor. If a simple one-dimensional one-group diffusion theory is assumed, the approximate versions of both Eqs. (1) and (5) can be solved analytically for a homogeneous slab using a Helmholtz eigenfunction expansion, and then α_S can be easily calculated. Tables 1 and 2 report the results for two slabs having the same multiplication constant, far away from unity ($k = 0.81654$), but significantly different spatial dimensions. The effect of the number of eigenfunctions used in the representation of the solution for different positions of the localized source can be observed. In the elementary theory referring to the fundamental eigenfunction only, the amplification would be estimated by $k/(1-k) = 4.4508$. From Table 2 it can be noticed that for larger

systems the dependence of the amplification factor on the position of the source is less important and its value approaches $k_\infty/(1-k_\infty) = 4.5785$. Table 3 reports results for a system close to criticality ($k = 0.98750$). The neutron distribution approaches the fundamental eigenstate: results are almost independent from the number of eigenfunctions employed and the dependence on the position of the source follows almost exactly the shape of the fundamental eigenfunction.

Table 1: Amplification factor for a largely subcritical 40 cm thick slab calculated with different numbers N of eigenfunctions and for different positions x_S of the localized source; $x_S = 0$ corresponds to source symmetrically placed within the slab.

N	$x_S = 0$	$x_S = 10cm$	$x_S = 15cm$
1	5.67	4.01	2.17
2	4.66	4.71	3.10
3	4.97	4.50	3.39
4	4.84	4.41	3.44
5	4.91	4.45	3.41
10	4.88	4.46	3.36
20	4.88	4.46	3.36
50	4.88	4.46	3.36

Table 2: Same as Table 1 for an 80 cm thick slab.

N	$x_S = 0$	$x_S = 20cm$	$x_S = 30cm$
1	5.67	4.01	2.17
2	4.12	5.10	3.60
3	4.80	4.62	4.22
4	4.46	4.37	4.36
5	4.65	4.51	4.28
10	4.57	4.55	4.12
20	4.58	4.53	4.13
50	4.58	4.53	4.13

2.2 Source Dominance

Subcritical systems require an external source to maintain a steady state, and the resulting neutron distribution is *dominated* by source injections, [11] rather than by

Table 3: Amplification factor for a 20 cm thick slab having $k = 0.98750$.

N	$x_S = 0$	$x_S = 5cm$	$x_S = 7.5cm$
1	100.57	71.11	38.49
2	99.90	71.59	39.11
3	100.03	71.50	39.24
4	99.98	71.46	39.26
5	100.0	71.48	39.25
10	100.0	71.48	39.23
20	100.0	71.48	39.23
50	100.0	71.48	39.23

the neutrons emitted through the inner multiplication process. As observed above, for largely subcritical assemblies the shape of the neutron flux may largely differ from the critical eigenstate. Peaking factors will certainly constitute a concern for engineering design.

A consequence of subcriticality is clearly that accelerator-driven systems are intrinsically much safer than conventional reactors, with respect to sudden insertions of positive reactivity. [13] As long as the system remains subcritical, the insertion of a perturbation would simply lead the system to a different steady situation at a different power level. However, two important issues need to be considered. The first one concerns the rapidity of power responses to external or accident perturbations, it is strictly connected to the role of delayed neutrons and is considered in the following section; the second one concerns the possibility to cause spatial and spectral flux distortions which might pose safety problems by themselves.

It is then worth to investigate the spatial effects of localized perturbations. Using codes recently developed, [14]' [15] it is possible to show that subcritical systems are particularly forgiving with respect to localized accidents. It has been verified [16] that, for systems characterized by multiplication constants around 0.95, strongly-localized perturbations implying even large power jumps (e.g., a doubling of the total power) result in only slight changes of the flux nearby the perturbed region of the system, with an almost irrelevant modification of the peaking factor of the system (e.g., of the order of few percents). In another work [3] it has been verified for a three-dimensional system in a configuration similar to the proposed energy-amplifier [6] that the distortion associated to a fixed perturbation is sharply increasing approaching criticality.

A straightforward consequence of the above aspect is very important for further dynamic applications and method development. Quasi-static techniques have to be regarded as very powerful tools for time-dependent analyses. The second portion of the paper is devoted to such numerical procedures. Furthermore, for many applications, such as the development of efficient methods for the interpretation of measurements for subcriticality monitoring, [17] or for preliminary safety eval-

uations, [18] even simple point-models may be sufficient. Of course, reference has always to be made to the source-dominated neutron shape characterizing the initial system.

2.3 Role of the Delayed Neutron Source

As is well-known, delayed emissions play a fundamental role in determining the kinetics of multiplying systems. Prompt neutrons are characterized by very short lifetimes, while delayed neutrons, although being only a small fraction of the total emissions, are released by radioactive processes associated to much longer decay times, with a consequent lag effect and a significant lengthening of the response time following a system perturbation. Mathematically, the evolution of the system is described by stiff sets of coupled differential equations. On the other hand, although it is obvious that the consequences of a given perturbation are much less dangerous in a source-driven system than in a critical reactor, the rapidity of the response can be a rather important parameter for safety evaluations. Therefore, it is worth investigating the role of delayed emissions.

A few recent papers have been devoted to the analysis of this problem. It appears that the relative weight of the delayed neutron source is significantly reduced with the subcriticality level and the inhomogeneous source overwhelms the delayed neutron lag effect. As a consequence, the power response to sudden perturbations is almost instantaneous, [11] for largely subcritical systems. This feature renders more easily solvable the dynamic system of equations, as neutron and precursors are almost decoupled. The situation can be even much worse in the case the system is devoted to actinide transmutation, since most actinides are characterized by a very low delayed neutron fraction, and that may cause concern for safety.

The effect of the reduction of the relative weight of delayed emissions can be understood by resorting to an elementary point system configuration. In the simplified case of one delayed-neutron family the evolution is described by the differential system of equations:

$$\begin{cases} \dfrac{dP}{dt} = \dfrac{\rho - \beta}{\Lambda} P + \lambda C + S, \\ \dfrac{dC}{dt} = \dfrac{\beta}{\Lambda} P - \lambda C, \end{cases} \tag{7}$$

where ρ is the system negative reactivity, and effective quantities and usual system parameters are introduced. The steady-state solution for a subcritical level given by ρ_0, which constitutes the initial condition to Eqs. (7), satisfies to the property:

$$\frac{\lambda C_0}{S_0} = -\frac{\beta}{\rho_0}, \tag{8}$$

which evidences that the weight of the delayed source with respect to the effective external source reduces as subcriticality ($|\rho_0|$) increases, owing to the reduced weight of fission neutrons in the overall balance. This feature is observed also in more realistic space-energy models.

In case of time-dependent problems following the insertion of a step perturbation, the solution to Eqs. (7) is obtained as a superposition of two exponential functions, whose time constants are given by the negative roots of the inhour equation. The value of the largest root is limited on the left by the value $-\lambda$ that is approached as the reactivity decreases. The other root is much smaller and its value sharply decreases with reactivity: hence, the contribution of the corresponding exponential disappears in a very short time. Consequently, the power shows a sudden jump while the delayed precursor population evolves on a much slower scale, the two populations behaving almost independently, as decoupled.

Table 4 reports some results [12] for the evolution in a realistic three-energy group cylinder where multiplicativity is step-wise reduced. The calculation is performed using a numerical code recently developed. [14] Also in this case it is observed that the total fission source, defined as:

$$S_f(t) = \int_V d\mathbf{r} \sum_{g'} \Sigma_{f,g'} \Phi_{g'}(\mathbf{r}, t), \tag{9}$$

shows an abrupt change, while the delayed source

$$S_c(t) = \int_V d\mathbf{r} \sum_i \lambda_i C_i(\mathbf{r}, t) \tag{10}$$

follows a much softer evolution affected by the decay constants of the delayed families, and the two evolutions are almost independent. At the very beginning of the transient the power reaches its asymptotic value in a very short time, while the delayed generation is almost unchanged. Even at very large times ($\backsim 10s$) when the power reaches definitively its new steady-state level, the delayed population is not yet at its equilibrium.

2.4 Fluid-Fuel Systems

Fluid-fuel reactors were considered in the early stage of nuclear engineering. Today the concept is considered interesting for the purpose of the efficient transmutation of long-lived radioactive products, since it presents several advantages, specially connected to the drastic simplifications of the fuel cycle. In fact no fabrication of the fuel elements is required, refueling is automatic and can be carried out along the operation of the system, and power distortions caused by burnup are eliminated. Also, the fissile material can be used as heat-removal fluid, circulating it outside the core through suitable heat exchangers. However, a few serious shortcomings cannot be overlooked. Firstly, molten salts, which are generally proposed as fluid fissile materials, are almost incompatible with standard structural materials; secondly, when transmuting actinides, non-fissile nuclides may accumulate and reach the solubility

Table 4: Evolution of the total fission source and of the delayed source in a transient; the initial system is characterized by $k = 0.99038$ and the perturbation leads to a system having $k = 0.96973$.

$t[s]$	$S_f(t)/S_f(0)$	$S_c(t)/S_c(0)$
0.0	1.0000	1.0000
4.0E-6	0.4236	1.0000
1.0E-4	0.3492	1.0000
5.0E-2	0.3484	0.9877
1.0E-1	0.3478	0.9764
1.0	0.3398	0.8450
2.0	0.3350	0.7577
5.0	0.3270	0.6316
7.0	0.3240	0.5816
10.0	0.3210	0.5323
∞	0.3074	0.3074

product, thus causing safety concerns and preventing the material to be circulated indefinitely. [19] For these reasons, fluid-fuel systems seem to be interesting and are studied worldwide, although their realization does not look to be imminent.

Many aspects of the physics of fluid-fuel systems are interesting and fascinating and have stimulated many new research activities. [20],[21] The circulation of the fuel, and consequently of the fission products, including the delayed neutron precursors, has to be accounted for. That implies a modification of the structure of the balance equations for the delayed neutrons and a coupling with the continuity equations for the fluid. Recent studies have been devoted to the full modelling of these systems. [22] A simplification can be attained supposing a *slug-flow* regime, [23] in which the motion of the fluid is established and fixed by circulation devices. To show some physical aspects of the problem, a homogeneous slab ($x \in [0, a]$) system is assumed in a two discrete-direction model for the transport of neutrons in a one-dimensional velocity field. [2] The corresponding time-dependent balance equations in the presence of a source with only one delayed neutron family shall

read:

$$\begin{cases} \dfrac{1}{v}\dfrac{\partial\varphi(x,t)}{\partial t} + \mu\dfrac{\partial\varphi(x,t)}{\partial x} + \Sigma\varphi(x,t) = \dfrac{\nu\Sigma_f + \Sigma_s}{2}(\varphi(x,t) + \psi(x,t)) + \\ \qquad\qquad \frac{1}{2}\lambda C(x,t) + \frac{1}{2}S(x,t), \\ \dfrac{1}{v}\dfrac{\partial\psi(x,t)}{\partial t} - \mu\dfrac{\partial\psi(x,t)}{\partial x} + \Sigma\psi(x,t) = \dfrac{\nu\Sigma_f + \Sigma_s}{2}(\varphi(x,t) + \psi(x,t)) + \\ \qquad\qquad \frac{1}{2}\lambda C(x,t) + \frac{1}{2}S(x,t), \\ \dfrac{\partial C(x,t)}{\partial t} + u\dfrac{\partial C(x,t)}{\partial x} = \beta\nu\Sigma_f(\varphi(x,t) + \psi(x,t)). \end{cases} \tag{11}$$

Forward and backward neutron fluxes are here indicated by φ and ψ, respectively, and emissions are supposed isotropic. It can be observed that the delayed neutron precursor equation assumes a typical transport structure, with a streaming term due to the motion of the fissile material. Boundary conditions are standard for neutrons and must be formulated to account for the decay of the delayed neutron precursors in the external circuit supposed neutronically isolated from the core itself, as

$$\varphi(0,t) = 0, \qquad \psi(a,t) = 0, \qquad C(0,t) = C(a,t-\tau)e^{-\lambda\tau}. \tag{12}$$

Obviously, τ is the time spent by the fluid in the recirculation duct. For the time-dependent problem (11) also proper initial conditions must be specified. It is worth mentioning that in stationary situations the model for the neutron balance in Eqs. (11) is equivalent to one-group diffusion theory.

In general, by introducing the vectors $\mathbf{X}(x,t)$ having angular group fluxes and delayed neutron precursor concentrations as components, the system of equations (11) can be given the following operator formulation:

$$\frac{\partial\mathbf{X}}{\partial t} + \hat{T}\mathbf{X} = \hat{\mathbf{\Theta}}\mathbf{X} + \mathbf{S}, \tag{13}$$

where the streaming operator $\hat{T}$ and the emission operator $\hat{\mathbf{\Theta}}$ are introduced, with obvious definitions. The initial conditions for this problem are determined by solving the steady state system of equations:

$$\hat{T}_0\mathbf{X}_0 = \hat{\mathbf{\Theta}}_0\mathbf{X}_0 + \mathbf{S}_0. \tag{14}$$

The multiplication eigenvalue problem can be also considered, by taking the stationary version of Eqs. (11) and eliminating the external source with the simultaneous introduction of the eigenvalue $1/k$ as a factor in front of the fission terms, as usual in reactor physics. Therefore, the following homogeneous system of equations

is obtained:

$$\begin{cases} \mu\dfrac{\partial\varphi(x)}{\partial x} + \Sigma\varphi(x) = \dfrac{\dfrac{1}{k}\nu\Sigma_f + \Sigma_s}{2}(\varphi(x) + \psi(x)) + \frac{1}{2}\lambda C(x), \\ -\mu\dfrac{\partial\psi(x)}{\partial x} + \Sigma\psi(x) = \dfrac{\dfrac{1}{k}\nu\Sigma_f + \Sigma_s}{2}(\varphi(x) + \psi(x)) + \frac{1}{2}\lambda C(x), \\ u\dfrac{\partial C(x)}{\partial x} = \dfrac{1}{k}\beta\nu\Sigma_f(\varphi(x) + \psi(x)). \end{cases} \tag{15}$$

The solution can be worked out by standard techniques, such as the use of the eigenvectors of the problem. [2], [19] It can be observed that in this case the eigenvalue is depending on the delayed neutron fraction, since the equation for the balance of precursors cannot be eliminated as for solid-fuel systems. Obviously, it is also depending on the fluid velocity u and on the recirculation time τ. A typical set of results is reported in Table 5, where it can be seen that the effect connected to the delayed neutron fraction is quite appreciable, while the effect due to the fluid velocity is less important. It is found that delayed precursors assume a spatial distribution by far different from what can be expected in solid-fuel systems.

Table 5: Multiplication eigenvalues k for a system characterized by $\Sigma = 1$, $\Sigma_s = 0.9$, $\nu\Sigma_f = 0.18$, and $a = 5$ mfp. The recirculation time is such that $\exp(-\lambda\tau) = 0.1$. The eigenvalue for a solid fuel system is 0.95866.

β	$u = 10[mfp/s]$	$u = 30[mfp/s]$	$u = 50[mfp/s]$	$u = 100[mfp/s]$
0.002	0.95680	0.95676	0.95675	0.95675
0.003	0.95586	0.95581	0.95580	0.95579
0.004	0.95493	0.95486	0.95485	0.95484
0.005	0.95400	0.95391	0.95389	0.95388
0.006	0.95307	0.95296	0.95294	0.95292
0.007	0.95213	0.95201	0.95199	0.95197

Time dependent calculations can be performed by solving numerically the system of differential equations (13). The time integration can be performed making advantage of the fact that the evolution of the neutron population is much faster than the transit time of the fluid through the system. Therefore, two scales can be introduced, with two much different time discretization intervals, [15] using a scheme which is often exploited in Particle-in-Cell (PIC) techniques for fluid or

plasma simulations. On a fast scale the neutron equations are discretized by an implicit scheme, supposing that the fuel is not moving, and thus eliminating the streaming term from the delayed precursor equations and introducing the new transport operator $\hat{T}^*$ which accounts for streaming of neutrons only. Hence, the vector $\mathbf{X}$ is updated on passing from time t_n to time $t_{n+1} = t_n + \Delta t$ solving the system:

$$\frac{\mathbf{X}^{n+1} - \mathbf{X}^n}{\Delta t} + \hat{T}^* \mathbf{X}^{n+1} = \hat{\mathbf{\Theta}} \mathbf{X}^{n+1} + \mathbf{S}^{n+1}. \tag{16}$$

To that end, the space variable has to be also discretized. After time ΔT the position of the fluid is updated by the displacement formula

$$C_i^{new}(z_p(\Delta T)) = C_i^{old}(z_p), \tag{17}$$

where z_p is the original position of the spatial grid point where the precursor density is known, and $z_p(\Delta T)$ is the position reached by the grid point moving at velocity u after time ΔT. It might be necessary to introduce an interpolation procedure to obtain values at grid points in order to proceed to the following neutron calculation.

An interesting investigation concerns the effect of circulation on the value of the effective delayed neutron fraction, which is significantly reduced, owing to spatial redistribution inside the core and decay in the external circuit. As usual, the effective delayed neutron fraction can be defined as the ratio between the importance associated to delayed emissions and the total importance associated to fission neutrons. In the present case, the importance of delayed precursors has to be introduced, owing to the motion of the fissile material. The importance is defined as the total number of fission neutrons produced in the system by a neutron or a delayed precursor, as the case may be. The balance can be worked out consistently, ending up with the formal adjoint of Eq. (14), once the inner product is introduced in the standard manner including a spatial integration, as:

$$\hat{T}_0^\dagger \mathbf{X}_0^\dagger = \hat{\mathbf{\Theta}}_0^\dagger \mathbf{X}_0^\dagger + \mathbf{S}_0^\dagger. \tag{18}$$

As can be seen, an adjoint source is needed. With the above definition of importance, the components of $\mathbf{S}_0^\dagger$ are simply the number of neutrons emitted per unit path in each direction and energy; no importance source is introduced for delayed neutrons.

As an example, the explicit formula for the effective delayed neutron fraction for each family $\tilde{\beta}_i$ is reported, for a diffusion multigroup model: [24]

$$\tilde{\beta}_i = \frac{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} \lambda_i C_i \right\rangle}{\sum_{i=1}^{R} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} \lambda_i C_i \right\rangle + (1-\beta) \sum_{n=1}^{G} \sum_{g=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_n \left(\nu \Sigma_f\right)_g \Phi_g \right\rangle} \tag{19}$$

The following Table 6 reports the ratios between the total effective delayed fraction and the physical value for a three-group calculation in one-dimensional slab geometry. The effects of velocity, subcriticality level and recirculation time are reported. A gaussian spatially distributed source is symmetrically placed within the slab. The results for the purely academic case characterized by $T_R = 0$ are also shown, to evidence the effect of the spatial redistribution of precursors within the core separately from recirculation, which can be seen to amount at about 20%. The effect of recirculation can be rather large and reduce significantly the role of delayed emissions: thus the response to system perturbations shall be much prompter summing up to the effect characterizing subcritical systems, as described in the previous section. In Figs. 1 and 2 graphs for the neutron and importance shapes are drawn for the three-energy group calculations mentioned above.

Table 6: Ratio $\tilde{\beta}/\beta$ as a function of τ and k when $u = 60$ and 100 cm/s. Results for a critical system are also shown.

T_R [s] $\rightarrow$	0	5	10	15	u [cm/s] $\downarrow$
$k_{eff} = 0.95009$	0.842	0.542	0.470	0.443	60
	0.835	0.422	0.363	0.332	100
$k_{eff} = 0.97048$	0.843	0.540	0.469	0.441	60
	0.834	0.420	0.353	0.330	100
$k_{eff} = 0.99028$	0.842	0.539	0.467	0.440	60
	0.833	0.419	0.352	0.329	100
$k_{eff} = 1.00000$	0.843	0.539	0.468	0.440	60
	0.834	0.420	0.352	0.329	100

Finally, by making use of the extension of the projection technique used in the separation procedures in neutron dynamics, [25] a consistent point model can be constructed: [2]

$$\begin{cases} \dfrac{dP}{dt} = \left(\dfrac{\rho_S - \tilde{\beta}}{\Lambda_P}\right) P + \displaystyle\sum_{i=1}^{R} \lambda_i \tilde{C}_i + \tilde{S}, \\ \dfrac{d\tilde{C}_i}{dt} = \left(\dfrac{\tilde{\beta}_i + \rho_i}{\Lambda_i}\right) P - (\lambda_i + \delta_u)\tilde{C}_i \ ; \ i = 1, ..., R. \end{cases} \tag{20}$$

The structure of the above system is completely equivalent to the standard point model; however, additional terms appear, to account for the fuel motion and the possibility of perturbation of the velocity field. Also, a source-reactivity term is appearing, as is discussed in a following section.

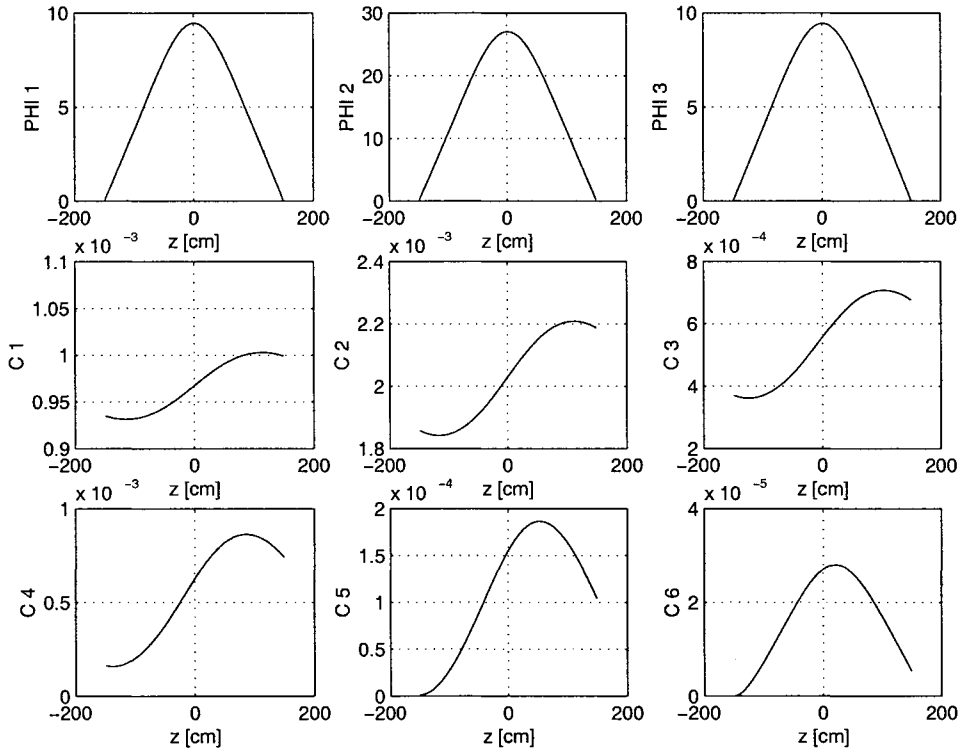

Figure 1: Flux and delayed neutron precursor concentration in a three-group diffusion calculation. The source is gaussian shaped centered on the symmetry plane of the system.

2.5 High Energy Neutrons and Transport Effects

Spallation reactions produce neutrons at energies which are generally higher than for typical fission reactions. [26] This fact poses new problems. Firstly, reliable cross-section data in unusual portions of the energy axis are needed. Hence, a strong effort in both experimental activities and in model development for interpretation of results is to be undertaken. On the other hand, spallation neutrons are characterized by large mean free paths in comparison with the size of the structure. For that reason, the neutron field in source-dominated systems is substantially different from the typical distribution of critical configurations and it can be foreseen that an accurate neutronic description requires the utilization of the transport equation, both in static and transient situations, rather than the diffusion approximation. It must be observed that the time-dependent transport equation describes phenomena associated with very different time scales. In particular, the sharp wavefronts that are typical of transients following localized perturbations propagate on a very short time scale and are not of practical interest, for their effect disappears before the

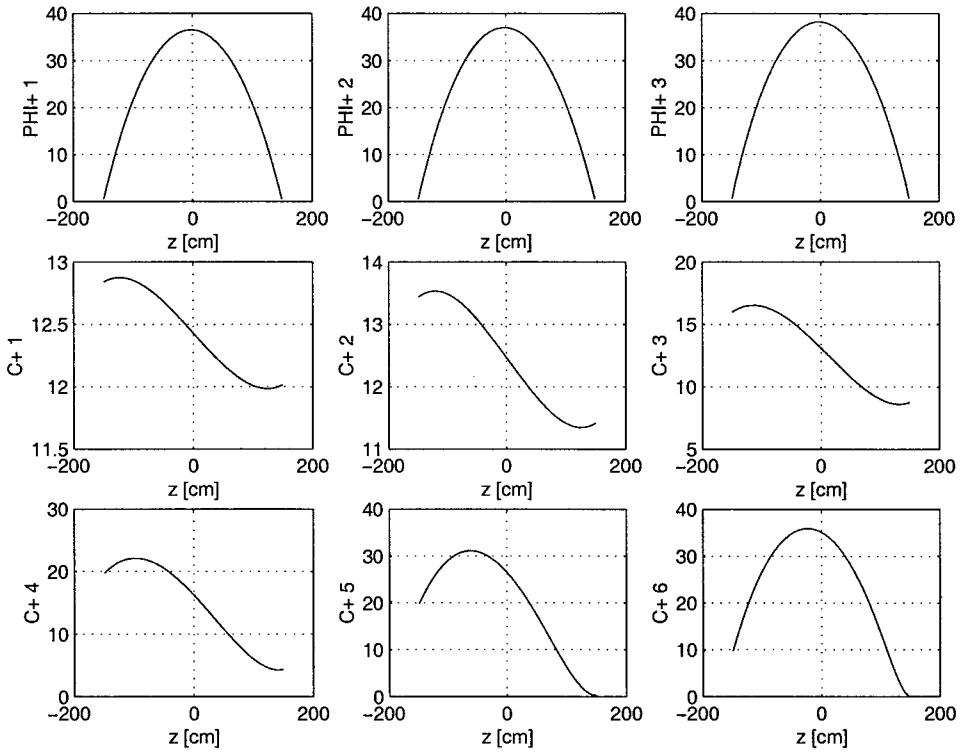

Figure 2: Adjoint shapes for neutrons and delayed precursors.

neutron level experiences a significant macroscopic variation. Thus, when a time-dependent transport problem has to be solved, the level of detail required to capture very short time-scale phenomena is usually not necessary. In such situations, time-implicit discretization schemes are the most appropriate. The classic Backward Euler implicit method has been analyzed. [27] In the referenced paper many aspects were considered, to assess the limits of applicability of the implicit scheme in different source configurations and for different materials. A time-harmonic analysis illustrates the separate effects of the time and space-angle discretizations. The analysis shows that for slowly varying sources (as in the proposed subcritical systems) the implicit Euler scheme provides accurate results when appropriate time steps are employed. To avoid the rapid phenomena induced by the propagation of sharp wavefronts, the numerical algorithm can be optimized through a suitable modification of the diffusion synthetic procedure, usually used for the acceleration of source iteration schemes, carrying out a coupled diffusion-transport calculation.

2.6 The Importance of Thermal Feed-back

Non-linear effects are of great importance in the analysis of subcritical systems and must be carefully studied for safety assessments. There are differences with respect to conventional reactors whose consequences need to be investigated. For instance, the requirement of a negative temperature coefficient is strictly adhered to for nuclear reactors, to guarantee stability in the event of a reactivity insertion. However, effects due to a negative temperature coefficient may be of some concern in accelerator-driven systems. In fact, it is proposed that power control is performed by operating directly on the level of the source and shut-down by simply turning off the proton beam. To clarify the physics of the phenomenon, a reduction of the source is taken into consideration. Following such an event, the system responds with a reduction of the power with a possible consequent reduction of the temperature. Therefore, a negative feed-back will counteract the temperature change with the insertion of a positive reactivity into the system, thus reducing the subcriticality level. Consequences may be important.

Some preliminary simplified studies have highlighted the importance of non linearities in both control and safety transients. [14] More recently, an intense research activity is going on in order to couple neutronic kinetic modules to realistic thermal-hydraulic codes. Within a collaboration between ENEA-Casaccia, Rome, and Politecnico di Torino, activity is going on to introduce coupling with a channel code, initially developed for fast reactors and now adapted for lead-bismuth cooled systems of the energy amplifier type. [28]

The thermal code can represent a single reactor channel with fuel, clad and coolant. It can consider a central fuel hole (if any) and a fuel-clad gap. The code does not consider the axial heat propagation in the fuel and the clad, thus assuming the temperature axial gradient negligible with respect to the radial one, and it solves the time dependent Fourier equation in the fuel and clad in a one-dimensional cylindrical configuration at any axial mesh point. The thermal source in fuel and, if significant, the gamma heating, present also in the clad and the coolant, are evaluated from the power given by the neutronic calculation. The coolant temperature is considered as a boundary condition. Once the axial clad temperatures are known, it is easy to carry out the calculation of the clad-coolant heat flux and of the axial coolant temperatures, at each axial position; hence, these temperatures are used as boundary conditions for the fuel-clad temperature calculation in the next time step. The code carries out only a single-phase calculation, which seems reasonable for molten lead-bismuth systems, since the boiling temperature is very high (1943 K at atmospheric pressure), even higher than the stainless steel melting point (1643 K), which constitutes the clad material.

The code takes into account the forced coolant circulation condition, which means that the velocity must be given among input data. As an alternative, natural coolant convection can also be considered. In that case the velocity is estimated step by step as a result of the thermal conditions of the whole coolant circuit (density

difference between the hot and cold leg).

The thermal calculation is coupled with the neutronic calculation as follows:

- at the end of a thermal time step the radial average of the axial profile of the power density is evaluated for a fixed number of reactor zones, characterized by a thermal channel;

- the axial power distributions are input to the thermal code which performs the temperature calculations for each channel;

- the channel average temperatures are assigned to each reactor zone, and used to modify cross sections, according to assigned interpolating functions, and a new neutronic step is initiated.

The steady-state configuration has to be determined coherently with the thermal feed-back, thus determining also the equilibrium initial temperatures. The calculation is started from a tentative temperature distribution inside the system followed by an iteration sequence with the neutronic calculation at all corresponding to an effective dynamic calculation with a fixed source, until an asymptotic steady condition is reached. [29]

3 The Quasi-Static Method

This work is dedicated to the memory of the late Jacques Devooght (Université Libre de Bruxelles), who passed away in 1999. It was an honor and a privilege for me to have known him and appreciated his great contributions to many fields in nuclear engineering, and especially to reactor physics. Many of his works are universally recognized as landmarks in reactor physics. His papers and his presentations always joined deep mathematical rigor with clarity. In his talks he had the singular gift to make even arduous mathematical aspects approachable, in the continuous effort to convey the physical meaning to the whole audience. For me many of his interpretations of transport models and equations are just unforgettable. He never wanted to address his talks to specialists but to fellow-researchers to whom he was always willing to give a full insight into his work, without unnecessary details, aiming at the essential aspects.

Quasi-static methods have constituted for several years a very powerful tool in reactor dynamics computations. In this field his contributions to the mathematical understanding of the procedure and its implications and to its sound founding are of fundamental value.

The whole reactor physics community owes him a lot and we all feel his death as a great loss. He will continue to be and example as a man, a scientist and a professor.

Thank you, Jacques.

3.1 Fundamentals of the Quasi-Static Method

The problem of the neutron dynamics of multiplying systems amounts to the numerical solution of the following system of time-dependent equations

$$\begin{cases} \mathbb{V}^{-1}\dfrac{\partial \mathbf{\Phi}}{\partial t}=\mathbb{H}_p\,\mathbf{\Phi}+\displaystyle\sum_{\alpha=1}^{6}\chi_\alpha\lambda_\alpha C_\alpha+\mathbf{S}\,, \\ \dfrac{\partial C_\alpha}{\partial t}=-\lambda_\alpha C_\alpha+\beta_\alpha\mathbf{F}\cdot\mathbf{\Phi}\,,\ \alpha=1,...,6\,. \end{cases} \tag{21}$$

In the framework of multigroup diffusion, the equations are written in vectorial form for the quantities $\mathbf{\Phi}(\mathbf{r},t)$ and $\mathbf{S}(\mathbf{r},t)$ as the vectors of group fluxes and external source, respectively. However, formally the same structure of the problem can be maintained also for different models, such as discrete ordinate transport, where flux and source vector components include angular and group fluxes. In the following, explicit reference to multigroup diffusion is made. $\mathbb{V}$ is the diagonal matrix of group velocities, while the balance operator, $\mathbb{H}_p$, is defined as $\mathbb{H}_p=\mathbb{L}+R+(1-\beta)\chi_p\mathbf{F}$, $\mathbb{L}$ being the diagonal leakage operator and $\mathbb{R}$ the group-to-group transfer operator. The vectors χ_p and χ_α are the prompt and the α-th family delayed fission spectra, respectively, while $\mathbf{F}$ is a vector with components $(\nu\Sigma_f)_g$. Standard boundary conditions with group-dependent extrapolation lengths and initial conditions define uniquely the solution of Eqs. (21). In the most general situation, material parameters may be time dependent.

The standard quasi static technique was developed as a multiscale scheme for the time integration of Eqs. (21), especially for the case with no source, [25], [30], [31], [32] where transients are studied starting from an initial critical system, usually referred to as the reference reactor. In such steady-state situation, a total balance operator can be defined as

$$\mathbb{H}_0=\mathbb{L}_0+\mathbb{R}_0+\chi\,\mathbf{F}_0\,, \tag{22}$$

where $\chi=(1-\beta)\chi_p+\sum_{\alpha=1}^{6}\beta_\alpha\chi_\alpha$ is the effective fission spectrum, and the neutron distribution is given by the solution of the homogeneous problem

$$\mathbb{H}_0\,\mathbf{\Phi}_0=0\,. \tag{23}$$

For the initial system an adjoint problem can be introduced as:

$$\mathbb{H}_0^\dagger\,\mathbf{\Phi}_0^\dagger=0\,. \tag{24}$$

whose solution is the importance function for neutrons in a critical reactor. [9]

According to the multiple time-scale quasi-static methodology, the neutron flux in Eq. (21) is factorized as the product of a rapidly-varying amplitude $A(t)$ and a slowly-varying shape vector $\varphi(\mathbf{r},t)$, as

$$\mathbf{\Phi}(\mathbf{r},t)=A(t)\varphi(\mathbf{r},t)\,. \tag{25}$$

assuming $\varphi(\mathbf{r},t=0)=\mathbf{\Phi}_0$. In order that the factorization be made unique a suitable weigh function, $\mathbf{w}(\mathbf{r})$, is introduced, requiring that

$$\langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle = \text{Const}\ , \tag{26}$$

where the inner product is defined as $\langle \mathbf{f}|\ \mathbf{g}\rangle = \int \mathbf{f}(\mathbf{r})\cdot\mathbf{g}(\mathbf{r})d\mathbf{r}$. The choice of the weight function is important for the efficiency of the method. In classic quasi-static procedures, $\mathbf{w}(\mathbf{r})$ is assumed to be the solution of the adjoint reference critical problem (24). Introducing the factorization (25) into Eq. (21), projecting on $\mathbf{w}$ and making use of the normalization condition, Eq. (26), one obtains a system of equations governing the evolution of the amplitude $A(t)$ and the effective precursor concentration $c_\alpha(t)=\langle \mathbf{w}|\ \chi_\alpha C_\alpha\rangle\ /\ \langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle$:

$$\begin{cases} \dot{A} = A\dfrac{\langle \mathbf{w}|\ \mathbb{H}_p\varphi\rangle}{\langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle}+\displaystyle\sum_{\alpha=1}^{6}\lambda_\alpha c_\alpha+\frac{\langle \mathbf{w}|\ \mathbf{S}\rangle}{\langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle}\ , \\ \dot{c}_\alpha = -\lambda_\alpha c_\alpha+\beta_\alpha A\dfrac{\langle \mathbf{w}|\ \chi_\alpha\mathbf{F}\varphi\rangle}{\langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle}\ ,\ \alpha=1,...,6\ , \end{cases} \tag{27}$$

while the shape function φ and the precursor distributions C_α satisfy the equations

$$\begin{cases} \mathbb{V}^{-1}\dfrac{\partial\varphi}{\partial t}=\mathbb{H}_p\varphi-\dfrac{\dot{A}}{A}\mathbb{V}^{-1}\varphi+\dfrac{1}{A}\left(\displaystyle\sum_{\alpha=1}^{6}\chi_\alpha\lambda_\alpha C_\alpha+\mathbf{S}\right)\ , \\ \dfrac{\partial C_\alpha}{\partial t}=-\lambda_\alpha C_\alpha+\beta_\alpha\mathbf{F}\cdot\varphi A\ ,\ \alpha=1,...,6\ . \end{cases} \tag{28}$$

Equations (27) and (28) constitute the basis of the quasi-static technique. Equations (27) govern the evolution of the amplitude, once the shape is specified, starting from the distribution for the reference reactor, and are solved on a fast time scale, introducing an adequate time-discretization interval Δt. Equations (28) are used to update the shape function, on a much slower scale (time interval ΔT for the discretization of the time derivative) with a consequent saving in computational effort. In order to obtain a solution for φ which fulfils the normalization condition (26), a suitable iterative procedure is employed.[33] In fact, if $\dot{A}$ appearing in Eqs. (28) is supposed to be given by the first of Eqs. (27), the resulting problem is obviously non-linear. Therefore, eqs. (28) are solved firstly using the shape adopted along the previous time step ΔT for the evaluation of and then iterating. It follows that the amplitude is continuous, while its derivative is discontinuous at the instants of the shape recalculation.

Equations (27) have the typical structure of the point-kinetic model. It is interesting to observe that the term multiplying A in the first Eq. (27) can be expressed in terms of the instantaneous kinetic parameters of the reactor, as:

$$\frac{\langle \mathbf{w}|\ \mathbb{H}_p\varphi\rangle}{\langle \mathbf{w}|\ \mathbb{V}^{-1}\varphi\rangle}=\frac{\rho-\overline{\beta}}{\Lambda}\ , \tag{29}$$

where the quantities Λ, $\overline{\beta}$ and ρ, defined as

$$\Lambda = \frac{\langle \mathbf{w}|\, \mathbb{V}^{-1}\varphi\rangle}{\langle \mathbf{w}|\, \chi\cdot\mathbf{F}\varphi\rangle}, \overline{\beta} = \sum_{\alpha=1}^{6} \beta_a \frac{\langle \mathbf{w}|\, \chi_\alpha\cdot\mathbf{F}\varphi\rangle}{\langle \mathbf{w}|\, \chi\cdot\mathbf{F}\varphi\rangle}, \rho = \frac{\langle \mathbf{w}|\, \mathbb{H}\varphi\rangle}{\langle \mathbf{w}|\, \chi\cdot\mathbf{F}\varphi\rangle}\,, \tag{30}$$

represent the mean generation time, the effective delayed neutron fraction and the total reactivity, respectively. The operator $\mathbb{H}$ can be written as $\mathbb{H}_0 + \delta\mathbb{H}$, where $\mathbb{H}_0$ refers to the unperturbed reactor and $\delta\mathbb{H}$ to the perturbation introduced.

It is worth observing that in real cases it is required that both time steps ΔT and Δt are adapted during the transient, in order to capture the full features of the evolution and to enhance the efficiency of the scheme. Many variations of the basic technique presented have been developed, e.g., allowing some space dependence to be included within the amplitude function, to account for one-dimensional distortions of the neutron distribution. [33]

3.2 Generalization of the Quasi-Static Method to Source-Driven Systems

The neutron distribution $\mathbf{\Phi}$ in the source-injected system in steady-state situation is the solution to the following problem:

$$\mathbb{H}_0\,\mathbf{\Phi}_0 + \mathbf{S}_0 = 0\,, \tag{31}$$

and it constitutes the initial state for time-dependent calculations. Contrary to classic procedures employed for nuclear reactor dynamics, [25] no critical reactor can be used as reference in the present case. The reference steady-state system is more suitably identified as the initial source-injected system, i.e., the solution of Eq. (31). For largely subcritical systems, this solution is significantly different from the one of the multiplication eigenproblem

$$\left(\mathbb{L}_0 + \mathbb{R}_0 + \frac{1}{k}\chi\,\mathbf{F}_0\right)\mathbf{\Phi}_{crit} = 0\,. \tag{32}$$

The quasi-static method requires the introduction of a weight function suitable for source-injected situations. A possibility is to choose the solution of the stationary adjoint problem:

$$\left(\mathbb{L}_0^\dagger + \mathbb{R}_0^\dagger + \frac{1}{k}\,\mathbf{F}_0\chi\right)\mathbf{\Phi}_{crit}^\dagger = 0\,. \tag{33}$$

However, when considering source-injected problems, the weight can be assumed as the solution of the adjoint equation for the subcritical system with a 'detector' $\mathbf{S}^\dagger$:

$$\mathbb{H}^\dagger\,\mathbf{\Phi}_{source}^\dagger + \mathbf{S}^\dagger = 0\,. \tag{34}$$

The identification of the adjoint source is a question still under investigation; however, there are physical motivations to justify the choice $\mathbf{S}^\dagger(\mathbf{r}) = \mathbf{F}(\mathbf{r})$. [8]

Following the classic procedure, introducing the factorization (25) into Eq. (21), projecting on $\mathbf{w}$ and making use of the normalization condition, Eq. (26), one obtains a system of equations governing the evolution of the amplitude $A(t)$ and for the averaged precursor concentration $c_\alpha(t)$ having the same structure as Eq. (27), while the shape function φ and the precursor distributions C_α satisfy the Eqs. (28) [3].

In this case, the reactivity term has to be written as $\varrho_{pert} + \rho_0$, where

$$\rho_0 = \frac{\langle \mathbf{w} |\ \mathbb{H}_0 \varphi \rangle}{\langle \mathbf{w} |\ \chi \cdot \mathbf{F} \varphi \rangle}, \rho_{pert} = \frac{\langle \mathbf{w} |\ \delta \mathbb{H} \varphi \rangle}{\langle \mathbf{w} |\ \chi \cdot \mathbf{F} \varphi \rangle} \ . \tag{35}$$

When $\mathbf{w}$ is chosen as the adjoint critical flux for the unperturbed reactor, $\mathbf{\Phi}^\dagger_{crit}$, the reactivity ρ_0 can be evaluated as

$$\rho_0 = \frac{k-1}{k} \frac{\left\langle \mathbf{\Phi}^\dagger_{crit} \mid \chi \cdot \mathbf{F}_0 \varphi \right\rangle}{\left\langle \mathbf{\Phi}^\dagger_{crit} \mid \chi \cdot \mathbf{F} \varphi \right\rangle} \ . \tag{36}$$

In usual quasi-static procedures referring to a critical reactor ($k = 1$), ρ_0 vanishes. According to Eq. (36), ρ_0 is a negative term due to the initial subcriticality of the reference reactor. When the weight function is chosen as the adjoint flux for the unperturbed reactor, $\mathbf{\Phi}^\dagger_{source}$, the term ρ_0 becomes:

$$\rho_0 = \frac{\left\langle \mathbf{\Phi}^\dagger_{source} \right|\ \mathbb{H}_0 \varphi \rangle}{\left\langle \mathbf{\Phi}^\dagger_{source} \mid \chi \cdot \mathbf{F} \varphi \right\rangle} = -\frac{\langle \mathbf{F} |\ \varphi \rangle}{\left\langle \mathbf{\Phi}^\dagger_{source} \mid \chi \cdot \mathbf{F} \varphi \right\rangle} \ . \tag{37}$$

The discussion above shows that the quasi-static methodology is applicable to subcritical source-driven systems, provided suitable modifications are introduced in the equations, coherently with the weight function assumed. A final observation is also in order. It is remarked above that in source-dominated systems even large perturbations cause small distortion of the neutron population, which is mainly determined by the presence of the source. As a consequence, quasi-static procedures seem particularly appropriate and efficient, provided the shape is chosen in accordance with the solution of the steady-state source-injected problem (31).

3.3 Discussion of Numerical Results

It is worth presenting some numerical results illustrating the features of the quasi-static algorithm when applied to source-injected systems. [3] The calculations concern a three-dimensional system with hexagonal fuel elements. The code has been developed for the analysis of configurations similar to the proposed energy amplifier.

Figure 3 shows the effect on the total power evolution of the time interval adopted to update the shape of the solution. It is clearly seen that the choice of such discretization parameter is crucial for the accuracy of the results. The transient at hand is started in an assembly having five rings of hexagonal elements,

where the source is located in the central channel. Four rings are constituted by multiplying material, while the fifth is the reflector. The source adjoint (solution of Eq. (34)) is employed. The jump of the initial static reactivity is 2020 pcm. For $N = 1$ the point kinetic solution is obtained.

It is clearly seen that the point kinetic model yields results which foresee a power significantly smaller than the actual one. A discontinuity in the derivative of the power curves is seen at instants corresponding to shape recalculations. That is a direct consequence of the previously discussed iterative procedure used to obtain shapes which fulfill the normalization requirement (26). Such discontinuities attenuate as the evolution proceeds, and as the reactivity approaches its final steady value.

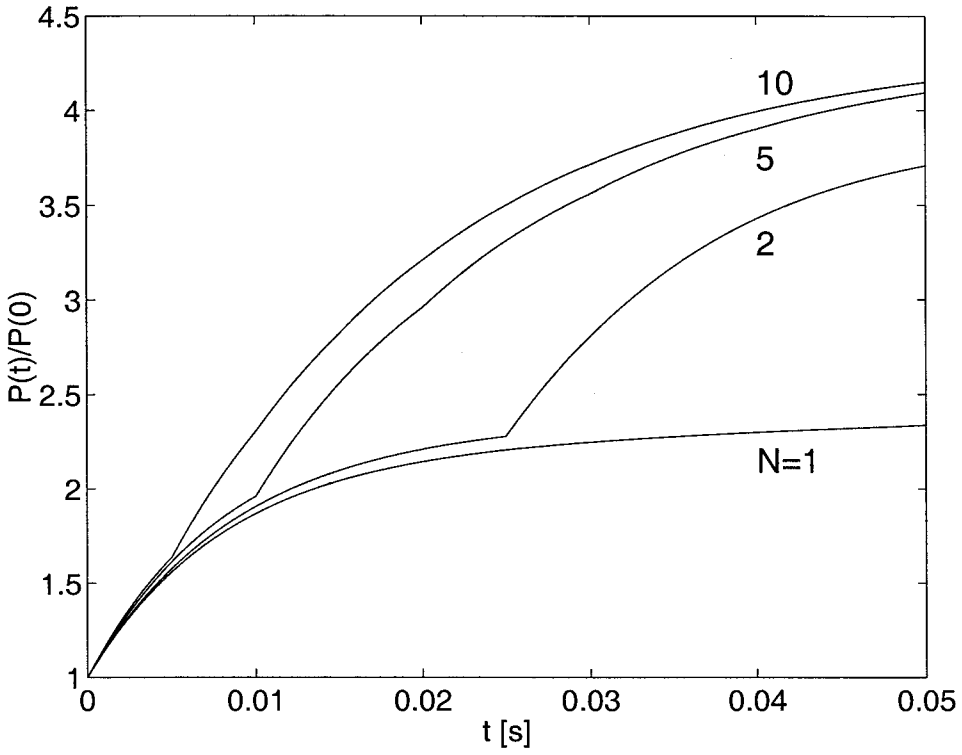

Figure 3: Power evolution computed with different time intervals between shape recalculations.

4 Summary and Conclusions

Source-driven systems pose many new challenges to modern reactor physics. Their study constitutes a new and very interesting chapter which is not yet fully written down. Many methods and procedures developed for standard reactors cannot simply be extended and employed for their neutronic analysis without suitable adaptation. Furthermore, there are many novel physical effects which need to be studied and interpreted.

In the present paper some typical features of interest for time-dependent neutronic analyses have been briefly addressed, such as the source dominance, the reduction of the role of delayed neutron emissions and the importance of non-linear feed-back phenomena. Having observed the effectiveness of the quasi-static procedure for source-dominated systems, its extension to source-driven systems has been presented in some detail, illustrating the modifications that are required.

In this field, fluid fuel systems give the opportunity to consider new and fascinating mathematical models, which can stimulate the interest and collaboration of the reactor physics community with both applied mathematicians and numerical analysts.

5 Acknowledgments

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