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Quasilocal Quark Models as Effective Theory of Non-perturbative QCD

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Abstract

We consider the Quasilocal Quark Model of NJL type (QNJLM) as effective theory of non-perturbative QCD including scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) four-fermion interaction with derivatives. In the presence of a strong attraction in the scalar channel the chiral symmetry is spontaneously broken and as a consequence the composite meson states are generated in all channels. With the help of Operator Product Expansion the appropriate set of Chiral Symmetry Restoration (CSR) Sum Rules in these channels are imposed as matching rules to QCD at intermediate energies. The mass spectrum and some decay constants for ground and excited meson states are calculated.

1 Introduction

The QCD-inspired quark models with four-fermion interaction are often applied for the effective description of low-energy QCD in the hadronization regime. The local four-fermion interaction is involved to induce the dynamical chiral symmetry breaking (DCSB) due to strong attraction in the scalar channel. As a consequence, the dynamical quark mass m_{dyn} is created, as well as an isospin multiplet of pions, massless in the chiral limit, and a massive scalar meson with mass $m_\sigma = 2m_{dyn}$ arise. However it is known from the experiment [1] that there are series of meson states with equal quantum numbers and heavier masses, in particular, $0^{-+}(\pi, \pi', \pi'', ...)$; $0^{++}(\sigma(f_0), \sigma', \sigma'', ...)$; $1^{--}(\rho, \rho', \rho'', ...)$. Due to confinement, one expects an infinite number of such excited states with increasing masses. Therefore in order to describe the physics of those resonances at intermediate energies one can extend the quark model with local interaction of the Nambu-Jona-Lasinio

(NJL) type [2] taking into account higher-dimensional quark operators with derivatives, i.e. [3, 4, 5, 6, 7], [8], [10] quasilocal quark interactions. For sufficiently strong couplings the new operators promote the formation of additional new meson states. Such a quasilocal approach (see also [11, 12, 14]) represents a systematic extension of the NJL-model towards the complete effective action of QCD where many-fermion vertices with derivatives possess the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the perturbative QCD effective action.

Another idea is to impose CSR Sum Ruls at high energies [8]. In paricular, at intermediate energies the correlators of QNJLM can be matched to the Operator Product Expansion (OPE) of QCD correlators [9]. This matching realizes the correspondence to QCD and improves the predictivity of QNJLM. It is based on the large-N_c approach which is equivalent to planar QCD. In this approximation the correlators of color-singlet quark currents are saturated by infinite number of narrow meson resonances. Namely, the two-point correlators of scalar, pseudoscalar, vector and axial-vector quark currents are represented by the sum of related meson poles in Euclidean space-time:

$$\Pi^C(p^2) = \int d^4x \exp(ipx)\langle T(\bar{q}\Gamma q(x)\bar{q}\Gamma q(0))\rangle_{planar} = \sum_n \frac{Z_n^C}{p^2+m_{C,n}^2} + D_0^C + D_1^C p^2, \tag{1}$$

where $C \equiv S, P, V, A$; $\Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$; $D_0, D_1 = const.$ The last two terms both in the scalar-pseudoscalar and in the vector-axial-vector channels represent a perturbative contribution with D_0 and D_1 being contact terms required for the regularization of infinite sums. On the other hand the high-energy asymptotics is provided [9] by the perturbation theory QCD and the OPE due to asymptotic freedom of QCD. Therefrom the above correlators increase at large p^2: $\Pi^C(p^2)\mid_{p^2\to\infty} \sim p^2 \ln\frac{p^2}{\mu^2}$. When comparing the two expressions one concludes that the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotics.

Meantime the differences of correlators of opposite-parity currents rapidly decrease at large momenta [8, 12]: $(\Pi^P(p^2)-\Pi^S(p^2))_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4}+O(\frac{1}{p^6})$, $\Delta_{SP} \simeq 24\pi\alpha_s < \bar{q}q >^2$ and [10, 11]: $(\Pi^V(p^2)-\Pi^A(p^2))_{p^2\to\infty} \equiv \frac{\Delta_{VA}}{p^6}+O(\frac{1}{p^8})$, $\Delta_{VA} \simeq -16\pi\alpha_s < \bar{q}q >^2$, where $< \bar{q}q >$ is quark condensate and we have defined for V,A fields $\Pi_{\mu\nu}^{V,A}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\mu p_\nu)\Pi^{V,A}(p^2)$, and the vacuum dominance hypothesis [9] in the large-N_c limit is adopted.

Therefore the chiral symmetry is restored at high energies and the two above differences represent a genuine order parameter of CSB in QCD. As they decrease rapidly at large momenta one can perform the matching of

QCD asymptotics by means of few lowest lying resonances that gives a number of constraints from the CSR. They may be used both for obtaining some additional bounds on the model parameters and for calculating of some decay constants (see in [13, 14] and references therein). In the present talk the QNJLM is considered with two channels where two pairs of SPVA-mesons are generated. Respectively it is expected to reproduce the lower part of QCD meson spectrum in the planar limit and the leading asymptotics of CSR for higher energies.

2 Quasilocal Quark Model of NJL-type

The minimal n-channel lagrangian of the QNJLM has [4, 5, 6] the following form, $L = \bar{q}i\hat{\partial}q + L^I_{SPVA}$, where

$$L^I_{SPVA} = \frac{1}{4N_c\Lambda^2}\sum_{k,l=1}^{2}\{a_{kl}[\bar{q}f_kq\cdot\bar{q}f_lq + \bar{q}f_ki\gamma_5q\cdot\bar{q}f_li\gamma_5q]$$

$$+b_{kl}[\bar{q}f_ki\gamma_\mu q\cdot\bar{q}f_li\gamma_\mu q + \bar{q}f_ki\gamma_\mu\gamma_5q\cdot\bar{q}f_li\gamma_\mu\gamma_5q]\} \qquad (2)$$

and a_{kl}, b_{kl} represents here a symmetric matrixes of real coupling constants and f_k are formfactors. We will restrict ourselves by the case $n = 2$ and describe the ground meson states and their first excitations only. This model interpolates the low-energy QCD action it is supplied with a cutoff Λ (of order of the CSB scale) for virtual quark momenta in quark loops. For simplicity we neglect the isospin effects encoded in quarks of different flavor, $N_f = 1$. Moreover the chiral limit $m_q = 0$ is implied throughout.

Following the standard procedure we introduce auxiliary scalar, pseudoscalar, vector and axial-vector fields,

$$L^I_{aux} = \sum_{k=1}^{2} i\bar{q}(\sigma_k + i\gamma_5\pi_k + i\rho_{k,\mu}\gamma_\mu + ia_{k,\mu}\gamma_\mu\gamma_5)f_kq$$

$$+N_c\Lambda^2\sum_{k,l=1}^{2}\{\sigma_ka^{-1}_{kl}\sigma_l + \pi_ka^{-1}_{kl}\pi_l + \rho_{k,\mu}b^{-1}_{kl}\rho_{l,\mu} + a_{k,\mu}b^{-1}_{kl}a_{l,\mu}\}. \qquad (3)$$

The observables should not depend on the cutoff Λ. The scale invariance is achieved by appropriate prescription of cutoff dependence for effective coupling constants a_{kl}, b_{kl}. Namely, we require the cancellation of quadratic divergences and parametrize the matrices of coupling constants in the vicinity of polycritical point as follows: $8\pi^2a^{-1}_{kl} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}$; $16\pi^2b^{-1}_{kl} = \delta_{kl} -$

$\frac{4}{3}\frac{\Delta_{kl}}{\Lambda^2}$; $\Delta_{kl}, \bar{\Delta}_{kl} \ll \Lambda^2$ The last inequalities provide the masses to be essentially less then the cutoff.

For strong four-fermion coupling constants $a_{kl} \geq 8\pi^2\delta_{kl}$ in the scalar channel the above interaction induces the dynamical chiral symmetry breaking. The parameters Δ_{kl} just describe the deviation from a critical point and determine the physical mass of scalar mesons. The CSB is generated by the dynamic quark mass function corresponding to nontrivial v.e.v. of scalar fields σ_1, σ_2, $M(\tau) = \sigma_1 f_1(\tau) + \sigma_2 f_1(\tau)$; $M_0 \equiv M(0) = 2\sigma_1$; $\tau \equiv -\frac{\partial^2}{\Lambda^2}$.

When integrating out the quark fields one comes to the bosonic effective action where the quadratic parts in boson field fluctuations are retained only to describe the meson mass spectrum,

$$S_{eff} \simeq \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\sum_{k,l=1}^{2}[\sigma_k\Gamma_{kl}^{\sigma\sigma}\sigma_l+\pi_k\Gamma_{kl}^{\pi\pi}\pi_l+\rho_{k\mu}\Gamma_{kl}^{\rho\rho\mu\nu}\rho_{l\nu}+a_{k\mu}\Gamma_{kl}^{aa\mu\nu}a_{l\nu}+2\pi_k\Gamma_{kl}^{\pi a\mu}a_{l\mu}].$$

Herein Γ_i (except the last one - $\Gamma^{\pi a}$, which corresponds to the $\pi - a$ mixing) have the structure: $\hat{\Gamma} = \hat{A}p^2 + \hat{B}$, where the two symmetric matrices - the kinetic term $\hat{A}$ and the momentum independent part $\hat{B}$ - have been introduced. Several comments are in order.

(i) We neglect terms $O(\frac{1}{\Lambda^2})$ in all calculations.

(ii) The large-log approximation $\ln\frac{\Lambda^2}{M_0^2} \gg 1$ is adopted, where M_0 denotes the dynamical mass at zero external momentum. It is compatible with confinement.

(iii) In order that the physical masses were insensitive to Λ, the effective coupling constants should be weakly dependent on the cutoff, $\Delta_{kl}, \bar{\Delta}_{kl} \sim M_0^2 \ln\frac{\Lambda^2}{M_0^2}$.

The physical mass spectrum can be found from solutions of the corresponding secular equation, $det(\hat{A}p^2 + \hat{B}) = 0$; $m_{phys}^2 = -p_0^2$. Let us display the mass-spectrum for ground meson states and their first excitations. We introduce the notations, $\sigma^2 \equiv \sigma_1^2 + \frac{2\sqrt{3}}{3}\sigma_1\sigma_2 + 3\sigma_2^2 > 0$, $d \equiv 3\bar{\Delta}_{11} + 2\sqrt{3}\bar{\Delta}_{12} + \bar{\Delta}_{22}$, and take into account the consistency inequalities $\Delta_{22} < 0, \bar{\Delta}_{22} < 0$. Spectrum for scalar and pseudoscalar mesons are: $m_\sigma = 4\sigma_1 = 2M_0$; $m_\pi = 0$; $m_{\pi'}^2 \simeq -\frac{4}{3}\Delta_{22} + \sigma^2$; $m_{\sigma'}^2 - m_{\pi'}^2 \simeq 2\sigma^2 > 0$. Spectrum for vector and axial-vector mesons are: $m_\rho^2 \simeq -\frac{\det\bar{\Delta}_{kl}}{2\bar{\Delta}_{22}\ln\frac{\Lambda^2}{M_0^2}}$; $m_a^2 \simeq m_\rho^2 + 6M_0^2$; $m_{\rho'}^2 \simeq -\frac{4}{3}\bar{\Delta}_{22} - \frac{d}{6\ln\frac{\Lambda^2}{M_0^2}} - m_\rho^2$; $m_{a'}^2 - m_{\rho'}^2 \simeq \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2) \simeq 3\sigma^2 > 0$. The prime labels everywhere the corresponding excited meson state.

We identify σ with $f_0(400-1200)$, σ' with $f_0(1370)$, π' with $\pi(1300)$, ρ with $\rho(770)$, ρ' with $\rho(1450)$ and a with $a_1(1260)$ [1]. The first excitation of the a_1 - the particle a_1' - is not found yet. This second axial-vector state, perhaps, could be found from hadronic τ decays, although there are the strong phase space limitations [1].The experimental data give us: $m_\sigma = 400 \div 1200$Mev; $m_{\sigma'} = 1200 \div 1500$MeV; $m_{\pi'} = 1300 \pm 100$ MeV; $m_\rho = 770 \pm 0.8$ MeV; $m_{\rho'} = 1465 \pm 25$ MeV; $m_{a_1} = 1230 \pm 40$ MeV. The prediction for the mass of σ-meson is then $m_\sigma \simeq 800$ MeV, which is close to the averaged experimental value. Furthermore we have the following prediction for the mass of a_1'-particle, $m_{a_1'} \cong 1465 \div 1850$ MeV. The large range for a possible mass of a_1'-meson is accounted for by a big experimental uncertainty for the mass of σ' and π' mesons. If we accept the averaged values for them, then $m_{a_1'} - m_{\rho'} \approx 30$ MeV.

3 Chiral Symmetry Restoration Sum Rules

Let us exploit the constraints based on chiral symmetry restoration at QCD at high energies. Expanding the meson correlators (1) in powers of p^2 one arrives to the CSR Sum Rules. In the scalar-pseudoscalar case (1) they read: $\sum_n Z_n^S - \sum_n Z_n^P = 0$; $\sum_n Z_n^S m_{S,n}^2 - \sum_n Z_n^P m_{P,n}^2 = \Delta_{SP}$, and in the vector-axial-vector (1) one obtains: $\sum_n Z_n^V - \sum_n Z_n^A = 4f_\pi^2; \sum_n Z_n^V m_{V,n}^2 - \sum_n Z_n^A m_{A,n}^2 = 0$, $\sum_n Z_n^V m_{V,n}^4 - \sum_n Z_n^A m_{A,n}^4 = \Delta_{VA}$. The first two relations are the famous Weinberg Sum Rules, with f_π being the pion decay constant. The residues in resonance pole contributions in the vector and axial-vector correlators have the structure, $Z_n^{(V,A)} = 4f_{(V,A),n}^2 m_{(V,A),n}^2$, with $f_{(V,A),n}$ being defined as corresponding decay constants.

In the scalar-pseudoscalar case it has been obtained [12, 15] that the residues in poles are of different order of magnitude; the second CSR Sum Rules constraint results in the estimation for splitting between the σ'- and π'-meson masses: $m_{\sigma'}^2 - m_{\pi'}^2 \simeq \frac{1}{6}m_\sigma^2$; are calculated and its value $L_8 = (0.9 \pm 0.4) \cdot 10^{-3}$ from [16] accepts $m_\sigma \simeq 800$ MeV.

In the vector-axial-vector case all residues are found to be of the same order of magnitude in contrast to the scalar-pseudoscalar channel [18]. The first and the second Sum Rules are fulfilled identically in the large-log approach. The third one takes the form: $Z_1(m_{a_1'}^2 - m_{\rho'}^2) \simeq 16\pi\alpha_s < \bar{q}q >^2$. The structure of $Z_{\rho'}$ and $Z_{a_1'}$ shows that if $m_{a_1'} \simeq m_{\rho'}$ then $Z_{a_1'} \simeq Z_{\rho'}$ and therefore $f_{a_1'} \simeq f_{\rho'}$. As a consequence these residues approximately cancel each other in Sum Rules and after evaluating we get $f_\rho \approx 0.15$ and $f_a \approx 0.06$ to be compared with the experimental values [16] $f_\rho = 0.20 \pm 0.01$, $f_a = 0.10 \pm 0.02$. We have also a reasonable prediction for the chiral constant L_{10} for the ρ, a_1-mesons and their first exitations (n=2) one gets $L_{10} =\approx -6.0 \cdot 10^{-3}$, which is consistent with that [16] from hadronic τ decays: $L_{10} = -(6.36 \pm 0.09) \mid_{expt} \pm 0.16 \mid_{theor}) \cdot 10^{-3}$. It is worth to mention also that within the four-resonance ansatz (n=2) and using two first Weinberg sum rules one obtains the estimation of electromagnetic pion-mass difference $\Delta m_\pi^4 \mid_{em} \simeq (3.85 \pm 0.16)$ MeV, (see [17]) which improves the agreement between theoretical predictions and the experimental value of $\Delta m_\pi \mid_{em}^{expt} \simeq (4.42 \pm 0.03)$ MeV.

4 Summary

We have shown that the QNJL model truncating low energy QCD effective action in VASP-sector can serve to the interpolation of more complicated gauge theory of QCD-type in hadronization regime. They contain the sufficient set of phenomenological coupling constants to describe an infinite spectrum of resonances with the same quantum numbers in vicinity of polycritical point. The QQM yield a broad set of the mass-splitting relations for the ground and excited VASP-meson states nealy independent of model parameters.

The matching to non-perturbative QCD (via OPE) based on Chiral Symmetry Restoration at high energies enhances the predictability of such models for the VASP-mass spectrum. (the agreement with the experiment for $SU(2)$ sector [17] is within 4% for P-mesons, $\leq$ 30% for S-mesons, $<$ 11% for V-mesons and $<$ 19% for A-mesons;). The CSR Sum Rules are well saturated by four resonances. Let us summarize main results presented in this talk.

(i) The mass of the second axial-vector particle with $I = 1$ is predicted. It is comparable with the mass of the vector counterpartner: $m_{a_1'} = 1465 \div 1850 MeV$ and the most plausible value of the mass difference is $m_{a_1'} - m_{\rho'} \approx 30 MeV$;

(ii) The estimation on the mass of the σ-meson does not contradict to existing experimental data [1]: $m_\sigma \simeq 800$ MeV;

(iii) The couplings f_ρ, f_a and the chiral constant L_{10} as well as the electromagnetic pion-mass difference $\Delta m_\pi^4 \mid_{em}$ [17] are evaluated from CSR Sum Rules as matching rules for QNJLM to QCD at intermediate energies.

Finally we would like to mention possible applications of QNJLM. Firstly, such models are thought of as relevant for investigations of behaviour of hadron matter at high temperatures and nuclear densities in the region near the restoration of chiral symmetry. One could expect that for increasing quark densities the mass-splitting is collapsing and therefore excitations become lighter and more important in hadron kinetics. Secondly, that the QNJL Models can be used to describe Higgs particles in extensions of the Standard Model. The QNJL Models of such extension can be found see [15, 19].

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