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OPTIMIZED NON RELATIVISTIC POTENTIAL FOR QUARKONIA

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Abstract:

For non relativistic quarkonia description, we consider a wide class of quark antiquark potentials in the form of power law. A systematic study is made by optimizing the potential parameters with a fit on quarkonia vector mesons that lie below the threshold for strong decays. Implications of the obtained results are discussed.

Keywords: *potential model, quarkonia, and spectroscopy.*

1. INTRODUCTION

The copious production of b quarks in Z^0 decays at the Large Electron-Positron (LEP) collider and in 1.8 TeV proton-antiproton collisions at the Fermilab Tevatron opens for study the rich spectroscopy of mesons and baryons containing b and c quarks. Furthermore, the recent experimental observation [1-5] of the $B_C^{\pm}$ meson has inspired new phenomenological and theoretical interest in the quark antiquark potential and in the quarkonia spectroscopy (see for example [6-19]).

In the last two decades, dozens of non relativistic and relativistic potential models have been proposed in the literature (see [20-21] and references therein)[†]. The quark antiquark potential is a key ingredient for the phenomenological description of hadronic observables and other quantities related to the phenomenology of strong interactions like: the $b\bar{c}$ mesons [22-24], the toponium production [25], the heavy quark production rate [26-27], *etc.* It is thus of great importance to determine, at least, a phenomenological effective potential which can reproduces quantitatively the data with its experimental errors.

The aim of the present work is to determine, in the framework of the Non Relativistic (NR) potential model approach, an effective phenomenological interquark potential.

We consider, following Motyka and Zalewski [28], a wide class of quark antiquark potentials in the form:

$$V(r) = -\alpha\, r^{-a} + \sigma\, r^{b} - U_0 \quad a,b \geq 0 \tag{1}$$

The potential parameters are optimized by a χ^2 test on quarkonia vector mesons that lie below the threshold of strong decay. We discuss the implications and the limitations of the obtained potential.

2. MODEL AND RESULTS

In the potential model approach, the *constituent* quark masses and the form of the potential vary from paper to paper [20-21]. Moreover, this feature is independent of the wave equation used: non relativistic, semi relativistic or relativistic [20-21]. In fact, for a given form of the interquark potential, the range of variation of the *constituent* quark masses is related to the determination of the overall additive constant, like U_0 in equation (1). The method used to fix this overall constant varies from paper to paper [20-21]. Indeed, the overall constant U_0 defines the zero of the potential and is related to the quark masses which are adjustable parameters in the potential model approach. It is partly for this reason, and also partly because different wave equations are used, that various potentials differ somewhat in the region: $0.1 \le r \le 1\ fm$, which is the most important region for quark antiquark bound states spectroscopy [20-21]. It is well known that potentials differ in this interval mostly by an overall constant which arises from different choices of quark masses. To a certain extent, *for heavy mesons*, we can compensate a change in quark masses by a change of the constant term in the potential [20-21]. However, if one wants a potential which is also applicable to heavy-light and light-light mesons, one has *less latitude to adjust this constant term*. Indeed, the light quark masses are more constrained than those of the heavy quarks, partly because a given change in mass is a larger percentage change for the light quarks and partly because one wants to be able to calculate baryon magnetic moments with *the same constituent quark masses used to calculate meson spectra*.

In this work, following the above described ideas, the overall additive constant U_0 *is not a free parameter*. Indeed, it is rather fixed by imposing that our set of parameters reproduces the value of the ground state energy for the ρ meson with the constituent masses of u and d quarks $m_u = m_d = 336\,\text{MeV}$, deduced from the well established experimental value of the magnetic moment of the proton [29][†]. Thus the constant U_0 is obtained as an implicit function of the parameters of the potential (1).

Let us make few comments about our method to fix the constant U_0. One can worry about the use of a *NR approach* to reproduce the ρ meson. In fact, if one restricts the discussion to *a few excited states of mesons containing light quarks*, a NR approach can be justified theoretically [20-21, 30]. Indeed, a semi relativistic or relativistic Hamiltonian can be interpreted as an effective NR Hamiltonian with dressed or effective parameters which are assumed to take into account the relativistic effects [20-21]. For example, in the case of the spinless Salpeter wave equation, the semi relativistic Hamiltonian can be described as a NR Hamiltonian one with quark masses being *level dependent* [30]. For heavy quarks, this dependence is very weak and the semi relativistic description is equivalent to a NR one with level independent heavy quark masses [20-21, 30]. For light quark masses, one has to restrict the NR description to a few excited quark antiquark bound states, otherwise the NR description becomes meaningless.

In the present paper, in the framework of the NR potential model approach, we have considered the interquark interaction in the form given by equation (1). This choice is guided by the fact that it is the simplest parameterization of the potential. Moreover, various models proposed by previous works [28, 31-37], correspond to the general form (1) with specific values of the parameters a and b. Among them, the famous and useful Cornell model, which incorporates the two main features of QCD, namely asymptotic freedom and linear confinement [31], has $a = b = 1$.

[†] As far as experimental data are concerned, we refer to the Particle Data Group Tables 2004 [29].

The free parameters of our model are the c and b quark masses, the parameters of the potential (1), excepted the overall constant U_0 which, as explained above, depends on α, σ, a and b. These parameters are fixed by minimizing the quantity:

$$\chi^2 = \frac{1}{5} \sum_j \left(\frac{M_j^{th} - M_j^{exp}}{\sigma_j + \delta_{th}} \right)^2 \quad (2)$$

where the indices j runs over the $b\bar{b}$ and $c\bar{c}$ *vector meson states* that lie below the threshold of strong decay: $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, J/Ψ and Ψ'. The constants σ_j represent, for a given meson j, the experimental mass error given by the PDG data table [29]. The notation δ_{th} stands for a theoretical error introduced by the numerical resolution of the Schrödinger equation. For all mesons, we take $\delta_{th} = 0.1\ MeV$ †.

The choice of observables in the fit requires some comments. We consider vector meson states instead of the spin averaged S states, since the pseudo scalar mesons η_b of the Υ family have not been yet observed [29]. We have not included quarkonia with masses above the threshold for strong decays. Indeed, their analysis would require coupled channel calculations which are beyond the scope of the present paper. Furthermore, many works (see for example [28]) include in the fit the leptonic decay widths of vector mesons. These quantities are related to the wave functions at the origin $\psi(0)^2$ through the Van Royen-Weisskopf formula with radiative corrections (see [21] and references therein). The factor taking into account the radiative corrections is not free of theoretical ambiguities [21, 28], even with the recent second order calculation [38]. In addition, from the phenomenological point of view, the experimental knowledge of quarkonia leptonic decay widths is given with a precision of about *2 % (in the most favorable case)* to be compared to the experimental knowledge of quarkonia masses given with a precision of about 0.005 *% (in the worst case)* [29]. For all these reasons, in this work, we have ignored in the fit the leptonic decay widths of vector mesons.

The results given by the fit, for the potential (1), are displayed in Table 1. We have also considered various potentials given in the literature [28, 31-37] which are particular cases of the general form (1). Using our method to fix the U_0 constant, we have optimized their parameters on 2004 data [29]. The results of the χ^2 test, for these potentials, are displayed in Table 2.

Let us make some comments about technical details concerning the fit procedure. For our potential (1), we have explored the region:

$$0 \le a \le 2 \quad and \quad 0 \le b \le 2 \quad (3)$$

In this work, we have used the Powell algorithm [39] to minimize the quantity (2), rather than the usual, in a χ^2 test problem, Levenberg-Marquardt algorithm [39]. Moreover, with the Powell algorithm, we have been able to implement the constraint (3) in the following way. If the parameters a and b lie in the region of interest (3), the quantity χ^2 is defined through equation (2), otherwise χ^2 is considered as infinity. The drawback of the Powell method (like the Levenberg-Marquardt one) lies in the fact that one finds a *local minimum*. Strictly

† We can increase the accuracy of the numerical resolution of the Schrödinger equation and reduce the theoretical error to *0.01 MeV*. However, this accuracy is too CPU time consuming and does not change significantly the results obtained.

speaking, the results displayed in Table 1, do not correspond *necessarily to a global minimum* of the χ^2 quantity in the region (3). Indeed, if one changes the initial values of *a* and *b*, one can obtain an other set of parameters with a χ^2 of the *same order of magnitude* ($\chi^2 << 1$). Moreover, note that *for a and b fixed*, the results of the fit are *not sensitive* to the initial values given to the others parameters.

In this work, we have adopted a practical point of view: we have reported in Table 1, the results corresponding to *only one local minimum*. Indeed, the results of fit do not change significantly. *The main result*, is that, the parameters *a* and *b* are *necessarily strictly smaller than one*. This feature means that the quarkonia spectroscopy can not "see" both the *1/r* behavior corresponding to one gluon exchange and the linear confinement part of the potential. Note that this property can be, also, seen from Table 2.

The order of magnitude of the χ^2 quantity, we obtained in this work (Table 1) comparatively to previous potentials (Table 2), indicates clearly the quality of our results.

Table 1. Parameters (in GeV units) obtained by fit for the potential (1) with the corresponding χ^2, the U_0 constant being fixed on the ρ meson mass with $m_u = m_d = 336\,\text{MeV}$.

a	b	m_b	m_c	α	σ	U_0	χ^2
0.501941	0.694158	5.176971	1.807397	0.93722	0.311722	0.5278999	$1.5\ 10^{-8}$

Table 2. Parameters (in GeV units) obtained by fit for previous potentials [28, 31-37] with the corresponding χ^2, the U_0 constant being fixed on the ρ meson mass with $m_u = m_d = 336\,\text{MeV}$.

Model	m_b	m_c	α	σ	U_0	χ^2
Ref. [28] *a=1 ; b=1/2*	5.180532	1.807564	0.2978829	0.7389483	1.583837	3.58
Ref. [31] *a=b=1*	5.195055	1.8127684	04681625	01904476	0.8459376	48.98
Ref. [32] *a=b=3/4*	5.182712	1.809108	0.5916824	0.3145243	0.866481	5.45
Ref. [33] *a=b=1/2*	5.173898	1.806058	0.7711087	0.5965644	0.98179639	1.05
Ref. [34] *a=b=2/3*	5.179384	1.808002	0.6422009	0.3813370	0.88774854	1.13
Ref. [35] *a=0 ; b=0.1*	5.166204	1.803314	/	6.714499	7.8781629	24.22
Ref. [36] *a=0.045 ; b=0*	5.17464	1.804126	16.82645	/	-15.651932	1049
Ref. [37] *a=1 ; b=2/3*	5.185582	1.809243	0.3688667	0.4419155	1.2085480	14.32

3. CONCLUSION

In this work, in the framework of the NR potential model approach, we have considered the quark antiquark interaction in the general form given by equation (1). The constant term U_0 has not been taken as a free parameter or taken arbitrarily equal to zero. We rather fix this constant term by imposing that our potential reproduces the ρ meson mass with $m_u = m_d = 336\,\text{MeV}$.

We have explored the quality of the fit in the region (3). From the phenomenological point of view, we can conclude that is possible to reproduce, ***with a very high accuracy***, the quarkonia spectra in the framework of the NR potential model approach. Furthermore, we obtain that the optimized values of the power parameters *a* and *b* are ***necessarily strictly smaller than unity***.

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