EG0700172

Ninth Degree Polynomial Fit Function for Calculation of Efficiency Calibrations for Ge(Li) and HPGe Detectors

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ABSTRACT

A new 9th degree polynomial fit function has been constructed to calculate the absolute γ-ray detection efficiencies (η_{Th}) of Ge(Li) and HPGe Detectors, for calculating the absolute efficiency at any interesting γ–energy in the energy range between 25 and 2000 keV and distance between 6 and 148 cm. The total absolute γ -ray detection efficiencies have been calculated for six detectors, three of them are Ge(Li) and three HPGe at different distances. The absolute efficiency of the different detectors was calculated at the specific energy of the standard sources for each measuring distances. In this calculation, experimental (η_{Exp}) and fitting (η_{fit}) efficiency have been calculated. Seven calibrated point sources Am-241, Ba-133, Co-57, Co-60, Cs-137, Eu-152 and Ra-226 were used. The uncertainties of efficiency calibration have been calculated also for quality control. The measured (η_{Exp}) and (η_{fit}) calculated efficiency values were compared with efficiency, which calculated, by Gray fit function (η_{fitG}). The results obtained on the basis of (η_{Exp}) and (η_{fit}) seem to be in very good agreement.

Keywords: *Gamma Ray, Detection Efficiency,* Ge(Li) and HPGe Detectors

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INTRODUTION

It is traditional in laboratory applications to calibrate gamma spectroscopy detectors in terms of absolute efficiency. Mathematical calibrations have been successfully performed in the past by several methods. Some mathematical formulas were used to compute the intrinsic response of the detector. Many Monte Carlo techniques exist but these are complex.

To calculate the activities of gamma- and X- ray emitters from a spectrum measured, the counting efficiency must be known. Usually its uncertainty is the main factor influencing the uncertainty of the calculated activity. Therefore the measurement and/or calculation of the counting efficiency must be done with the greatest care and outmost precision.

This efficiency depends on detector properties, sample properties and the relative sample-detector position. For any detector the efficiency is a function of the sample shape, size, composition, the distance of the sample from the detector and the photon energy. In the spectrum all the photons that interact with the material within the sensitive volume of the detector are registered. Since radioactive atoms emit a discrete spectrum of photons, peaks in the spectrum occur at these photon energies due to interactions leading to full absorption of the photon energy within the sensitive volume.

When a point source of radiation is located on the extended axis of a detector, a mathematical expression [1] for calculating the total absolute detection efficiency as in equation :

$$\eta_{Exp} = \frac{N_p \cdot 100}{I_\gamma[\%]} \cdot \frac{1}{A_{ref} \cdot e^{-\lambda \cdot t_d}} \text{ ....(1)}$$

with: N_p = net peak area at E_γ I_γ = intensity of emitted γ-ray,
A_{ref} = activity of the standard source, t_d = decay time.
λ = decay constant for γ-ray

The absolute efficiency of the different detectors was calculated at the specific energy of the standard sources for each measuring distances. But, we need some fitting function to calculate the absolute efficiency for any considered γ-energy. We used for this purpose a new fit function for calculating the absolute efficiency at any interesting γ-energy in the energy range between 25 and 2000 KeV and detector-sample distance between 6 and 148 cm.

Efficiency function was obtained by applying 9th degree polynomial equation to the experimental efficiency data, which is in the next form:

$$\eta_{Th} = (a + bE_\gamma + cE_\gamma^2 + dE_\gamma^3 + eE_\gamma^4 + fE_\gamma^5 + gE_\gamma^6 + hE_\gamma^7 + iE_\gamma^8 + jE_\gamma^9) \text{ ....(2)}$$

Where E_γ represents energy in KeV, and (*a ,b , c, d, e, f ,g ,h ,i and j)are* coefficient data. By Eq. (2), we can determine the absolute efficiency, η_{Th}, at any specific energy E_γ if we know the energies and coefficient data. The efficiencies were repeatedly checked for every detector, and from the experimental efficiency curves, the coefficient data were determined at each distance for every detector by using a curve fitting system so-called (Curve Expert 1.34) [2].

The Gray and Ahmad fit function

Efficiency functions were obtained by fitting a double-logarithmic linear function to the experimental efficiency data above 300 KeV. Below this energy a fit function according to Gray and Ahmad [3] was chosen. These authors constructed two linear classes of Ge(Li) detector efficiency functions and assessed their range of applicability using a large number of independent efficiency data sets. They use that class is defined by,

$$Eff = \frac{1}{E_\gamma} \sum_{i=1}^{8} a_i \cdot \left(\ln \frac{E_\gamma}{E_0} \right)^{i-1} \text{ .... (3)}$$

Where E_γ represents energy in MeV,
E_0 is the energy of calibrated point source, and
a_i are Gray-Parameters.

THE UNCERTAINTY OF EFFICINENCY CALIBRATION

If a quantity Y being measured, called the measured, often is not measured directly, but is determined from N other quantities $X_1, X_2, \ldots, X_N$ through a functional relation f, often called the measurement [4] as shown in Eq.(4),

$$Y = f(X_1, X_2, \ldots, X_N) \text{ ....(4)}$$

Included among the quantities X_i are corrections or correction factors as well as quantities that take into account other sources of variability, such as different observers, instruments, samples, laboratories, and times at which observations are made, e.g. different days of production and measurements. Thus, the function f of Eq.(4) should express not simply a physical law but a measurement process, and in particular, it should contain all quantities that can contribute significantly to the uncertainty of the measurement result.

An estimate of the measured or *output quantity* Y, denoted by y, is obtained from Eq. (4) Using *input estimates* $x_1, x_2, \ldots, x_N$ for the values of the N *input quantities* $X_1, X_2, \ldots, X_N$. Thus, the *output estimate* y, which is the result of the measurement, is given by,

$$y = f(x_1, x_2, \ldots, x_N) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The combined standard uncertainty of y is then given by,

$$u^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial X_i}\right)^2 \cdot u^2(x_i) \;\ldots\ldots\ldots\ldots\ldots(6)$$

The absolute efficiency was calculated from next equation,

$$Eff = \frac{100 \cdot N_p}{I_\gamma \cdot TOC \cdot A_{BOC}} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

with: N_p= net peak area at E_γ, I_γ = intensity of emitted γ-ray, TOC = time of counting and A_{BOC} = activity of the standard source at the start of counting ,which calculated by Eq.(8)

$$A_{BOC} = A_{DOC} \cdot \exp(-\lambda \,.\, (BOC\text{-}DOC)) \ldots\ldots\ldots\ldots(8)$$

Where A_{DOC} is the activity of the standard source at date of calibration DOC, and λ is the decay constant.

The combined standard uncertainty of absolute efficiency $u(EFF)$ is consists of $u(N_p)$, $u(I_\gamma)$, $u(TOC)$ and $u(A_{BOC})$ so,

$$\left[\frac{u(EFF)}{EFF}\right]^2 = \left[\frac{u(N_p)}{N_p}\right]^2 + \left[\frac{u(I_\gamma)}{I_\gamma}\right]^2 + \left[\frac{u(TOC)}{TOC}\right]^2 + \left[\frac{u(A_{BOC})}{A_{BOC}}\right]^2 \;\ldots\ldots\ldots\ldots(9)$$

Because of $u(TOC) \ll TOC$, we neglected $u(TOC)$,while $u(A_{BOC})$ was calculated by Eq. (10)

$$\left[\frac{u(A_{BOC})}{A_{BOC}}\right]^2 = \left[\frac{u(A_{DOC})}{A_{DOC}}\right]^2 + (BOC - DOC)^2 \cdot u^2(\lambda) \;\ldots\ldots\ldots(10)$$

SET- UP, CALIBRATION, AND DATA COLLECTION

The measurements were performed using six detectors [indicated with GeLi (PW), GELIU4, GELIU2, GE_02, GE_NEU and GE_01] three of them germanium-lithium (Ge (Li)) with aluminum cap , two high purity germanium (HPGe) with aluminium cap and one (HPGe) with beryllium cap. The properties and measuring distances for all used detectors used are shown in table (1).

Table (1): The properties, shields and measuring distances for used detectors.

GeLi(PW)	Ge(Li) planar	1.9/2.4	Al	7. 5 to 148
GELIU4	Ge(Li) planar	0.9/2.0	Al	6 and 11
GE_02	HPGe planar	0.9/1.9	Al	6 and 11
GE_NEU	HPGe planar	1.2/2.1	Al	6 and 11
GELIU2	Ge(Li) planar	2.6/3.7	Al	6 and 11
GE_01	HPGe planar	0.8/1.8	Be	6 and 11

Each detector was connected via pre- and spectroscopy amplifiers to computer-controlled multi-channel buffers whose built-in ADCs digitized the pulses into spectra of usually 4096 channels as shown in Fig. (1). The amplification was chosen for the registration of γ-quanta with energies between some tens of keV and about 2 MeV. Typical resolutions ranged from about 1 KeV at 122 KeV of ^{57}Co to about 2 KeV at 1332 KeV of ^{60}Co.

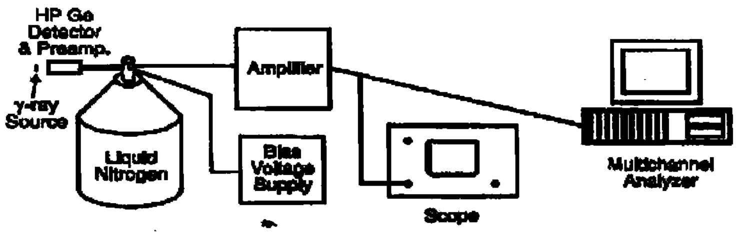


Fig: (1) Germanium detector and associated counting equipment.

In order to determine the peak efficiencies of each detector, several gamma-spectra were collected for several counting periods. Each detector was connected to an MCA and was used to collect long count spectra three hours from each source at each distance. By ^{241}Am, ^{133}Ba, ^{137}Cs, ^{60}Co, and ^{226}Ra with certified accuracies of ≤ 2 % point sources, the MCA was energy calibrated using the 59.54 KeV, 356 KeV, 661.7 KeV, 1172 KeV , 1332.5 KeV and 1764.49 KeV gamma-ray peaks of these sources. After separately calibrating each of the detectors, the face of each detector was positioned against the surface of point sources to the alignment marks, to collect the spectra as shown in fig (2).

Measurements were performed with calibrated point source samples (standard sources), which contain a known activity of one or more gamma-ray emitters of the radionuclides ^{241}Am, ^{133}Ba, ^{57}Co, ^{60}Co, ^{137}Cs, ^{152}Eu, and ^{226}Ra with certified accuracies of ≤ 2 % (PTB Braunschweig, Germany). We used the commercially available code Gamma-W [5,6] to analyze the γ-spectra. This code evaluates the γ–spectra and provides more information about the spectra, specially the energies, net peak areas (Np) and uncertainty of net peak areas $u(N_p)$

for all peaks in the spectra, while $u(\lambda)$ and $u(I_\gamma)$ were taken from the compilation of Reuss and Westmeier [7].

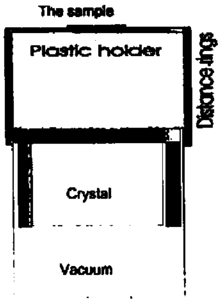


Fig. (2): The position of the sample in front of the detector

The absolute efficiency of the different detectors was calculated at the specific energy of the standard sources for each measuring distances. We used for this purpose the so-called Gray and Ahmad fit function [3] for calculating the absolute efficiency at any interesting γ–energy.

By Eq. (2), we can determine the absolute efficiency, ε_γ, at any specific energy E_γ if we know the energies and efficiencies of the calibrated point source (E_0) and the coefficient data. The efficiencies were checked for each geometry for every detector, and from the efficiency curves, the coefficient data were determined for each geometry for every detector.

In order to determine the peak efficiencies of each detector, several gamma-spectra were collected for several counting periods. Each detector was connected to an MCA and was used to collect long count spectra three hours from each source at each distance. After separately calibrating each of the detectors, the face of each detector was positioned against the surface of point sources to the alignment marks, to collect the spectra as shown in fig (2).

RESULTS AND DISCUSSIONS

The dependence of the efficiency on the radiation energy was determined at different distances experimentally and by using the Gray Ahmad fit function. From experimental efficiency curves the coefficient data were calculated by (Curve Expert 1.34) program for every detector at different distances. In table (2) the coefficients date for detector GeLi (PW) at different were listed.

Table (2): The coefficients date for detector GeLi (PW) at different distances

Coefficient Data	at 7.5cm	at 20cm	at 40cm	at 60cm	at 100cm	at 148 cm
a	-1.76E-02	-2.28E-03	-7.94E-04	-6.49E-04	-1.75E-04	-5.76E-05
b	5.72E-04	7.26E-05	2.56E-05	1.76E-05	4.96E-06	1.60E-06
c	-4.34E-06	-5.22E-07	-1.92E-07	-1.31E-07	-3.58E-08	-1.00E-08
d	1.60E-08	1.85E-09	7.10E-10	4.87E-10	1.29E-10	3.03E-11
e	-3.36E-11	-3.78E-12	-1.52E-12	-1.04E-12	-2.68E-13	-5.27E-14
f	4.27E-14	4.77E-15	2.00E-15	1.37E-15	3.44E-16	5.63E-17
g	-3.32E-17	-3.77E-18	-1.63E-18	-1.12E-18	-2.76E-19	-3.77E-20
h	1.53E-20	1.81E-21	8.08E-22	5.57E-22	1.35E-22	1.54E-23
i	-3.75E-24	-4.82E-25	-2.22E-25	-1.53E-25	-3.66E-26	-3.52E-27
j	3.68E-28	5.49E-29	2.58E-29	1.78E-29	4.23E-30	3.45E-31

By using these coefficient data and applying the new 9th degree polynomial fit function the efficiency were determined. The relative deviations (R.D.) between fitting (η_{fit}) and the experimental (η_{exp}) efficiency calibration values are calculated as $\{(\eta_{exp} - \eta_{fit}) / \eta_{exp}\}$. The obtained data for detector GeLi (PW) at 7.5 to 148cm and relative deviations (R.D.) presented in % are illustrated in figures (3 and 4).

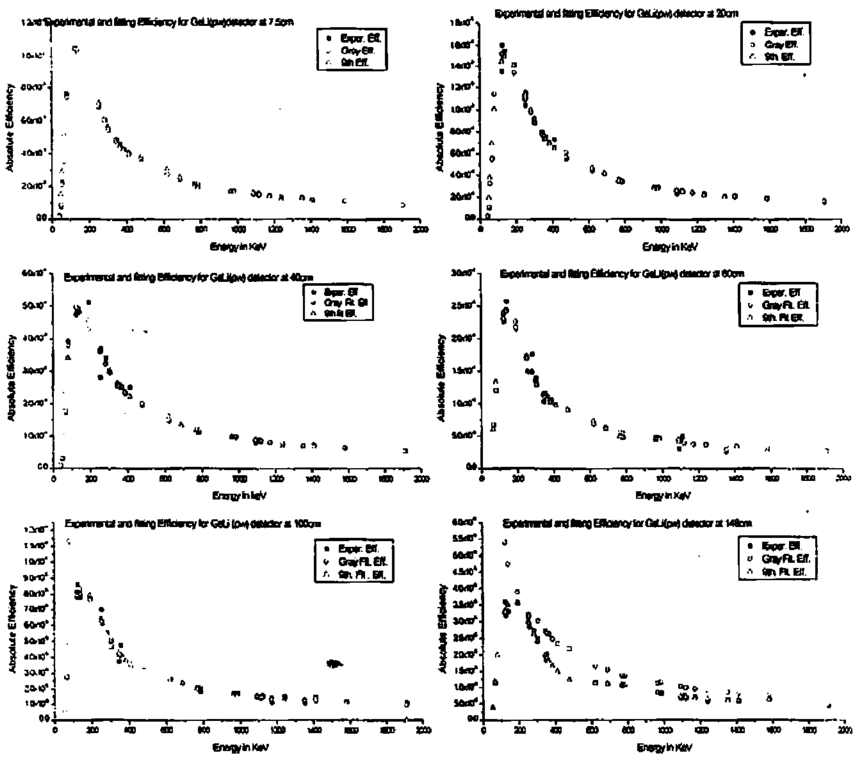
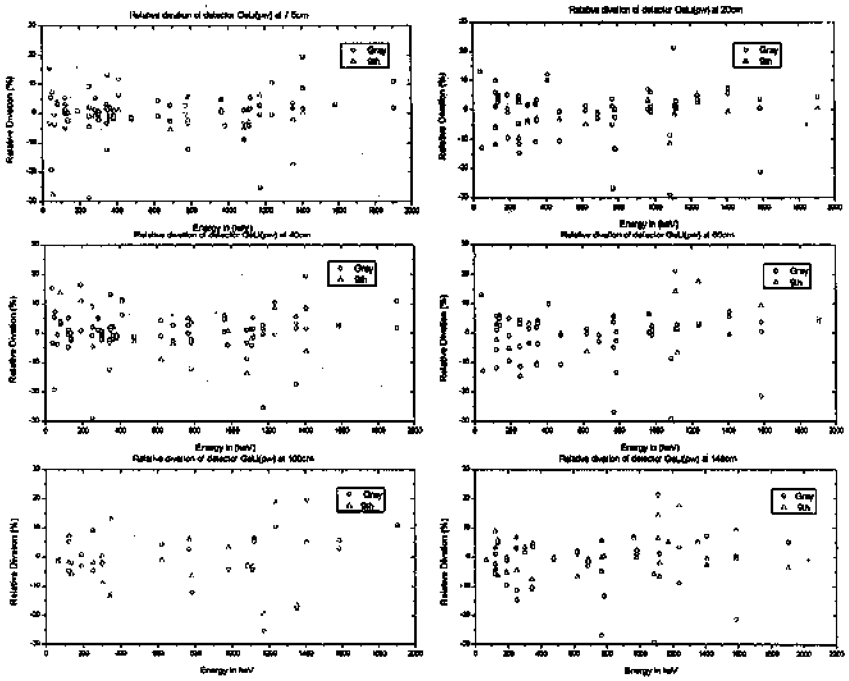


Fig. (3): The absolute efficiency curve for detector GeLi (PW) at different distances.

As it appears in figures (3), the absolute efficiency values were very sensitive to energy at the range 25-300 KeV while they have a smooth change at E > 300 KeV. In these figures one can see a very good agreement between experimental and fitting date (Gray and 9th fit function) at all distances except at distance (148cm) , the 9th fit function is better than Gray fit function.

The relative deviations as it appears in figure (4) are between 1 to 20 % for both Gray fit and 9th fit function for distance 7.5 to 100cm. At distance (148cm) the relative deviations are between 1 to 10% for 9th fit function, while (R.D.) in case of Gray fit function are between 1 to 50%, this give us a sense that the applying of 9th fit function is better than Gray fit function specially at large distances.



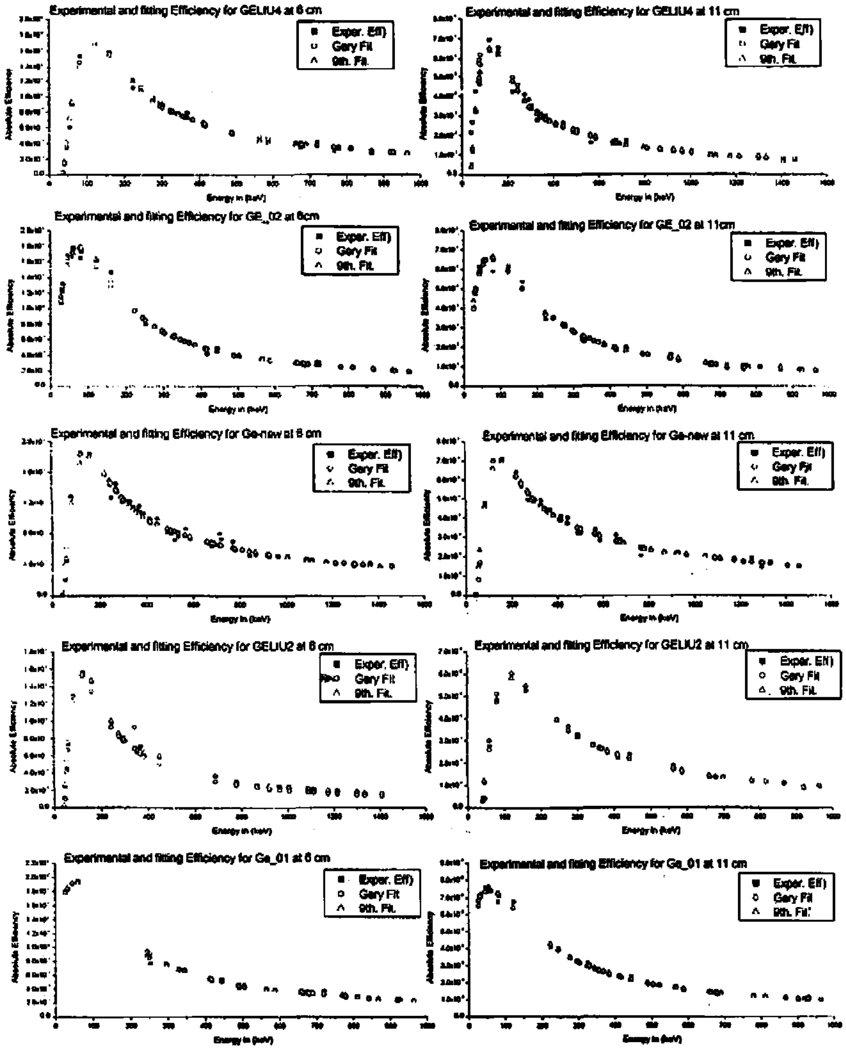
Fig(4):Relative deviations between experimental and fitting efficiency calibration values for detector GeLi(PW) at different distances.

In the next table (3) the coefficients date for GELIU4, GE_02,GE_NEW, GELIU2 and GE_01 detectors were listed.

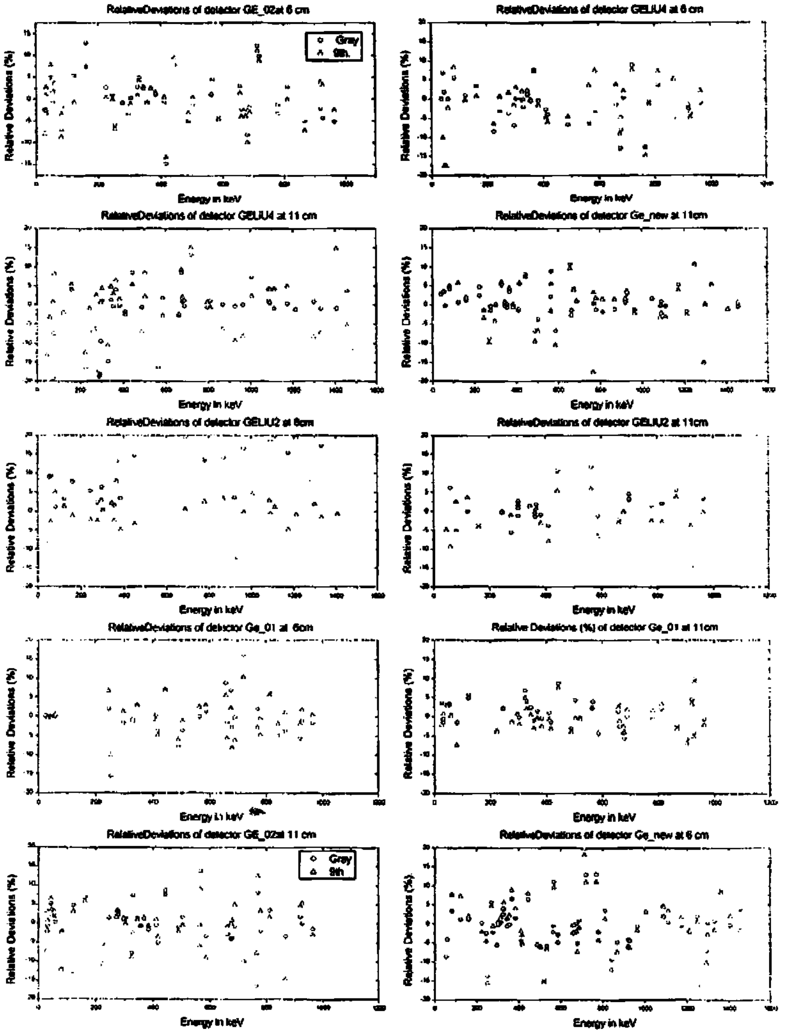
Table (3): The coefficients date for the other five detectors at different distances

Coefficient Data	GELIU4		GE_02		GE_NEU		GELIU2		GE_01	
	at 6 cm	at 11cm	At 6cm	at 11cm	at 6 cm	at 11cm	At 6cm	at 11cm	at 6cm	at 11cm
a	-2.72E-02	-1.06E-02	-6.73E-04	-3.02E-05	-2.87E-02	-1.10E-02	-2.68E-02	-1.06E-02	1.28E-02	5.31E-03
b	1.02E-03	3.89E-04	6.27E-04	2.38E-04	9.10E-04	3.46E-04	9.53E-04	3.95E-04	2.94E-04	9.00E-05
c	-8.55E-06	-3.32E-06	-7.65E-06	-3.10E-06	-6.79E-06	-2.54E-06	-8.03E-06	-3.68E-06	-4.35E-06	-1.27E-06
d	3.29E-08	1.40E-08	4.42E-08	1.95E-08	2.58E-08	9.42E-09	3.32E-08	1.79E-08	2.56E-08	7.33E-09
e	-6.03E-11	-3.42E-11	-1.50E-10	-7.28E-11	-5.81E-11	-2.06E-11	-7.91E-11	-5.29E-11	-8.28E-11	-2.42E-11
f	2.74E-14	5.16E-14	3.19E-13	1.70E-13	8.22E-14	2.79E-14	1.16E-13	1.02E-13	1.61E-13	5.00E-14
g	8.75E-17	-4.86E-17	-4.31E-16	-2.49E-16	-7.38E-17	-2.38E-17	-1.06E-16	-1.29E-16	-1.90E-16	-6.57E-17
h	-1.66E-19	2.80E-20	3.60E-19	2.24E-19	4.09E-20	1.24E-20	5.92E-20	1.04E-19	1.35E-19	5.32E-20
i	1.16E-22	-8.95E-24	-1.69E-22	-1.12E-22	-1.27E-23	-3.63E-24	-1.85E-23	-4.82E-23	-5.15E-23	-2.43E-23
j	-3.02E-26	1.22E-27	3.41E-26	2.39E-26	1.70E-27	4.55E-28	2.47E-27	9.85E-27	8.14E-27	4.78E-27

By using above coefficient data and applying the new 9th degree polynomial fit function the efficiency were determined. The obtained efficiency data for detectors GELIU4, GE_02, GE_new, GELIU2 and GE_01 at distances 6 and 11cm and its relative deviations (R.D.) presented in % are illustrated in the figures (5 and 6).



Fig(5): The absolute efficiency curve for detectors GELIU4, GE_02, GE_NEW, GELIU2 and GE_01 detectors different distances.



Fig(6):Relative deviations between experimental and fitting efficiency calibration values for detectors *GELIU4, GE_02,GE_NEW, GELIU2 and GE_01 detectors different distances.*

In figure (5) one can observe a very good agreement between experimental and fitting date (Gray and 9th fit function) at all distances. The majority values of the relative deviations (R.D.) as it appears in figure (6) are less than 10% for all detectors at different energies and distances, except at some energy it was greater than this value.

CONCLUSIONS

With available data for the total absolute detection efficiencies of point source γ -rays have been calculated (η_{fit}) for (HPGe and GeLi) detectors and compared with measured efficiencies (η_{Exp}). From the result, conclusions are drawn up as follows:

1-The results obtained on the basis of (η_{Exp}) and (η_{fit}) seem to be in very good agreement.

2-We can use this new 9th polynomial fit function to calculate the absolute efficiency for any considered γ–energy at any distances.

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