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PROGRESS REPORT ON REACTOR PHYSICS RESEARCH PROGRAMM JANUARY 1963 - FEBRUARY 1964

BORIS KIDRIČ INSTITUTE
OF
NUCLEAR SCIENCES
NPY-PROJECT

Progress report on reactor physics research program, January 1963 - February 1964

1. Introduction

This progress report is a part of the annual report of the Departement of Reactor Physics, prepared for the Boris Kidrič Institute of Nuclear Sciences. It is a review of research activities in the field of theoretical and experimental reactor physics in the year 1963. The research program was sponsored by the Yugoslav Federal Nuclear Energy Commission, on the basis of fire research contracts. A part of this program was included in the NPY - Cooperative Program in reactor physics.

1.1. Research Program

The research program in reactor physics at the Boris Kidrič Institute was based on two lines.

One line was mainly concentrated on the study of physics of the GCR type power reactors. It is a part of the effort of Boris Kidrič Institute in connection with the nuclear energy power program of the Yugoslav Federal Nuclear Energy Commission.

The main features of the research activities in reactor physics on this line are:

a) Development of calculation methods for the analysis of the static parameters and long term reactivity changes for a gas cooled power reactor.

b) Development of subcritical experimental techniques for the determination of graphite - natural uranium lattice parameters, and preparations for the construction of a graphite subcritical assambly.

c) Determination of the nuclear properties of the home-made nuclear materials.

d) Stulies of the dynamic behavior of the GCR power reactors by analog methods.

The second line in the research program was more general in the nature. It was concentrated on the development of theoretical and experimental methods for the analysis of the basic reactor parameters. Owing to the fact that D_2O zero power reactor and 7 MW D_2O research reactor are available, the most of the program was done on D_2O reactors.

According to the NPY - Joint Commitee Agrement this part of research activity in the Boris Kidrič" Institute is included in NPY - Cooperative Program in reactor physics in accordance with the tasks formulated by the Committee. Owing to the fact that at present not all of the tasks are covered by group of the Boris Kidrič Institute the following activities should be mentioned.

a) Calculations of the thermal neutron distribution and reaction rate in a reactor cell and comparisons with experiments.

b) Buckling measurements and interpretation

c) Thermalisation and slowing down of neutrons

d) Pulsed neutron source techniques

e) Reactor kinetics

Although there is some interest in other tasks involved in the NPY - Cooperative program there has been no contribution so far.

2. Reactor theory and calculations

The work on reactor theory covered three main topics - the transport the slowing down and thermalization of neutrons. The calculation work has been concentrated on two reactor types - the heavy water experimental reactors and GCR - type graphite power reactors.

2.1. Reactor theory

The P_3 approximation to the transport theory has been extensively used in studying fine neutron flux distributions. A P_3 multiregion code for a ZUSE Z-23 computer has been developed. The flux distribution and the corresponding disadventage factors for a number of cells has been determined (for instance: RB, NORA, Calder Hall etc.). The standard reflecting boundary conditions have given good results for larger lattice pitches, but failed for smaller lattices. In these cases the use of white boundary conditions has moved to be quite satisfactory. The woid regions were treated as media with very small cross sections corresponding to the gas under pressure.

In treating the slowing-down of neutrons, a correct method of evaluating the average energy of neutrons affer n - collisions has been developed. The method is applicable to both isotropic and unisotropic scattering.

An analitical method for studying neutron thermalization in heterogeneous reactors has been developed. The moderator is treated according to a heavy gas model in diffusion approximation The slowing down of neutrons in the fuel is neglected and the angular distribution of neutrons at the fuel boundary is assumed to

be isotropic. The non - Maxwellian factor of the distribution in the moderator is expressed as a sum of one - Velocity solution and a series of Laguerre polynomials, multiplied by Bessel functions. The use of the diffusion approximation in the fuel region has been avoided by a energy boundary condition dependent on the fuel surface. This boundary condition is connected with corresponding escape probabilities, whose energy dependence is determined from the basic cross-section data.

Explicit formulae for calculating disadventage factors of the fuel and moderator, the average neutron energy and the neutron temperature, have been derived.

The convergency of the method and its applicability to graphite systems have been studied. Some practical adaptation for calculation with different fuel radii and moderator temperatures have been also done .

The use of reflecting boundary conditions at the outer boundary of the cell in calculating the energy dependent term has proved to be satisfactory. Further improvement is possible by using for the first (one-velocity) term of the sum and nondiffusional solution (for instance P_3) of the onevelocity equation. Such interpretation seems to be physicaly reasonable and gives good results but it requires a more rigorous mathematical evidence.

2.2. Reactor calculations

A calculation of static parameters of the Calder Hall reactor type has been done, with special reference to the anisotropy and flux fratterning. The applicability of different methods has been discused.

The long term reactivity changes for the same reactor system have been studied too. Special analysis was made for the influence of the flux and temperature on the shape of the curve k_{eff} in dependence on the irradiation.

The influence of the axial temperature distribution on the reactivity of the chanal has been thoroughly studied and some practical conclusions drawn.

The efficiency of the control rods system of the RA reactor has been calculated. A new two-group method in the diffusion approximation has been applied and the results compared with the experiments.

In connection with the above mentioned calculations the following programs for the digital computer ZUSE Z-23 have been developed:

REPA - I - A program for the calculation of parameters in the four - factor formula. One calculates ε, f, p, L^2, η and k_∞ as well as the critical buckling. The program is based on a classical formula with correlative data for and resonance integral.

REPA - II - A new version of REPA - I. The program calculates the lattice parameters of a hot and poisoned reactor and takes into account the changes of isotope concentration of the fuel during irradiation.

RAF - A two-group one-dimensional diffusion theory calculation of critical parameters and flux distribution in multizone reactor systems.

HETERO - A heterogeneous method of calculating the critical dimensions of a reactor with a finite radial reflector in the two-group diffusion approximation and cylindrical geometry.

TOKAR - The program consists of three parts: the first gives point reactivity changes, the second the reactivity of the cannal, and the third the reactivity of a reactor, as a function of irradiation.

ABSORBER - A two - group one-dimensional calculation of the reactivity worth of a system of absorbers regularly distributed in a heterogeneous reactor.

3. Development of experimental techniques

The RB reactor was used intensively in the development of experimental techniques for the determination of reactor parameters. The main effort was concentrated on the development of suitable methods for buckling measurements and interpretation of experiments especially in highly reflected systems where the usual assumption that the flux follows the one group distribution can not be accepted. The substitution technique for the determination of reactor parameters with a small number of fuel elements has been studied, using both Swedish and French methods and a new method based on the heterogeneous reactor theory for the interpretation of experiments.

The measurement of neutron flux distribution inside a reactor cell has been used as a routine method to check the theoretical studies of neutron thermalization. The preliminary results have been obtained for neutron temperature distributions by using Lu-foils.

For the first time in this Institute the pulsed neutron techniques have been used for measuring the kinetics parameter of the reactor as well as diffusion parameters of various moderators.

In addition, a number of routine methods have been used for various measurements connected with reactor operation and safety. Some of those have been improved or specialy adapted to the RB reactor.

3.1 Interpretation of the buckling measurements on highly reflected systems by the two-group diffusion theory

Determination of buckling by flux mapping techniques in highly reflected small size reactor cores, is faced with several difficulties. There is no pronounced region in the core where the cadmium ratio is constant and where the reflector influence considerably changes the $J_o(B_r.R)$ shape of thermal neutron distribution in the core. This makes one-group diffusion theory interpretation not accepable.

An attempt was made to use the two-group diffusion theory interpretation of the thermal and epithermal neutron flux distributions in the core to obtain the radial buckling. For that purpose two-group diffusion equations

$$\phi_s = AJ_o(B_r.R) - CI_o(v_r.R)$$

$$\phi_{th} = S_1AJ_o(B_r.R) + S_2CI_o(v_r.R)$$

are normalized to give the unit thermal and epitermal neutron flux in the center of the reactor

$$\bar{\phi}_s = \frac{A}{A-C} J_o(B_r.R) - \frac{C}{A-C} I_o(v_r.R)$$

$$\bar{\phi}_{th} = \frac{S_1 A}{S_1 A + S_2 C} J_o(B_r.R) + \frac{S_2 C}{S_1 A + S_2 C} I_o(v_r.R)$$

From these two equations the transient term $I_o(v_r.R)$ can be eliminated to obtain

$$J_o(B_r.R) = \bar{\phi}_{th} - K\ (\bar{\phi}_{th} - \bar{\phi}_s)$$

where

$$K = \frac{1}{1 + R.\frac{1}{S_2}}$$

R is the ratio between the epithermal and the thermal fluxes in the reactor center and S_2 the coupling coefficient. Using the Westcott convention for the factor r obtained by Cd-ratio measurements, the ratio between fast and thermal flux can be written

$$R = \frac{r_o}{1-r_o} \ln \frac{E_o}{E_{th}}$$

Coupling coefficient S_2 has the usual meaning

$$S_2 = p. \frac{D_s}{D_{th}} . \frac{1}{1 + B^2\tau}$$

Using this interpretation it is possible, by measuring the thermal, and epitermal neutron flux distributions in the reactor and determining the absolute ratio between the fast and thermal neutron fluxes in the reactor center to synthesize $J_o(B_r.R)$

distributions and obtain B_r-radial buckling in the usual way.

Experiments

The thermal neutron flux distribution is measured by using copper foils. The epithermal neutron flux distribution is measured by gold foils in Cd-boxes, the cadmium ratio using gold foils is measured in the reactor center and r factor is determined.

In this way it is possible to determine $\bar{\phi}_s(R)$ and $\bar{\phi}_{th}(R)$ distributions in the core with the absolute ratio of two distributions.

Constant K is calculated using the expected value for B^2 and the $J_o(B_r.R)$ distribution synthesized. The fig. 3.1.1 shows the various stages in the synthesizing $J_o(B_r.R)$ function.

The whole analysis is performed by means of a Z-23 computer. The input data include the total neutron density distribution obtained by Cu-foils measurement, and the epithermal distribution obtained by Au-foils in Cd-boxes and r factor in the reactor center. The calculated values for p, D_s, D_{th}, and the expected value for B_2, are introduced also. The program includes calculations of the normalized $\bar{\phi}_s$ and $\bar{\phi}_{th}$ and the synthesis of $J_o(B_r.R)$. The least squares method gives B_r. An iterated procedure is used to correct B^2 in S_2 input, but the convergency is rather fast.

Results

The buckling measurements for 2% enriched uranium heavy water lattices are presented in Table I.

Owing to the fact that the reactor was bare in the axial direction the axial buckling was obtained by fitting the obtained thermal neutron flux distribution in the cos (B_z.H) function.

The radial buckling for most of the lattices was obtained by a two-group interpretation except for few with the long lattice pitches where the reflector thickness was small. To compare the experimental results the one-group interpretation is also given. The fig. 3.1.2. gives the radial axial and total buckling for lattice pitches ranging from 7 cm to 19,8 cm. The curve presenting the calculated value for B_r^2 is also given. The systematic deviation is mainly due to the error in the Al cross-section used.

Table I

a (cm)	B_z^2 (m^{-2})	B_r^2 (m^{-2}) theor.	experiment.	B^2 (m^{-2}) one-group	two-group
7	3.150±0.030	16,2	15.40±0.25	14.00±0.40	18.55±0.30
8	4.714±0.043	14,88	13.46±0.29	14.80±0.60	18.18±0.33
9,90	6.354±0.039	12,45	11.73±0.11	15.50±0.60	18.08±0.15
11,34	6.650±0.080	10,85	9.690±0.12	15.50±0.20	16.34±0.20
14	6.278±0.059	8.40	7.770±0.037	13.40±0.20	14.04±0.09
16	5.400±0.050	7.12	6.539±0.100	11.79±0.20	11.79±0.20
17,90	4.400±0.021	6.48	6.065±0.061	10.47±0.08	10.47±0.08
19,80	3.203±0.018	6.05	5.677±0.037	8.879±0.056	8.879±0.056

3.2. Reactor parameter determination by substitution techniques

The substitution technique was used to determine the basic reactor parameters of natural uranium fuel. As the reference lattice 2% enriched uranium was used. The critisal height ad D_2O was measured after inserting successively 2,4,8,12 and 16 elements in the central part of the reactor. Various lattice pitches were used rangind from 8 - 16 cm.

The experimental results were theoreticaly interpretatied in three ways.

1. By the Swedish and two-group diffusion theory interpretations.

2. By the Franch two-group diffusion theory interpretation.

3. By the Feinberg Galanin - heterogeneous method.

3.2.1. The Swedish version

The experimentaly determined difference in axial bucklings of the reference B_c^2 and the substituted core B^2, for the successive substitutions of 2,4,12 and 16 are presented according to the one-group perturbation theory

$$\frac{\bar{B}^2 - B_r^2}{W_r + \frac{1}{n}\sum W_i} = (B_c^2 - B_r^2) + \frac{\sum_1^0 \Delta B^2 W_i}{W_r + \frac{1}{n}\sum W_i}$$

where W_c is the statistical weight for the elementary cells in the test zone, W_i statistical weight of the intermediate cells, and $(B_c^2 - B_r^2)$ the difference in cukling of the reference and the test zone, n is a number which has the different value for the different type of the intermedium cell. It is so selected to give the miniumum value of B^2.

$\frac{B^2 - B_r^2}{W_r + \frac{1}{n}\Sigma_i}$ is presented as the function of

$\frac{\Sigma W_i}{W_r + \frac{1}{n}\Sigma W_i}$ and the extrapolated value of $B_c^2 - B_r^2$ is determined. This way for 2.5 fuel element rod canned in Al (Σ_q = 0.332) the following buckling was obtained

Table I

Lattice pitch	Reference B_c^2 (m^2)	Test B_c^2 (m^2)
8	18.68	5.40
9.9	18.08	6.95
11.3	17.35	7.71
14.0	14.04	6.82
15.65	12.60	5.96
16.00	12.30	5.75

3.2.2. The French version

The difference in buckling of the reference and substituted coresis presented according to the two-group perturbation theory in the form of the formula

$$\Delta B^2 = (1 + e)\Delta\alpha + IS'$$

where ΔB^2 is the difference in buckling of the reference and the test zone, $\Delta\alpha^2$ the obtained difference in axial bucklings after substitution, e is the reflector coefficient S is given by

$$S = \frac{(S_1 - S_1')(S_2 - S_2')}{(S_1 - S_2')(S_2 - S_1')}$$

and

$$I = \frac{(\beta I_0 I_1 + \gamma' J_0 I_1)(\gamma J_0 K_1 - \beta K_0 I_1)}{\beta Q_0 (J_0^2 - J_1^2)(\gamma' K_0 I_1 + \gamma I_0 K_1)}$$

S_1 and S_1' are the coupling coefficients for two zones. γ and γ' the transient bucklings for the two zons. The I is calculated using the expected values for in the test zone. S is matched to have the constant value through series of substitutions. The results for the same series of experiments presented above is given in Table II.

Table II

Lattice pitch	B^2_{test} (m^2)
8	5.20
9.9	6.85
11.3	7.98
14.0	6.95
15.65	6.10
16.00	6.90

3.2.3. Heterogeneous method

For any substitution the critical determinant formed on the principles of the heterogeneous method has been solved. The critical parameters specially in a the two-group diffusion theory, are found either for η the multiplication constant, γ , the thermal constant.

The critical determinant is formed from the coefficients in the equations discribing the thermal neutron flux at the position of individual fuel elements. From two successive substitutions the critical determinant can be found for both η and γ .

To obtain η_o and γ_o for the reference cores the same heterogeneous method was applied. The Table III shows the results obtained for η and k_∞ is obtained by multiplying by the ratio of the number of neutrons absorbed in fuel and these totally absorbed in the reactor, i.e. fuel and moderator.

Table III

Lattice pitch	Reference η	k_∞ heterogeneous	k_∞ homogeneous
7	1.334	1.329±0.005	1.343±0.02
8	1.378	1.371±0.004	1.373±0.01
9.9	1.442	1.430±0.004	1.425±0.01
11.3	1.475	1.459±0.004	1.462±0.01
14.0	1.516	1.490±0.004	1.484±0.008
15.65	1.528	1.495±0.004	1.498±0.008
16.00	1.533	1.498±0.003	1.500±0.008
17.9	1.541	1.497±0.003	1.500±0.008
19.8	1.539	1.485±0.003	1.488±0.008

The Table IV gives the results for the substitution measurements. The thermal constant γ was obtained by solving simultaneously the critical determinants for 8 and 12 fuel rod substitutions in a 16 cm lattice pitch. The obtained result $\gamma = 1.33$ is compared with the flux distribution measurement and a satisfactory agreement obtained.

Table IV experimental data for substituted lattices, k_∞

Pitch	Substitution 4 elements	Substitution 12 elements	Substitution 16 elements
8	1.178±0.03	1.151±0.013	1.142±0.01
9.9	1.198±0.03	1.183±0.013	1.188±0.01
11.3	1.240±0.03	1.217±0.013	1.215±0.01
14.0	1.247±0.03	1.237±0.013	1.237±0.01
15.65	1.251±0.03	1.242±0.013	1.246±0.01
16.0	1.230±0.03	1.245±0.013	1.245±0.01

3.3. Thermal neutron flux measurements in an elementary cell

A series of experiments were carried out with heavy water 2% enriched uranium and heavy water natural uranium fueled reactor systems in order to determine microdistributions in an elementary cell with different lattice pitches. The thermal neutron distribution measurements gave the values of average fluxes and the corresponding disadventage factors.

The results are reported in Tables I and II as well as in Fig. 3. The measurements with a natural uranium fueled cell are still in progress and the results are preliminary.

Description of the experiment

Small foils of dysprosium aluminium alloy were used to map the thermal flux. Owing to its suitable characteristics, dysprosium proved as a convenient detector for intracell measurements.

All thermal neutron distribution measurements within the elementary cell were carried out with a specially prepared fuel element which was mounted at the top of the central square lattice. The fuel element was drilled with a Ø 3 mm. diamond drill close to the middle of the active height and the hole was painted with colourless lacquer or canned with a thin wall Al tube to avoid a release of fission products. A special small tube with detector foils was placed in the hole.

To enable detection of particular thermal flux variations in different diffusion media specially in the case of 2% enriched tubular fuel we had to attain the highest possible resolutions between the points measured. According to the experimental results it was evident that a high statistical accuracy of the activaties

measured and strict positioning of the detector foils were required to make these micro-efects visitable.

To define more accurately the positions of the detector foils a special small tube was made of plexiglass with an 2.1 mm. internal and 2.8 mm. extermal diameters. Within the tube 0.15 mm thick foils , 2 mm. in diameter, were arranged in a sandwich type principle by using plexiglass cylindrical spacers with a 2 mm. diameter. The distance among the foils was kept constant in order that the shadowing factor remained the same. The positions of the detectors within the fuel element were changed by placing on the tube a 2.8/3.3 mm plexiglass ring whose thickness varied between 0.5 - 2.5 mm. All the elements.included in the sandwich were made with an accuracy of $\pm$ 0.01 mm. so that the relative position of the detector was known with a better than $\pm$ 0.1 mm. accuracy.

All the foils used were intercalibrated with a standard deviation of $\pm$ 0.3%. Intracell irradiation was carried out along the side and the diagonal of the square lattice in the case of enriched fuel, and with an angle of 25^{o} from the side of the square lattice with natural uranium fueled system. At least two irradiations were performed in each direction for each lattice. After the irradiations the foils were left to cool down for about an hour. The beta activities were measured in cycles with four independent GM channels of approximately identical efficiences. The measured intervals were chosen to obtain a standard error of about $\pm$ 0.3% out of all channels.

Region averaged fluxes and disadventage factors

If r_i and R_i are the inner and outer radius of the i^{th} region, its average flux F_i, based upon Gauss, formula can be written

$$\bar{F}_i = \frac{2}{R_i^2 - r_i^2} \int_{r_i}^{R_i} r F_i(r)\, dr$$

To elaborate the results a programme was made for the digital ZUSE Z-23 computer. The obtained results are given in Table I and II.

3.4. Pulsed neutron measurements on the heavy water critical reactor RB

Decay constant measurements were performed on the critical and subcritical reactor for three different lattice configurations. 52 rods of 2% enriched uranium in a square lattice with d = 9,9 cm, 14 cm and 19.8 cm. pitch were used.

For the same lattices the control and safety rod worth was measured.

In addition (to these lattices) the average prompt neutron lifetime was measured for several two-region systems.

In order to get acquainted with the experimental problems of a pulsed neutron technique, the measurements of diffusion parameters on light water were made. For the sample of lower geometrical buckling, time and space distribution of the neutron flux were measured. After Fourier analysis of the data the decay constant of the fundamental mode was obtained and afterwards the diffusion parameters: $L = 2{,}74 \pm 0.03$ cm $4850 + 50\ \text{sec}^{-1}$, $D_o = 35500 \pm 500\ \text{cm}^2\ \text{sec}^{-1}$ and $c = 4750 \pm 2660\ \text{cm}^4\ \text{sec}^{-1}$ are determined.

As a pulsed neutron source the Kaman Pulsatron was used. The Pulsatron has an output of 10^7 neutrons per burst and a 5 sec pulse length. Pulsed neutron source was located on the bottom of the reactor on central line.

Two neutron BF_3 counters connected in paralel were located on the side of the reactor at one and two thirds of its extrapolated height. With thit arrangement, the second and third modes of the vertical neutron distribution were eliminated.

Output pulses were amplified and fed into the analyser. The detectors and the electronic system had the dead time of 27 /usec.

The data from the analyser were corrected for dead time losses and background.

The decay constant α with a probable error resulting from conuting statistics were found by using a single experimental function in the data a numerical analysis was performed using the ZUSE Z-23 digital computer.

Measurements

1. The first group of measurements the decay constant α of the asymptotic neutron flux as a function of the backling B^2 is made with the ain of comparing it with the results obtained by the homogeneous two-group diffusion theory.

$$\alpha = \frac{1}{l_2} + \nu_2 D_2 B^2 - \frac{k_\infty (1-\beta)}{l_2 (1+\tau B^2 - \alpha l_1)}$$

where τ and $\nu_2 D_2 / l_2 = L^2$ are the diffusion areas, and 1, and l_2 are the infinite - medium lifetimes of fast and slow group neutrons.

The results of our measurements on a lattice pitch of 9.8 cm are given on Fig. 3.4.1. A correction on decay constant dueto the build-up and the decay constant of delayed neutrons is now in progress.

From the same measurement the relationship of the reactivity R versus the height H of the D_2O has been obtained. Reactivity is determined from $R = \frac{\alpha-\alpha_c}{\alpha_c}$ and given on Fig.3.4.2 where α_c is the decay constant of a critical reactor.

2. The reactivity worth of shim, control and safety rods and their effects have been measured. The results are given in Table 1. The reactivity in dollars has been determined from

$$R = \frac{\alpha-\alpha_c}{\alpha_c}$$

3. The neutron life time has been determined from the constant of a delayed critical reactor.

$$\alpha_c = \frac{\beta}{l\rho}$$

The results are given in Table 2.

3.5. Measurement of ρ by a source jerk method in heavy water reactor

The source jerk method previously developed in England for measurements of subcritical reactivities in graphite moderated reactors is adopted for antireactivity measurements in heavy water reactors.

According to this method, reactivity is determined from

$$\rho = \frac{\theta}{\tau}\left(\frac{C_0}{C_1} - 1\right)$$

where the C_o/C_1 is the measured ratio of the neutron intensity in the reactor with and without neutron source, and τ the fiction neutron decay period after the source jerk.

To examine the influence of the long-life photo-neutrons on the constant Θ a series of experiments were performed where the same reactivities were determined by chaning the time during which the source was held in the reactor. From these experiments it is concluded that in a 5 - 30 minutes time interval influence of long-life photo-neutrons can be negleced and the constant Θ can be obtained using the first four groups of photo neutrons. In this way the constant Θ is given by

where β_i and λ_i are yield and decay constants for delayed and photo neutrons respectively and γ the nonleakage γ - ray probability calculated by the usual procedure. The value of the constant Θ is found

$$\Theta = 0.1029 \text{ sec}$$

as compared to 0.0893 for graphite reactors.

By varing the level of the heavy water , the negative reactivities are measured and the results are compared with the data obtained by standard dp/dH measurements. A satisfactory agreement is found. The results of both measurements are given in the Tables below.

Table 1. From the source jerk measurement

ΔH	10	7,5	5	4	3	2
$-\rho$	1907.08	1478.40	971.12	785.76	541.98	375.19
$\frac{\Delta\rho}{\Delta H}$	190.71	197.12	194.28	196.44	180.66	187.60
$\left(\frac{d\rho}{dH}\right)$			191.14 $\pm$			

Table 2. $\frac{d\rho}{dH}$ from the standard measurement

ΔH	0,27	0,32	0,38	0,41	0,49
ρ	51.83	63.12	69.69	79.13	93.62
$\frac{\Delta\rho}{dH}$	191.96	197.25	183.40	194.00	191.06
$\left(\frac{d\rho}{dH}\right)$			191.33 $\pm$		

3.6. Analog computer applications in reactor physics

Although the principal concern of the group for dynamic analysis lies in the domain of control theory, the first stage in the synthesis of control systems for nuclear reactors, i.e. the stage of formulating adequate models for the process to be controlled, is very closely related to reactor physics. The work on reactor models has thus resulted in some very useful applications of analog computers in reactor physics. We shall

leave aside such applications as the analysis of isotope concentration and long term reactivity changes where analog computers are well established and where their use can be considered as classical, and refer briefly to an application which concerns the flux distribution problems and which has proved to be very practical and useful.

In determing statistically ewighed coefficients for an integral reactor model one must solve the flux distribution equations and their adjoint. Although the adjoint equations can be very easily programmed on an analog computer, considerable difficulties are encountered when the equations to be solved must satistfy certain initial and boundary conditions. In other words, one must first determine the parameters which satisfy boundary conditions, and with these parameter values solve the equations to obtain the flux distribution. When the reactor is composed of several different media and the two group approximation is used, it is impossible to solve the flux equations directly due to extremely large sensitivity to initial conditions and the necessity to adjust simultaneously two parameters to satisfy initial and boundary conditions. Such problems are therefore normally solved on a digital computer. By suitable transformations, however, it is possible to adapt the problem for an analog computer and use a non-iterative procedure which requires only a few computer components regardless of the number of reactor regions.

The transformation consists in reducing the original Bessel equations into a system of Riccati equations which are of first order and where only one parameter, i.e. the critical radius, has to be adjusted. Since the initial and boundary values for the solution of these equations are known at

the center and the extrapolated boundary of the reactor, it is possible to reduce the number of regions to only two and find the critical radius which will satisfy the conditions at the boundary between the two remaining regions. Once this critical radius is found, it is easy to determine the initial values of neutron flux and current at all boundaries and then generate the Bessel functions which determine the flux distribution. The resulting Riccati equations can be normalised and solved once for all for various values of constants. The sets of solutions thus obtained can be used to determine critical radii and flux distributions in multiregion reactors without using either analog or digital computers. Furthermore, using the coefficients determined by solving normalised Riccati equations, one can express the flux distribution by an analytic expression which is very simple from the computational aspect though the accuracy obtained approaches that of two group treatment.

Thus the analog computer has proved to be very useful in determining criticality conditions and plotting flux distribution curves in multiregion nuclear reactors. A detailed description of the methods developed is contained in the following papers

L. Radanović, S. Bingulac, B. Lazarević, M. Mataušek

AN ANALOG COMPUTER METHOD FOR SOLVING FLUX DISTRIBUTION PROBLEMS IN MULTIREGION NUCLEAR REACTORS; VIII Nuclear Congress, Rome, 1963.

S. Bingulac, L. Radanović, B. Lazarević, M. Mataušek

ONE-VELOCITY NEUTRON DIFFUSION CALCULATIONS BASED ON A TWO GROUP REACTOR MODEL, to be presented at III Geneva Conference.

T A B L E I
Section 3.3
System: D_2O - 2% enriched tubular fuel

	F_1 DF_1	F_2 DF_2	F_3 DF_3	F_4 DF_4	F_5 DF_5	F_6 DF_6	F_7 DF_7	F_8 DF_8	F_9 DF_9
8,0	1,09362	1,06303	1,04041	1,01598	1,00414	1,06702	1,09895	1,18421	1,28941
	1,08910	1,05864	1,03611	1,01179	1,00000	1,06262	1,09442	1,17932	1,28409
9,87	"	"	"	"	"	"	"	"	1,38422 1,37851
14,0	"	"	"	"	"	"	"	"	1,51469 1,50845
17,9	1,07341	1,05958	"	"	"	"	"	"	1,53751 1,53117
19,7	"	"	"	"	"	"	"	"	1,60975 1,60310

T A B L E II

Section 3.3

System: D_2O- Natural Uranium

	F_1 DF_1	F_2 DF_2	F_3 DF_3
8,0	1,1657 1,0000	1,3788 1,1828	1,7483 1,4998
9,9	"	"	1,8194 1,5608
11,3	"	"	1,9357 1,6605
15,7	"	"	2,0476 1,7565

	$(msec)^{-1}$ $.10^2$	Reactivity (%)				Dimension and position of rods
		Single	Coupled			
Reactor is critical	0.661±0.002					Ø 35 mm 50 cm from bottom
SŠ-1	1.254±0.006	0.39±0.02	2.04±0.13	2.77±0.37	6.70±0.87	F_e and Cd Ø 32 110cm from bottom
SŠ-2	1.254±0.006	0.89±0.02				"
KŠ	1.263±0.002	0.90±0.02	4.80±0.45			Fe Ø 24 50 cm from bottom
Ni	2.960±0.001	3.86±0.10				Cd Ø 34 12 mm from bottom

TABLE 1.

Section 3.4

Rc (cm)	d (cm)	c (1/msec)	c(%)	bp (exp)(msec)	Lattice pitch a	Fuel rods
65	35	$0.6627 \cdot 10^{-2}$	0.4	1.777	14	52
41	59	$0.8385 \cdot 10^{-2}$	0.4	0.930	9.87	52
74	26	$0.5190 \cdot 10^{-2}$	0.4	1.503	14-14 2*	24-28*
78	22	$0.4438 \cdot 10^{-2}$	0.4	1.758	7-14 2	4-48
53	47	$0.8484 \cdot 10^{-2}$	0.4	0.919	7 2-14	24-28

TABLE 2.

Section 3.4

Rc - core radius

d - reflector thickness

* First number is for the inner region and second one for outer

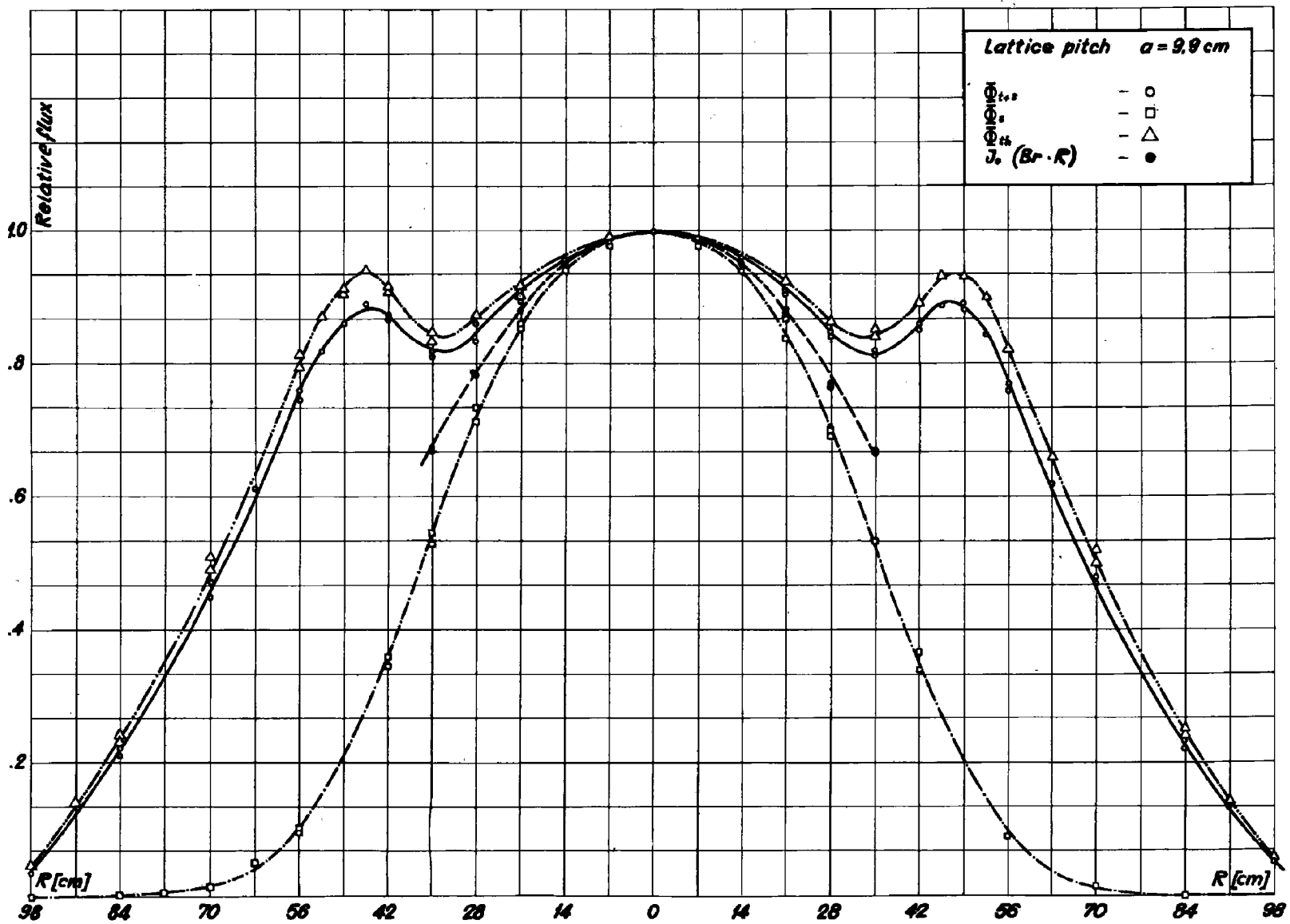

Lattice pitch a = 9.9 cm
J₀ (Br · R)
Relative flux
1.0
.8
.6
.4
.2
R [cm]
98
84
70
56
42
28
14
0
14
28
42
56
70
84
98
R [cm]

Fig.3.1.1

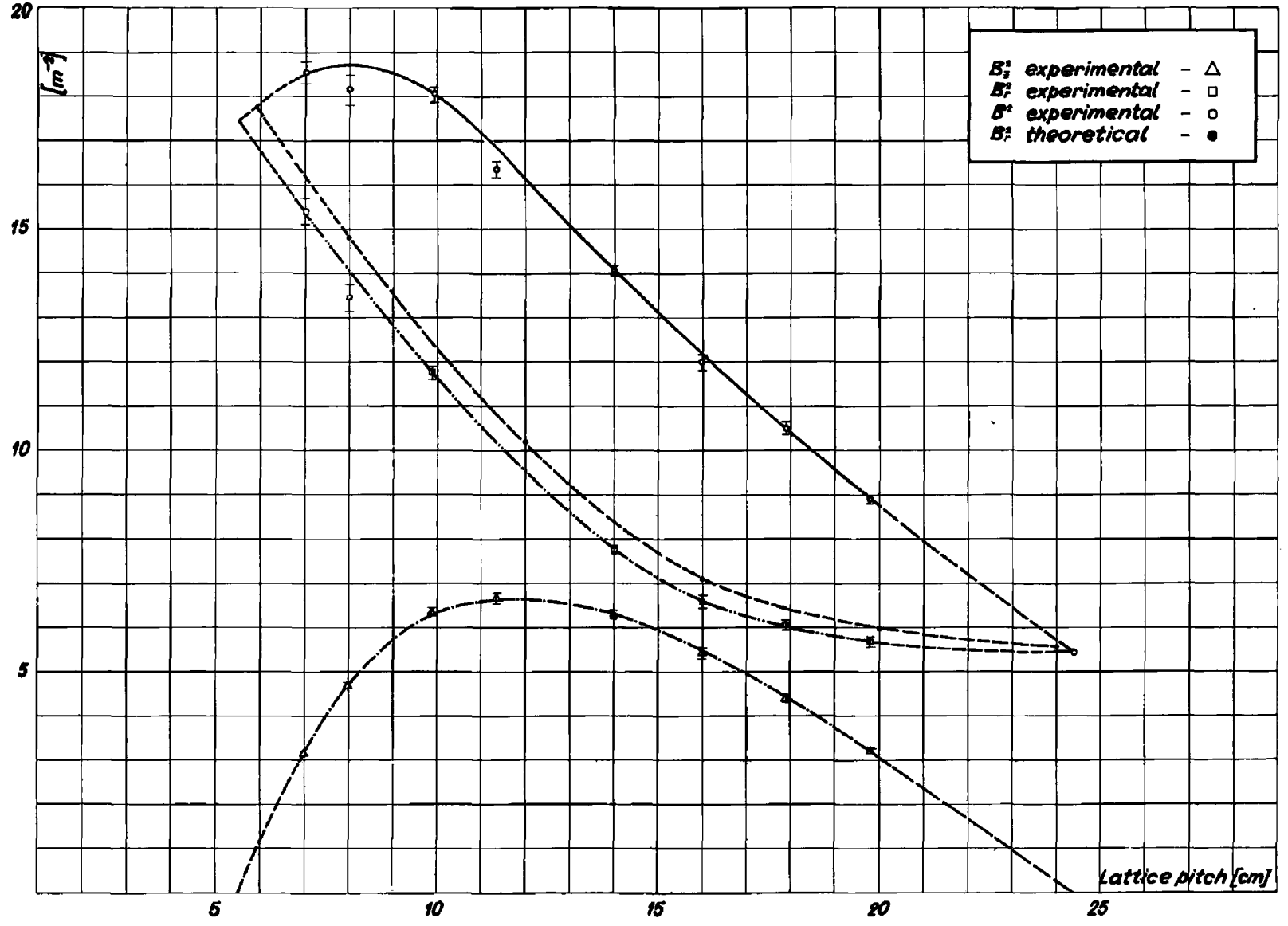

20
15
10
5
B_m^2 [m^{-2}]
B_1^2 experimental - △
B_r^2 experimental - □
B^2 experimental - o
B_r^2 theoretical - ●
5
10
15
20
25
Lattice pitch [cm]

Fig. 3.1.2

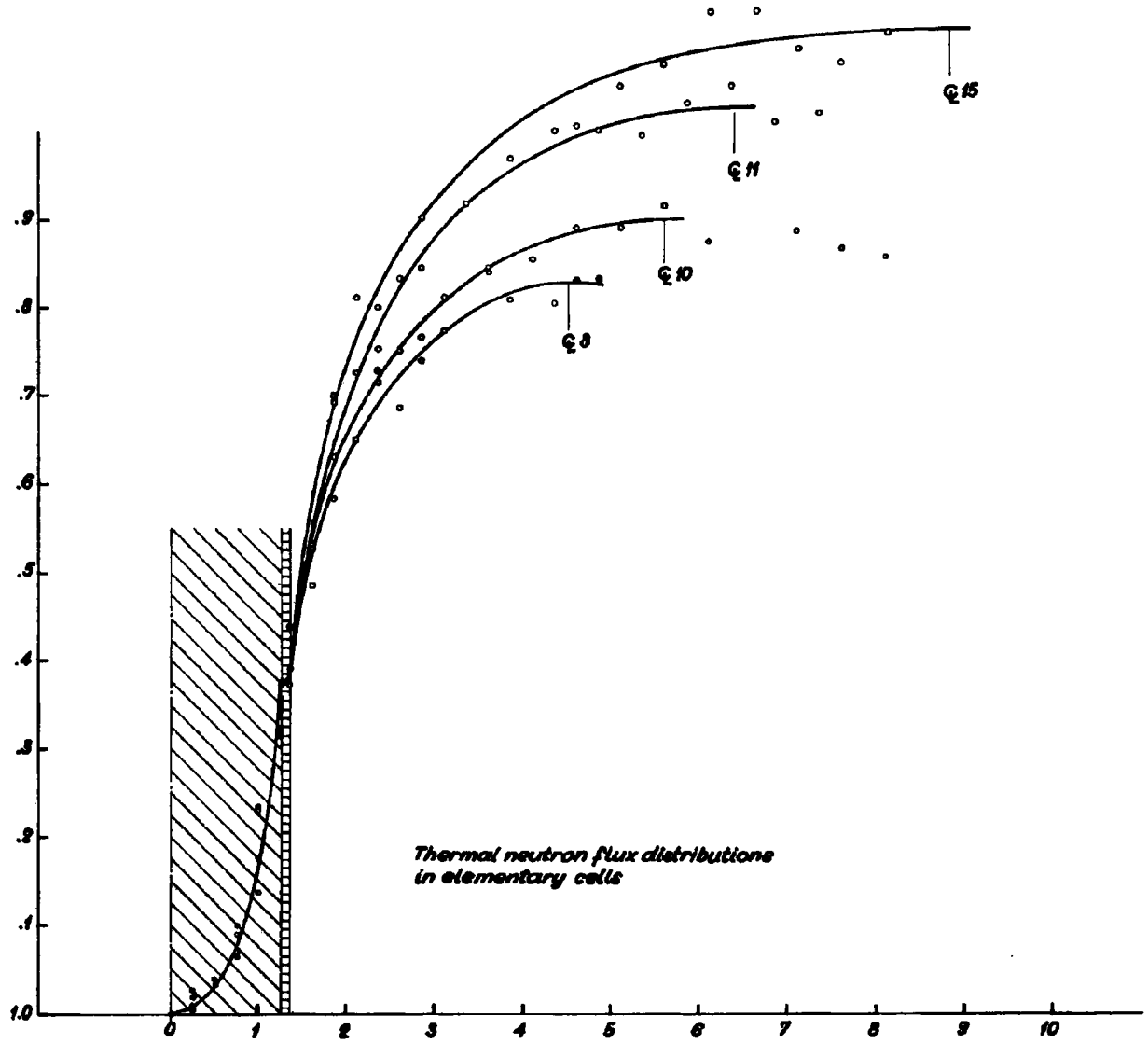
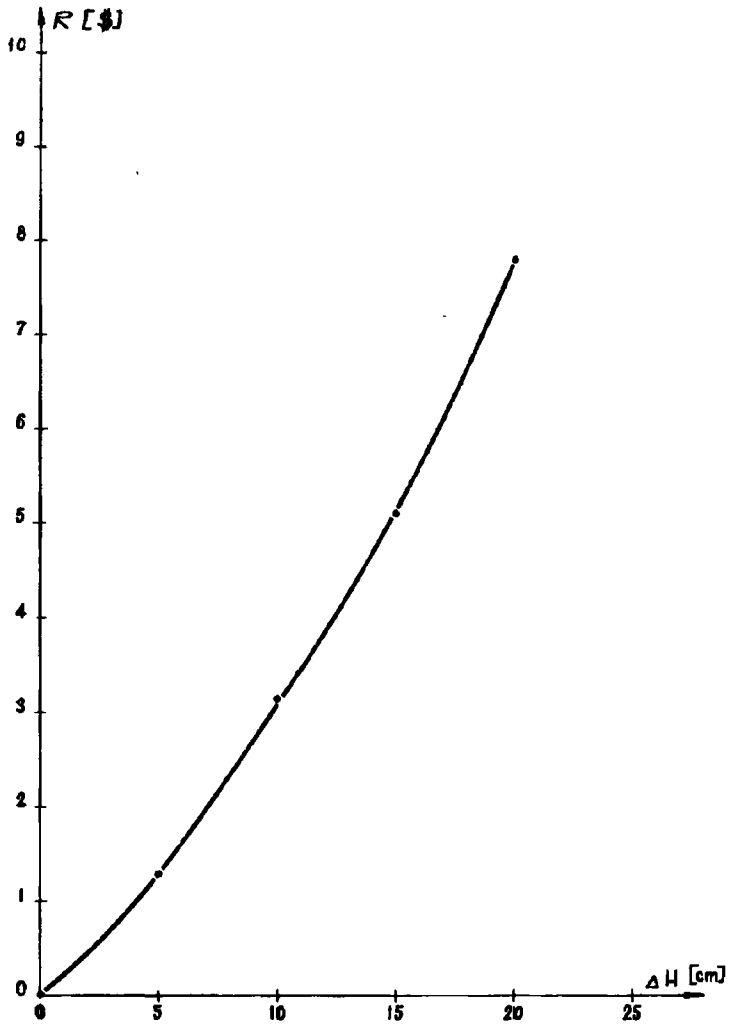
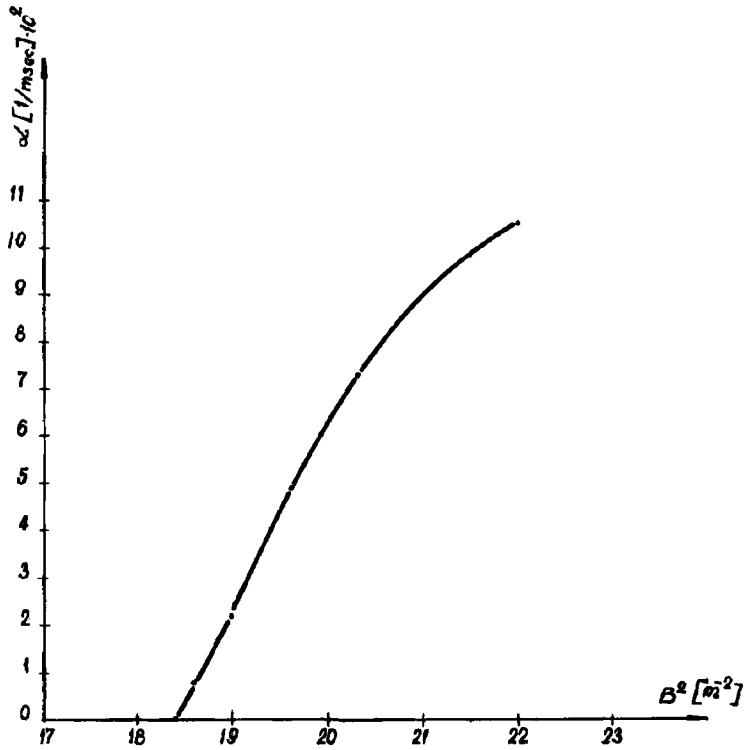

Thermal neutron flux distributions
in elementary cells
Ø 15
Ø 11
Ø 10
Ø 8
.9
.8
.7
.6
.5
.4
.3
.2
.1
1.0
0
1
2
3
4
5
6
7
8
9
10

Fig. 3.3.1



$R = f(\Delta H)$ for Lattice $a = 9.87$ cm

FIGURE 3.4.2



$\alpha = f(B^{2})$ for lattice $a = 9.87$ cm

FIGURE 3.4.1