AE-35

# The Multigroup Neutron Diffusion Equations/1 Space Dimension

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Summary.

A description is given of a program for the Ferranti Mercury computer which solves the one-dimensional multigroup diffusion equations in plane, cylindrical or spherical geometry, and also approximates automatically a two-dimensional solution by separating the space variables.

In section A the method of calculation is outlined and the preparation of data for two group problems is described. The spatial separation of two-dimensional equations is considered in section B. Section C covers the multigroup equations. These parts are self-contained and include all information required for the use of the program. Details of the numerical methods are given in section D. Three sample problems are solved in section E. Punching and operating instructions are given in an appendix.

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LIST OF CONTENTS

# The Multigroup Neutron Diffusion Equations/1 Space Dimension

## A. The two-group diffusion equations

### 1. Statement of the problem

The two-group diffusion equations can be written

$$D_1 \nabla^2 \varphi_1 - \sigma_1 \varphi_1 + (N_1 + \nu \theta_1) S^* = 0 \qquad (1a)$$

$$D_2 \nabla^2 \varphi_2 - \sigma_2 \varphi_2 + \sigma_{2,1} \varphi_1 + (N_2 + \nu \theta_2) S^* = 0 \qquad (1b)$$

$$S^* = \sigma_1^* \varphi_1 + \sigma_2^* \varphi_2 \qquad (1c)$$

$\varphi_1$ and $\varphi_2$ are the unknown functions; D, $\sigma$, N and $\theta$ are given step functions; $\nu$ is a parameter, the critical value of which is to be determined. The critical value is the smallest number $\nu$ for which the system has a solution not identically zero. We confine ourselves to the one-space-dimensional case and denote the space variable by x. Then the Laplacian operator $\nabla^2 = \frac{d^2}{dx^2} + \frac{p}{x} \frac{d}{dx}$, where p is a quantity determined by the geometry of the problem: p = 0, 1 or 2 in plane, cylindrical and spherical geometry respectively.

The following boundary conditions are to be satisfied for either function (dashes used to denote differentiation with respect to x):

a) at an outer boundary a relation of the form $a D\varphi' + A\varphi = 0$, where a and A are given constants,

b) at an interface between dissimilar regions $\varphi$ and $D\varphi'$ continuous.

The interpretation of the quantities is

$\varphi_g$ the neutron flux in group g, $\varphi_1$ the fast and $\varphi_2$ the thermal flux

$D_g$ the diffusion coefficient in group g

$\sigma_g$ the macroscopic absorption cross section in group g

$\sigma_{g,g'}$ the macroscopic cross section for transfer of neutrons from group g' to group g

$\sigma_g^*$ the macroscopic fission cross section in group g

$S^*$ the fission source

The functions N and $\theta$ can be used in various ways.

1. The critical multiplication factor for a simple core - reflector system is to be determined. Choose for instance the constants as follows (p = the resonance escape probability)

| core | reflector |
|---|---|
| $\sigma_2^* = \sigma_2$ | $\sigma_2^* = 0$ |
| $\theta_1 = 1/p$ | $\theta_1 = 0$ |

$\sigma_1^* = \theta_2 = N_1 = N_2 = 0$

The parameter $\nu$ calculated by the program is the critical multiplication factor.

2. The core of a reactor is divided into two regions A and B, and the critical multiplication factors, assumed to be in a given ratio $c_A : c_B$, are to be computed. Take $\theta_1 = c_A/p_A$ in medium A and $\theta_1 = c_B/p_B$ in medium B, $\theta_2 = N_1 = N_2 = 0$. The desired multiplication factors are $\nu\, c_A$ and $\nu\, c_B$.

3. One of the multiplication factors, $K_B$, is given, the value of the other needed to make the system critical is to be determined. Choose $\theta_1 = 1/p_A$, $N_1 = 0$ in medium A, $\theta_1 = 0$, $N_1 = K_B/p_B$ in medium B, and $\theta_2 = N_2 = 0$. The value of $\nu$ obtained is the required multiplication factor.

Eqs (1) have been chosen to fit the multigroup system considered in section C. That is the reason for including a term $(N_2 + \nu\ \theta_2)\, S^*$ in the second equation.

## 2. The numerical method

The program solves eqs (1) by a method of successive approximations. This consists in taking an arbitrary initial estimate of the solution, computing $S^*(x)$ and then finding from eqs (1a, b) a better approximation to the actual fluxes. An approximate value of $\nu$ is found from the ratio of successive estimates of $S^*(x)$. The estimates are improved step by step and the process is continued until the desired accuracy is obtained.

During each iterative cycle a couple of differential equations are to be solved, one for each energy group. The program employs a method based on finite difference approximations. We have to choose a set of points $\{x_i\}$:

$$x_a = x_o < x_1 < \ldots < x_{M-1} < x_M = x_b$$

which divide the range of integration ($x_a$, $x_b$) into M intervals and yield a suitably fine mesh for the differences.

It is not necessary to space the meshpoints at equal intervals. As an example let us consider a cylindrical, axially bare reactor with three regions of outer radii 116 cm, 151 cm and 156 cm and composed of media 1 (homogenized core), 2 (reflector) and 3 (steel wall) respectively. Let us space the meshpoints as follows:

| Medium | Range (cm) | Mesh length (cm) |
|---|---|---|
| 1 | 0 - 116 | 4.64 |
| 2 | 116 - 141 | 5.00 |
| 2 | 141 - 151 | 2.50 |
| 3 | 151 - 156 | 1.00 |

At interfaces between media of considerably different physical properties the fluxes are likely to vary rapidly and the mesh length should be kept small. Therefore medium 2 has been divided into two sub-regions, the outer one with a finer mesh.

Thus the range of integration is split into 4 Divisions, a division being defined as a sub-region of the reactor lying wholly within one medium and divided into meshes of equal length.

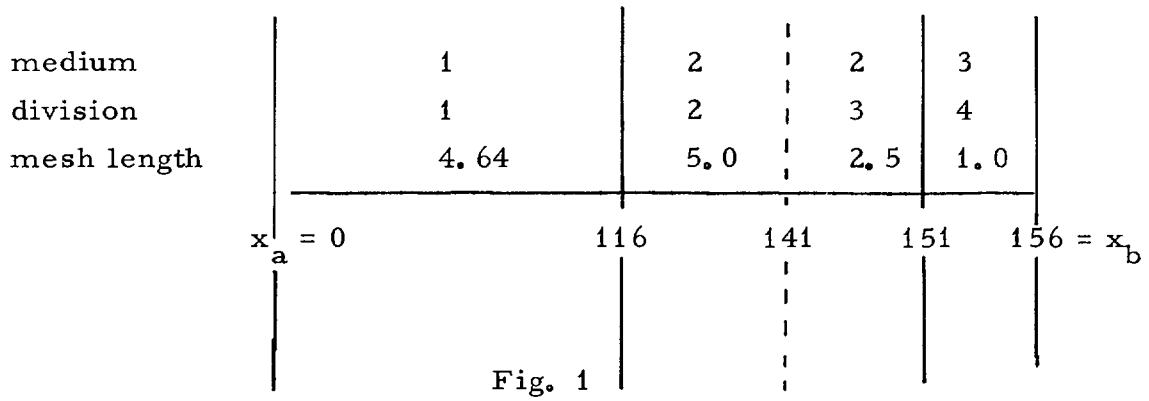



Fig. 1

Boundaries between divisions must coincide with meshpoints. It is difficult to give general rules for the choice of mesh in a medium. The mesh size is closely related to a quantity $L = \min_{(g)} \left\{\sqrt{D_g/\sigma_g}\right\}$ and it is recommended to choose the mesh length $h < 0.7\ L$. Near interfaces, however, it may be necessary to use a considerably finer mesh. The final choice must be based on numerical experiments.

## 3. Data preparation

The way in which data should be set out will be described by reference to the example above. The constants for the media are:

Medium 1

| g | $D_g$ | $\sigma_g$ | $\sigma_g^*$ | $N_g$ | $\theta_g$ | $\sigma_{g,g'}$ |
|---|---|---|---|---|---|---|
| 1 | 1.122 | 0.00320 | 0 | 0 | 1.264 | |
| 2 | 0.963 | 0.00617 | 0.00617 | 0 | 0 | 0.00253 |

Medium 2

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1.033 | 0.00361 | 0 | 0 | 0 | |
| 2 | 0.862 | 0.00017 | 0 | 0 | 0 | 0.00361 |

Medium 3

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0.654 | 0.00107 | 0 | 0 | 0 | |
| 2 | 0.378 | 0.150 | 0 | 0 | 0 | 0.00107 |

The boundary conditions are for either flux

$$\varphi'(x_a) = 0 \qquad \text{and} \qquad \frac{D\varphi'(x_b)}{\varphi(x_b)} = -\frac{1}{2.13}$$

which can be written

$$1\ D\ \varphi'(x_a) + 0\ \varphi(x_a) = 0, \qquad 2.13\ D\ \varphi'(x_b) + 1\ \varphi(x_b) = 0$$

Let us denote the coefficients by a, A, b and B respectively. Then we have

| a | A | b | B | |
|---|---|---|---|---|
| 1 | 0 | 2.13 | 1 | for $\varphi_1$ |
| 1 | 0 | 2.13 | 1 | for $\varphi_2$ |

The input data for this problem is as follows.

```
TITLE
REACTIVITY  AKG.  12.1.1960    ØØ
CONSTANTS
        2   11
1       1.122 0.963  0.00320  0.00617  0  0.00617  0  0  1.264  0  0.00253
2       1.033 0.862  0.00361  0.00017  0  0  0  0  0  0          0.00361
3       0.654 0.378  0.00107  0.150    0  0  0  0  0  0       0.00107

CYLINDRICAL
MESHPOINTS
0(4.64)116.0(5.0) 141.0(2.5)151.0(1.0)156.0
DIVISIONS
1  2  2  3
BOUNDARY CONDITIONS
1  0  2.13  1
1  0  2.13  1
START
PRINT TITLE
FLUX
WAIT
```

Each group of data should be preceded by a directive occupying a separate line on the printed page. On reading this information the computer will recognise the directives and take an appropriate action. If an unrecognised word is read a fault indication is printed out and the machine comes to a hoot stop.

Every problem must have a TITLE. The machine reads and stores the characters on the lines following this directive until it comes to two consecutive figure shift symbols, Ø Ø. The title is printed immediately and will be printed again if the machine finds the directive PRINT TITLE.

The word CONSTANTS is to be followed by two integers specifying the number of energy groups and the number of constants for each medium. After the integers the constants are listed, each block preceded by a material index chosen arbitrarily from the integers 1, 2, ..., 99. The constants are to appear in the order $D_1$, $D_2$, $\sigma_1$, $\sigma_2$, $\sigma_1^*$, $\sigma_2^*$, $N_1$, $N_2$, $\theta_1$, $\theta_2$, $\sigma_{2,1}$.

The geometry of the problem is specified by one of the directives CYLINDRICAL, SPHERICAL and, in plane geometry, X c, Y c or Z c, where c is a number such that z = c, say, is the "central position" on the axis.

No exactitude is required, a rough estimate will do.

Data after MESHPOINTS is self-explanatory. All division boundaries must appear in the sequence even if two adjacent divisions have the same mesh length. If the boundaries do not coincide with meshpoints the program will adjust the mesh lengths appropriately.

The integers following DIVISIONS specify the media of which the divisions are composed and refer to material indices given after CONSTANTS.

BOUNDARY CONDITIONS is to be followed by a list of the coefficients a A b B defined above, each group on a separate line. If the same conditions hold for either group, the first line may be terminated by ETC and the second line omitted:

BOUNDARY CONDITIONS
1 0 2.13 1 ETC
START

When a complete set of data has been run in, the word START initiates the calculation. During the first few seconds data is checked and arranged in a more convenient order and then, after a short hoot, the iterative process is started. For each iterative cycle the machine prints out the cycle number and the estimate of the critical parameter $\nu$. The estimates settle down gradually, and when the desired accuracy is obtained, a hand-switch on the control desk is set and after finishing the current cycle the machine will read and interpret the next directive on the data tape.

The directive FLUX gives a listing for each energy group of the space variable and the flux value at each meshpoint. WAIT provides a hoot stop which is passed by depressing and releasing a hand-switch.

Of the directives mentioned above only CONSTANTS can precede TITLE, and in fact it may come anywhere before BOUNDARY CONDITIONS. CYLINDRICAL, MESHPOINTS and DIVISIONS should come in this order.

## 4. The lateral leakage

In the preceding paragraph we have not taken the lateral (in this case axial) flux distribution into consideration. To do this we write the Laplacian

$$\nabla^2 = \nabla_x^2 - \alpha^2$$

where $\nabla_x^2$ is the one-dimensional operator as defined in paragraph 1 and $\alpha^2$

a constant, the lateral buckling. It can for instance be used to represent an axial flux distribution of the form $\cos(\alpha z)$ or a radial distribution of the form $J_o(\alpha r)$.

The term $-\alpha^2$ will be inserted in the operator by the directive

LATERAL LEAKAGE

$\alpha^2$

It is assumed that the lateral buckling is the same for both groups and for all values of x.

## 5. LOOP and ACCURACY

A reasonable estimate of $\nu$ and the principal features of the flux distribution will generally be obtained after a few iterations, but in some region the flux may converge more slowly. If details of the flux distribution are essential the machine operator must check the flux convergence using a directive

LOOP

$\xi$

where $\xi$ is a number specifying a position on the axis or radius. This directive, read before START, will cause the machine to print out the successive flux estimates at the point $x = \xi$ (or the nearest meshpoint). In spherical geometry, however, the values printed out will be not $\varphi_1$ and $\varphi_2$ but $\xi\varphi_1$ and $\xi\varphi_2$. Physical considerations generally indicate in which region the convergence is likely to be slow. The printing can be suppressed (for instance during the first few cycles) by setting a hand-switch on the control desk.

In many cases the estimates of $\nu$ decrease or increase monotonically after the first few iterations. Thus, if the detailed flux distribution is not required, it is reasonable to let the machine test the convergence of $\nu$ and terminate the process when a simple convergence criterion is satisfied. Each estimate of $\nu$ is compared with the values obtained during the two preceding cycles, and if

$$\nu_i(1-\epsilon) < \left\{ \begin{matrix} \nu_{i-1} \\ \nu_{i-2} \end{matrix} \right\} < \nu_i(1+\epsilon)$$

where suffix i denotes the current iteration, the process is automatically interrupted and the next directive on the tape is obeyed. The quantity $\epsilon$ is

set by a directive

ACCURACY

$\epsilon$

which is to come before START. If it is omitted, the program sets $\epsilon = 0$.

## 6. INTEGRALS

The word INTEGRALS will cause the machine to calculate and print out the following integrals for each division.

$$j \qquad \int_{\bar{x}_{j-1}}^{\bar{x}_j} \varphi_G \, x^p \, dx \qquad \int_{\bar{x}_{j-1}}^{\bar{x}_j} \varphi_G^2 \, x^p \, dx \qquad \int_{\bar{x}_{j-1}}^{\bar{x}_j} \varphi_G^3 \, x^p \, dx$$

p is the geometry constant; $j = 1, 2, \ldots$ is a division index; $\bar{x}_j$ is the outer boundary of division j, and $\bar{x}_o = x_a$. Subscript G refers to the thermal group.

In plane and cylindrical geometry the highest thermal flux and the shape factor F of the fission source are also printed. The shape factor is defined

$$F = \frac{\overline{S^*} \int_{x_a}^{x_b} \delta \, x^p \, dx}{\int_{x_a}^{x_b} S^* \, x^p \, dx}$$

where $\overline{S^*}$ is the highest value of $S^*$ and $\delta$ takes the value 0 in regions with all $\sigma_g^* = 0$ and the value 1 in regions with some $\sigma_g^* \neq 0$.

INTEGRALS must come after START on the data tape.

## 7. The adjoint equations

The program solves the adjoint of equations (1) if the word ADJOINT is read before START. If the normal as well as the adjoint equations are to be solved, it is not necessary to repeat the geometry specification, MESHPOINTS etc. to obtain the adjoint fluxes. The directives can be

arranged as follows.

TITLE

.

.

.

START The iterative procedure for solving the normal equations is initiated.

FLUX The normal fluxes are printed out.

ADJOINT

START The iterative procedure for solving the adjoint equations is initiated.

FLUX The adjoint fluxes are printed out.

The critical value $\nu$ is the same for the adjoint and the normal differential equations. However, errors arising from the use of finite differences will cause the values computed by the program to differ slightly. The discrepancy will increase with the mesh length.

For a system, the adjoint of which is to be solved, one of the following conditions must be satisfied in each region.

i) $\theta_g = 0$ for all g

ii) $N_g = 0$ for all g

Different conditions may be applied to different regions, however. Thus region 1 may have all $\theta_g = 0$, region 2 all $N_g = 0$ and region 3 $\theta_g = N_g = 0$ for all g.

In order to get the correct normalization of the adjoint fluxes $\theta_g$ and $N_g$ should be set = 0 for all g in media with all $\sigma_g^* = 0$.

Note that the order of the groups is reversed on the flux print out.

## 8. Series of problems

If several problems with similar configurations and physical parameters and approximately the same number of meshpoints are to be solved, it is an advantage to use the solution of one problem as initial estimate for another. The previous solution will be preserved if on the data sheet for the problem to be solved is the directive

RETAIN SOLUTION.

The same result can be produced by running in the data with hand-switch 0 set.

Suppose that a number of problems are to be solved in which the constants of medium 1 are modified, other media and the size of the system being the same. The following arrangement is recommended.

For the constants of media 2, 3 ... a separate tape is used:

```
CONSTANTS
        2       11
2       D1      D2 ...
3       D1      D2 ...
.
.
.
WAIT
```

The tape is fed into the machine immediately after the program. Generally the constants will not be wiped out between problems and this tape need not be included in the data for the following problems.

For the remaining data a second tape is used. Constants are listed for medium 1 only. The data for the first problem will be:

```
TITLE
......
CONSTANTS
        2       11
1       D1      D2 ...
CYLINDRICAL
.
.
.
START
```

The data for the following problems should include the directive RETAIN SOLUTION (or hand-switch 0 should be set during input).

```
TITLE
.......
        2       11
1       D1      D2 ...
CYLINDRICAL
.
.
.
RETAIN SOLUTION
START
```

## 9. Normalization

The fluxes are normalized so that

$$\frac{\int_{x_a}^{x_b} S^* \, x^p \, dx}{\int_{x_a}^{x_b} \delta \, x^p \, dx}$$

where p is the geometry constant and the quantity $\delta$ takes the value 0 in regions with all $\sigma_g^* = 0$ and the value 1 in regions with some $\sigma_g^* \neq 0$.

## 10. Limitations

Let M be the number of intervals in the mesh and d the number of divisions. The limitations of the current program used for two-group calculations are

$$d \leq 21$$

$$M + d \leq 319$$

## B. Approximation of two-dimensional solutions

### 11. Separation of the space variables

The methods available for solving the diffusion equations in two (or three) space dimensions are slow, and in many cases approximate one-dimensional methods can be useful, particularly if it is necessary to explore a wide range of reactor parameters. These methods are based on separation of the space variables and consist in representing the reactor with an equivalent bare system with identical reactivity. In addition to its conventional applications this program has been designed to approximate a two-dimensional solution by a method of spatial separation. The following iterative scheme is employed.

Let us consider a cylindrical reactor with all-round reflectors (fig. 2a). We write the diffusion equations

$$D_g \nabla^2 \varphi_g - \sigma_g \varphi_g + S_g = 0 ; \qquad g = 1, 2, \ldots \tag{2}$$

and the Laplacian

$$\nabla^2 = \nabla_r^2 + \nabla_z^2 ; \qquad \nabla_r^2 = \frac{\delta^2}{\delta r^2} + \frac{1}{r}\frac{\delta}{\delta r}$$

$$\nabla_z^2 = \frac{\delta^2}{\delta z^2}$$

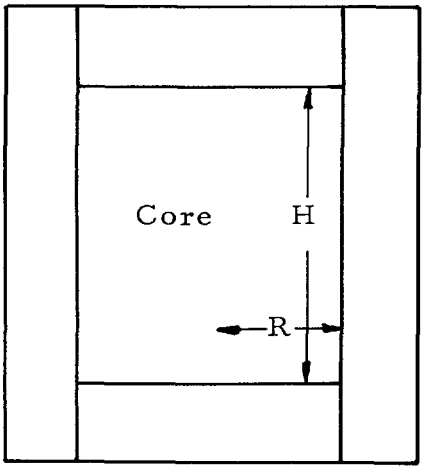
In the first stage we consider an axially bare system with the same radial configuration as the actual reactor (fig. 2b). Its height is taken to be $H_e = H + \Delta H$, where H is the height of the actual core and $\Delta H$ an estimated value of the axial reflector savings. The axial buckling for this system is

$$\alpha_z^2 = \left(\frac{\pi}{H_e}\right)^2$$

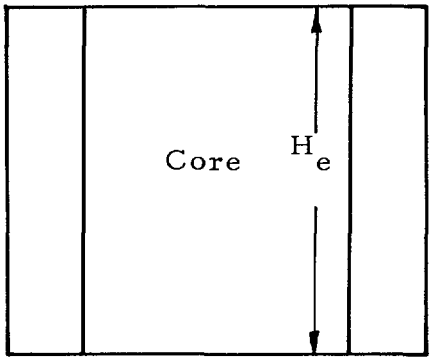
The program takes $\nabla_z^2 \varphi_g = -\alpha_z^2 \varphi_g$ and solves the radial equations

$$D_g(\nabla_r^2 - \alpha_z^2)\varphi_g - \sigma_g \varphi_g + S_g = 0 ; \qquad g = 1, 2 \ldots \tag{3}$$

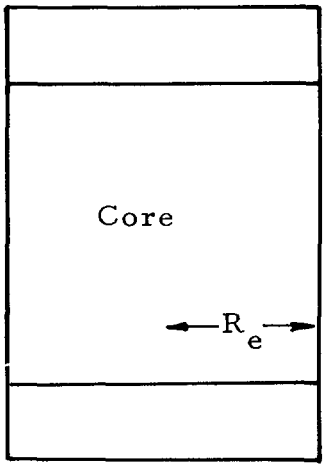
as described in section A and obtains a critical parameter $\nu_r$. Finally it calculates the radial buckling $\alpha_r^2$ for the equivalent bare reactor of extrapolated height $H_e$.




(a) The actual reactor




(b) The equivalent reactor without end reflectors




(c) The equivalent reactor without side reflectors

Fig. 2. Types of reactors discussed

In the second stage we consider a radially bare system with the same axial configuration as the actual reactor and an extrapolated radius $R_e$ corresponding to the radial buckling $\alpha_r^2$ just found (fig. 2c). The program solves the axial equations

$$D_g (\nabla_z^2 - \alpha_r^2) \varphi_g - \sigma_g \varphi_g + S_g = 0 ; \qquad g = 1, 2, \ldots \qquad (4)$$

and obtains a critical parameter $\nu_z$. Now it can find a better estimate of $\alpha_z^2$ for the equivalent bare reactor.

In the third stage we return to model 2b, the radial equations are solved with the new value of $\alpha_z^2$, a new estimate of the radial buckling is found, and so on. The process is continued until $\nu_r$ and $\nu_z$ are equal within the desired accuracy.

For an uncomplicated large system the one-dimensional technique provides a fairly good approximation. The errors cannot easily be assessed, however, and its applicability should be checked by solving a test problem by some more precise method.

## 12. Data preparation

Data for eqs (3) and (4) should be given as for two one-dimensional problems under the same title: see section E: Sample problem 2. A few points should be noted.

Constants for all media involved in the calculation can be brought together and given preferable before or immediately after the title.

Geometry specifications must be given separately for each direction. Thus for a cylindrical reactor the specification for the radial direction should be CYLINDRICAL, for the axial direction Z c.

If several media contain fissionable materials and have some $\theta_g \neq$ $\neq 0$, it must be specified in which of them the lateral buckling is to be evaluated. An asterisk after the material index is used as indicator:

```
DIVISIONS
1*    2     5     4
```

Generally the one-dimensional approximations are not suited for configurations of this kind.

If an approximate value of $\alpha_z^2$ or $\alpha_r^2$ is known from similar problems it can be used as initial estimate. Thus if an estimate of $\alpha_z^2$ is known, it should be given as LATERAL LEAKAGE for the radial direction.

The current estimates of $\alpha_z^2$ and $\alpha_r^2$ are preserved by RETAIN SOLUTION and the equivalent hand-switch setting.

The directives ACCURACY, ADJOINT, RETAIN SOLUTION, FLUX and INTEGRALS relate to both directions, LOOP to one direction only. If the successive flux estimates are required for a radial as well as an axial position, two directives should be given, one in the radial, the other in the axial data set.

## 13. INTEGRALS

The flux integrals and the highest thermal flux are listed for each direction.

The shape factor printed out is the product of the individual shape factors of the two directions as defined in paragraph 6.

## 14. Normalization

The radial and axial equations in the example above are solved separately and normalized separately as described in paragraph 9. Thus if the radial flux $\varphi_r(0)$ and the axial flux $\varphi_z(c)$, $z = c$ being the central position on the axis, are printed out by LOOP directives, the values will generally be different. Before the FLUX printing takes place, however, the solutions will be renormalized so that

$$\max \left\{ S_r^* \right\} = \max \left\{ S_z^* \right\} = \overline{S^*}$$

and

$$\frac{\int \frac{S_r^* \; S_z^*}{\bar{S}^*} \; r \, dr \, dz}{\int \delta \; r \, dr \, dz} = 1$$

the integrals taken over the reactor volume. The quantity $\delta$ is defined as in paragraph 9.

## 15. Limitations

For two-group calculations the conditions given in paragraph 10 must be satisfied for each direction separately. Thus 21 radial and 21 axial divisions are permitted in the same problem.

## C. The multigroup diffusion equations

### 16. The group equations

The basic group diffusion equation is

$$D_g \nabla^2 \varphi_g - \sigma_g \varphi_g + S_g' + (N_g + \nu \theta_g) S^* = 0 ; \quad g = 1 \ldots G \qquad (5)$$

$$S^* = \sum_{g=1}^{G} \sigma_g^* \varphi_g$$

$$S_g' = \begin{cases} 0 & \text{for } g = 1 \\ \sum_{g'=1}^{g-1} \sigma_{g,g'} \varphi_{g'} & \text{for } g = 2, 3, \ldots, G \end{cases}$$

The enumeration of the groups starts from the highest energy level; thus subscript G refers to the thermal group. The parameters and the boundary conditions are analogous to those used in the two-group equations. Generally $S_g'$ is the slowing down source in group g, $\sigma_{g,g'}$ being the macroscopic cross section for transfer of neutrons from group $g'$ to group g.

### 17. Data preparation

As an example let us consider a three-group problem with fission from groups 2 and 3 into group 1 and slowing down from one group to the next. The constants in one of the media are:

Medium

| g | $D_g$ | $\sigma_g$ | $\sigma_g^*$ | $N_g$ | $\theta_g$ |
|---|---|---|---|---|---|
| 1 | 1.3180 | 0.03056 | 0 | 0 | 1 |
| 2 | 0.9967 | 0.03161 | 0.00397 | 0 | 0 |
| 3 | 0.9638 | 0.04751 | 0.03512 | 0 | 0 |

| g | $\sigma_{g,g'}$ $g'$ 1 | 2 | 3 |
|---|---|---|---|
| 1 | | | |
| 2 | 0.03056 | | |
| 3 | 0 | 0.02701 | |

The constants should be listed in the order $D_1\ D_2 \cdots\ \sigma_1\ \sigma_2 \cdots\ \sigma_1^*\ \sigma_2^* \cdots$ $N_1\ N_2 \cdots\ \theta_1\ \theta_2 \cdots\ \sigma_{2,1}\ \sigma_{3,1}\ \sigma_{3,2} \cdots\ \sigma_{G,1} \cdots\ \sigma_{G,G-1}$.

```
CONSTANTS
      3     18
1     1.3180     0.9967     0.9638
      0.03056    0.03161    0.04751
      0     0.00397     0.03512
      0     0     0
      1     0     0
      0.03056     0     0.02701
```

Constants $\sigma_g^*$, $\sigma_{g,g'}$, $N_g$ and $\theta_g$ which are zero in all media can be omitted if two Pattern symbols are given to indicate the positions of the non-zero constants. In this case

| $\sigma_g^*$ | $N_g$ | $\theta_g$ | $\sigma_{g,g'}$ |
|---|---|---|---|
| 0 | 0 | 1 | |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 1 |

The constants are listed in the same order as above, the zeros omitted. Thus:

```
CONSTANTS
   3    11
1  1.3180       0.9967       0.9638
   0.03056      0.03161      0.04751
   0.00397      0.03512
   1
   0.03056      0.02701
PATTERN

001
100
100

1
01
```

The two integers following CONSTANTS are the number of groups and the number of constants in each medium.

The Pattern symbols are common to all media and only constants which are zero in all media can be entered as zeros in the Pattern symbols.

If no PATTERN directive is given, it is assumed that all constants are in the lists, i.e. the program will use patterns with 1's in all positions. The Pattern symbols can of course be used in two-group as well as in multigroup calculations, for instance to get rid of $\sigma_1^*$, $N_2$ and $\theta_2$.

PATTERN must come after CONSTANTS and may come before TITLE. The Pattern symbols will not be cleared between problems. Thus it is convenient to give them immediately after CONSTANTS and on the same tape if several are used.

Other directives are used as in two-group calculations.

## 18. Limitations

Let us define

G = the number of energy groups

M = the number of mesh intervals

d = the number of divisions

k = the number of constants in each medium (the second of the two integers following CONSTANTS)

q = the number of 1's in the $\sigma_{g,g'}$-pattern

m = the number of media for which constants are given

We write

$$s = 1 + \left[\frac{k-1}{32}\right]$$

$$t = 1 + \left[\frac{M+d-1}{32}\right]$$

The limitations are

(i) $G \leq 20$

(ii) $d \leq 21$

(iii) $M + d \leq 319$

(iv) $ms + q + G(6 + 2t) \leq 319$

Restriction (iv) relates to a computer with 512 sectors backing store. It is not, however, a very severe limitation. With $G \leq 9$ and a reasonable number of media conditions (ii) and (iii) will be sufficient.

For two-direction problems condition (iv) is replaced by a more complicated one. Generally there must be room for both systems simultaneously

in the backing store, but some sectors are used in common. In most practical cases, however, restrictions (ii) and (iii) applied separately to the two systems will be sufficient.

The machine prints out a fault indication if the capacity is exceeded.

## D. The methods of calculation

### 19. The geometry

In a one-dimensional space the Laplacian can be written

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{p}{x}\frac{d}{dx}$$

where p = 0, 1 or 2 in plane, cylindrical and spherical geometry respectively. In the spherical case it may be advantageous to apply the transformation $\varphi \rightarrow \psi = x\varphi$. The transformed group equation (5) will be

$$D_g \frac{d^2}{dx^2}\psi_g - \sigma_g \psi_g + S_g'(\psi) + (N_g + \nu\theta_g) S^*(\psi) = 0$$

which is the ordinary group equation in plane geometry. Since the numerical solution will be simplified in the plane case, the program makes use of this transformation and calculates the functions $\psi_g$ subject to boundary conditions appropriately transformed. LOOP printing gives the values of $\psi$, FLUX and INTEGRALS the values of $\varphi$.

### 20. The source iterations

The eigenvalue problem set by eqs (5) is solved by the source iteration process described for instance by Hassitt (ref. 1). A simple extrapolation from successive source estimates is used to speed up the convergence.

### 21. The buckling iterations

In the iterative process used in two-direction problems (section B) one has to determine the radial or axial buckling of an equivalent bare reactor. The following method is employed.

Suppose we have solved the radial equations (3) with an estimated axial buckling $\alpha_z^2$ and obtained a critical $\nu$ and a critical flux distribution. We assume the solution to be of the form

$$\nabla^2 \varphi_g + B^2 \varphi_g = 0$$

and substitue $-B^2 \varphi_g$ for $\nabla^2 \varphi_g$ in (5). $B^2$ can easily be shown to be an eigenvalue of a GxG matrix (G = the number of energy groups). The smallest eigenvalue is computed and the radial buckling of the equivalent bare reactor

is found from the relation

$$B^2 = \alpha_r^2 + \alpha_z^2$$

In the next stage of the process an axial buckling is determined analogously.

## 22. The differential equations

The iterative cycle involves the solution of several differential equations of the form

$$\lambda \nabla^2 \varphi - \sigma \varphi + S = 0$$

where the Laplacian operator $\nabla^2 = \frac{d^2}{dx^2} + \frac{p}{x} \frac{d}{dx}$

Let us consider the more general problem of solving an ordinary linear second-order equation of the form

$$y'' + f(x)\, y' + g(x)\, y + h(x) = 0 \qquad (6)$$

subject to the boundary conditions

$$a\, y' + A\, y + \alpha = 0 \qquad \text{for } x = x_a \qquad (7a)$$

$$b\, y' + B\, y + \beta = 0 \qquad \text{for } x = x_b \qquad (7b)$$

It is assumed that all functions are continuous.

The solution of a boundary value problem of this kind may be written

$$y = c\, u + v$$

where c is a constant, u is a solution of the homogeneous equation obtained by setting $h(x) \equiv 0$, $\alpha = \beta = 0$, and v is a particular solution of the original equation satisfying the boundary conditions at $x = x_a$. If the functions u and v cannot be found explicitly, they may be evaluated by a suitable step-by-step method. The constant c is then determined so that $(cu + v)$ satisfies the boundary condition at $x = x_b$.

In many cases, however, this method will give numerical troubles. Often the functions u and v increase rapidly and the values of (cu + v) will be obtained as small differences between large numbers. Thus the method will be unstable.

Another way of tackling the problem would be a "factorization" process. Using the differential operator notation we rewrite the equation

$$\left[ D^2 + f(x)\ D + g(x) \right] y + h(x) = 0$$

Let us take two first-order factors $[ D + r(x)]$ and $[ D + s(x) ]$ and try to define the functions $r(x)$ and $s(x)$ so that the product of the operators is equal to the second-order operator in the given equation.

$$\left[ D^2 + f\,D + g \right] y = \left[ D + r \right] \left[ D + s \right] y$$

We expand the right hand side and equate corresponding terms:

$$\begin{cases} s + r = f \\ [ D + r]\ s = g \end{cases} \tag{8}$$

Thus we have

$$[ D + r(x)]\ [ D + s(x)]\ y + h(x) = 0 \tag{9}$$

$r(x)$ and $s(x)$ subject to conditions (8). Now let us define a function $v(x)$ by the equation

$$[ D + s(x)]\ y + v(x) = 0 \tag{10}$$

Substitution in (9) gives

$$[ D + r]\ v - h(x) = 0 \tag{11}$$

From eqs (8), (10) and (11) we obtain

$$[ D - s + f]\ s - g = 0$$
$$[ D - s + f]\ v - h = 0$$
$$[ D + s]\ y + v = 0$$

which is the same as

$$s' - s^2 + f(x)\ s - g(x) = 0 \qquad (12a)$$

$$v' - sv + f(x)\ v - h(x) = 0 \qquad (12b)$$

$$y' + s(x)\ y + v(x) = 0 \qquad (12c)$$

Thus the second-order equation (6) is replaced by an equivalent system of three first-order equations. It remains to find the appropriate boundary conditions.

We assume that, in equation (7a), the coefficient $a \neq 0$. Then, taking

$$s(x_a) = A/a$$
$$v(x_a) = \alpha/a$$

the first boundary condition is satisfied. The initial values define $s(x)$ and $v(x)$ uniquely, thus also the values at $x = x_b$. Eqs (7b) and (12c) give

$$b\,y' + B\,y + \beta = 0$$
$$y' + s(x_b)\ y + v(x_b) = 0$$

If this system can be solved uniquely we obtain $y(x_b)$; if not, the conditions (7) permit an infinite number of solutions, or none. The end-point value and equation (12c) define uniquely a function $y$ which satisfies the original equation (6) subject to boundary conditions (7).

If the boundary condition at $x = x_a$ takes the form $y = c$, i. e. if in equation (7a) the coefficient $a = 0$, we have to try a different procedure. We impose the following conditions on $s(x)$ and $v(x)$:

$$\begin{cases} \lim\limits_{x \to x_a} s(x) = \infty \\ \\ \lim\limits_{x \to x_a} \dfrac{v(x)}{s(x)} = \alpha/A \qquad (A \neq 0, \text{ since } a = A = 0 \text{ has no meaning}) \end{cases} \qquad (13)$$

By considering the functions

$$U = \begin{cases} 1/s, & x \neq x_a \\ 0, & x = x_a \end{cases}$$

$$V = \begin{cases} v/s, & x \neq x_a \\ \alpha/A, & x = x_a \end{cases}$$

one may verify that these conditions define $s(x)$ and $v(x)$ uniquely in the interval $x_a < x \leq x_b$.

The required solution is the function $y$ defined for $x = x_a$ as $-\alpha/A$ and for $x \neq x_a$ as the solution of (12) with boundary conditions (7b) and (13).

The numerical procedure will be as follows. Using a step-by-step method we solve eqs (12a) and (12b) for $s(x)$ and $v(x)$ by integrating from $x_a$ to $x_b$. Eqs (12c) and (7b) give $y(x_b)$ and finally we obtain the required solution by integrating equation (12c) from $x_b$ to $x_a$, i. e. in the direction of $x$ decreasing.

This method of solving differential equations has been described by several authors. The notation above is due to Mrs E. Cicely Ridley (ref. 3).

For the factorization of the equation

$$\lambda \nabla^2 \varphi - \sigma \varphi + S = 0$$

let us use two functions $\bar{s}(x)$ and $\bar{v}(x)$. They are defined by the equations

$$\bar{s}' - \bar{s}^2 + \frac{p}{x}\bar{s} + \frac{\sigma}{\lambda} = 0$$

$$\bar{v}' - \bar{s}\bar{v} + \frac{p}{x}\bar{v} - \frac{S}{\lambda} = 0$$

$$\varphi' + \bar{s}\varphi + \bar{v} = 0$$

We substitute $\bar{s} = s/\lambda$ and $\bar{v} = v/\lambda$.

$$s' = s^2/\lambda - \frac{p}{x}\, s - \sigma$$

$$v' = sv/\lambda - \frac{p}{x}\, v + S \qquad (14)$$

$$\lambda\varphi' = -(s\,\varphi + v)$$

The functions $s(x)$ and $v(x)$ defined by (14) are continuous even at interfaces between regions of dissimilar physical properties.

To integrate the equations over a mesh interval the program makes use of step formulas obtained by Taylor expansions about the mid-point of the interval. The formulas are of the two-point type and involve only the function values at the end-points of the interval.

## E. Sample problems

### Sample problem 1: axially bare cylinder, two energy groups

Sample problem 1 is the example considered in paragraph 3 (fig. 1). The extrapolated height of the reactor is assumed to be 292 cm, which gives a lateral leakage factor $\alpha^2 = (\frac{\pi}{292})^2 = 1.16 \cdot 10^{-4}$. The input data is as follows.

```
TITLE
SAMPLE PROBLEM 1.   12.1.1960
REACTIVITY AKG Ø Ø

CONSTANTS
      2   11
1     1.122  0.963  0.00320  0.00617  0  0.00617  .0  0  1.264    0  0.00253
2     1.033  0.862  0.00361  0.00017  0  0  0    0  0  0  0.00361
3      .654   .378  0.00107   .150    0  0  0    0  0  0   .00107

CYLINDRICAL
MESHPOINTS
0(4.64)116(5)141.0(2.5)151(1.0)156.0

DIVISIONS
1  2  2  3

BOUNDARY CONDITIONS
1  0   2.13  1  ETC

LATERAL LEAKAGE
1.16,-4
ACCURACY
.2,-4
START
WAIT
```

Output:

SAMPLE PROBLEM I. 12.1.1960

REACTIVITY AKG

R

1 1.19306

2 1.18490

3 1.17963

4 1.17823

5 1.17778

6 1.17760

7 1.17753

8 1.17750

9 1.17749

10 1.17748

Running time: 1 min 7 sec

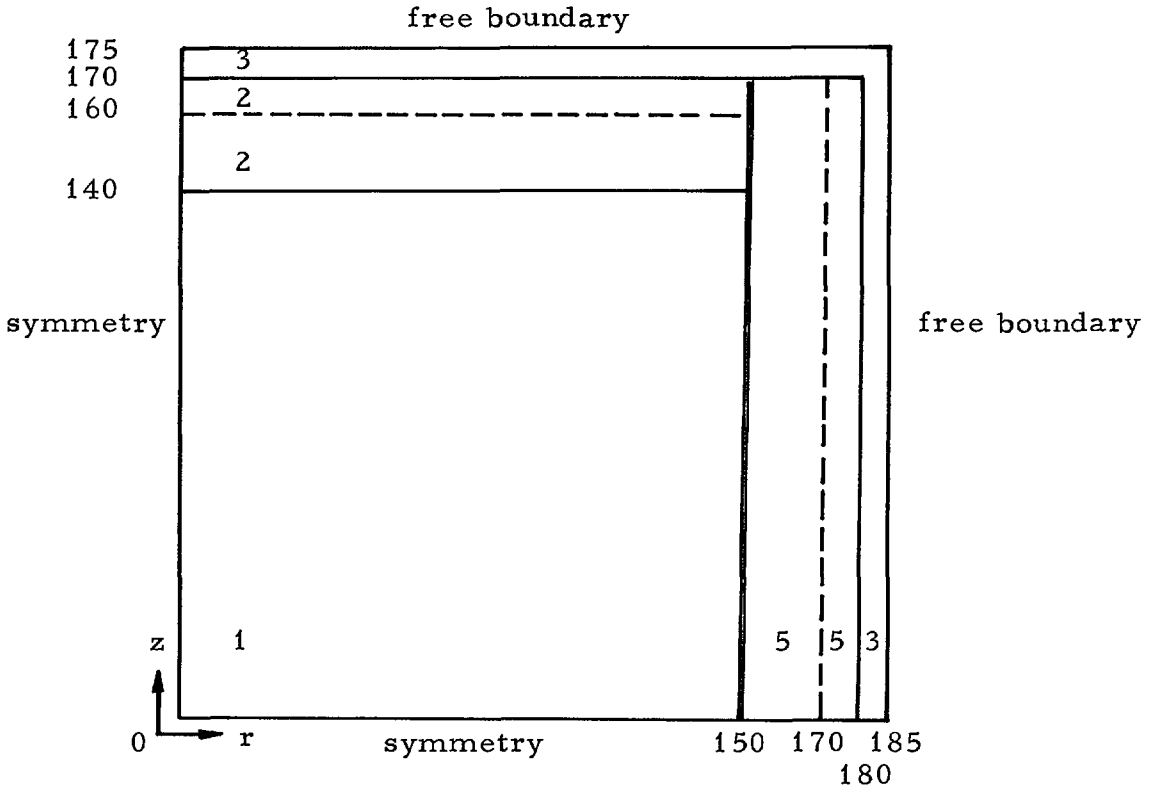



Fig. 3

Sample problem 2: two-direction problem, two energy groups

Consider a reactor with all-round reflectors (fig. 3). The program approximates it with an equivalent bare reactor. The input data is:

```
TITLE
SAMPLE PROBLEM 2    12.1.1960    ØØ

CONSTANTS
      2  11
1     1.411  1.034  .00887  .00433  0  .00433  0  0  1.128    0  .00786
2     1.344  1.084  .00888  .00008  0  0  0  0  0  0  0.00888
5     1.344  1.083  .00891  .00005  0  0  0  0  0  0  0.00891
3     0.654  0.378  .00107  .150    0  0  0  0  0  0  0.00107

CYLINDRICAL
MESHPOINTS
0(5.0)150. 0(5.0)170. 0(2.5)180. 0(1.0)185.0

DIVISIONS
1  5  5  3

BOUNDARY CONDITIONS
1  0  2.13  1  ETC

Z 0
MESHPOINTS
0(5.0)140. 0(5.0)160. 0(2.5)170. 0(1.0)175.0

DIVISIONS
1  2  2  3
BOUNDARY CONDITIONS
1  0  2.13  1  ETC

ACCURACY
0.00005
START
WAIT
```

Output:

```
SAMPLE PROBLEM 2    12.1.1960

R
        1     1.13070
        2     1.11703
        3     1.10870
        4     1.10567
        5     1.10440
        6     1.10385
        7     1.10361
        8     1.10350
        9     1.10345
Z
        1     1.11980
        2     1.11252
        3     1.10781
        4     1.10597
        5     1.10514
        6     1.10477
        7     1.10459
        8     1.10452
        9     1.10448
       10     1.10446
       11     1.10445

R
       10     1.10445
       11     1.10444
       12     1.10443
```

Running time: 2 min 18 sec.

Sample problem 3: axially bare cylinder, three energy groups

We consider a three-group problem with fission from groups 2 and 3 into group 1 and slowing down from one group to the next. The constants are

| | g | $D_g$ | $\sigma_g$ | $\sigma_g^*$ | $N_g$ | $\theta_g$ | $\sigma_{g,g-1}$ |
|---|---|---|---|---|---|---|---|
| | 1 | 1.3180 | 0.03056 | 0 | 0 | 1 | |
| medium 1 | 2 | 0.9967 | 0.03161 | 0.00397 | 0 | 0 | 0.03056 |
| | 3 | 0.9638 | 0.04751 | 0.03512 | 0 | 0 | 0.02701 |
| | 1 | 1.3180 | 0.03056 | 0 | 0 | 0 | |
| medium 2 | 2 | 0.9967 | 0.02682 | 0 | 0 | 0 | 0.03056 |
| | 3 | 0.9556 | 0.00929 | 0 | 0 | 0 | 0.02670 |
| | 1 | 1.4150 | 0.03882 | 0 | 0 | 0 | |
| medium 3 | 2 | 0.7069 | 0.04271 | 0 | 0 | 0 | 0.03882 |
| | 3 | 0.4918 | 0.01016 | 0 | 0 | 0 | 0.04063 |

For all media $\sigma^*$ in group 1, all N, $\theta$ in groups 2 and 3 and the cross section for slowing down from group 1 into group 3 are zero. They may be omitted if the positions of the non-zero constants are indicated by pattern symbols.

```
001
100
100

1
01
```

Using this device, the input data will be as follows.

```
TITLE
SAMPLE PROBLEM  3.    12.1.1960     ØØ
CONSTANTS
      3   11
1     1.318    0.9967    0.9638
      0.03056   0.03161   0.04751
      0.00397   0.03512
      1
      0.03056   0.02701
2     1.318    0.9967    0.9556
      0.03056   0.02682   0.00929
      0   0
      0
      0.03056   0.02670
3     1.415    0.7069    0.4918
      0.03882   0.04271   0.01016
      0   0
      0
      0.03882   0.04063

PATTERN

001
100
100

1
01

CYLINDRICAL
MESHPOINTS
0(3.0)63.0(2.0)75.0(2.0)95.0(1.0)101.0(1.0)106.0

DIVISIONS
1  1  2  2  3

BOUNDARY CONDITIONS
1  0   2.13  1  ETC

LATERAL LEAKAGE
4.1,-4

ACCURACY
0.5,-4
START
WAIT
```

Output:

SAMPLE PROBLEM 3. 12.1.1960

R

| | |
|---|---|
| 1 | 1.50514 |
| 2 | 1.48529 |
| 3 | 1.47237 |
| 4 | 1.46757 |
| 5 | 1.46546 |
| 6 | 1.46450 |
| 7 | 1.46406 |
| 8 | 1.46384 |
| 9 | 1.46373 |
| 10 | 1.46367 |
| 11 | 1.46364 |
| 12 | 1.46362 |

Running time: 1 min 57 sec.

References.

1. HASSITT A.
Progress in Nuclear Energy, Series I, vol. II (1958), p. 271.

2. HASSITT A.
Reports AERE T/R 2487 and AERE T/R 2859.

3. RIDLEY E. C.
Proc. Camb. Phil. Soc. 53 (1957), p. 442.

4. MARCHUK G. I.
J. Nuclear Energy 3 (1956), p. 238.

SL/AJ

## Appendix 1: The directives used in the program.

1. TITLE.

The program will read, store and print out the characters on the lines after the directive until it comes to two consecutive figure shifts. The directive, which must not be omitted, indicates that data for a new problem will follow.

The title can be printed out at any later date by the directive

2. PRINT TITLE.

3. CONSTANTS.

To be followed by two integers, the number of energy groups and the number of constants in each medium (the same for all media). The constants, preceded by a material index chosen from the integers 1, ..., 99, are listed in the following order.

$$D_1 \; D_2 \ldots \sigma_1 \; \sigma_2 \ldots \sigma_1^* \; \sigma_2^* \ldots \; N_1 \; N_2 \ldots \theta_1 \; \theta_2 \ldots$$

$$\sigma_{2,1} \; \sigma_{3,1} \; \sigma_{3,2} \ldots \sigma_{G,1} \ldots \sigma_{G,\,G-1}$$

Constants which are zero in all media can be omitted if the positions of the non-zero constants are indicated by a directive

4. PATTERN.

The Pattern symbols are described in paragraph 17.

The constants and the Pattern symbols will be wiped out only if the word CONSTANTS is read again and the following two integers are different from those read before.

5. Z c (or X c or Y c)
   CYLINDRICAL
   SPHERICAL

These words specify the geometry of the problem. The number c in the directives for plane geometry indicates a position on the axis such that the plane z = c is approximately the central plane of the reactor. It is used for some initial estimates and does not affect the final result of the calculation.

6. MESHPOINTS.

The mesh is defined by a sequence

$$\bar{x}_o\ (h_1)\ \bar{x}_1\ \ (h_2)\ \bar{x}_2 \ldots\ (h_d)\ \bar{x}_d$$

where $\bar{x}_o$, $\bar{x}_1$ ... are the values of the space variable at the division boundaries and $h_1, h_2$ ... are the mesh lengths. The program adjusts the mesh lengths so that the last meshpoint in each division coincides with the outer division boundary.

7. DIVISIONS.

The integers on the next line specify the materials of the reactor.

8. BOUNDARY CONDITIONS.

The boundary conditions are of the form

$$a\ D\ \varphi'(x_a)\ +\ A\ \varphi(x_a)\ =\ 0;\quad b\ D\ \varphi'(x_b)\ +\ B\ \varphi(x_b)\ =\ 0$$

The coefficients are listed for each group on a separate line.

$$\begin{array}{cccc} a_1 & A_1 & b_1 & B_1 \\ \vdots & & & \vdots \\ a_G & A_G & b_G & B_G \end{array}$$

If the same conditions hold for all the groups, one may write

a A b B ETC

9. LATERAL LEAKAGE

$a^2$

This directive adds a lateral leakage term $-a^2$ to the Laplacian:

$$\nabla^2 = \nabla_x^2 - a^2$$

The same value is used for all groups and all positions.

10. LOOP.

$\xi$

The current estimates of $\varphi$ or in spherical geometry $\xi\varphi$ at the point $x = \xi$ (or the nearest meshpoint) are printed out at the end of every cycle.

11. ACCURACY

$\epsilon$

This directive sets the quantity $\epsilon$ in the convergence criterion in paragraph 5. If the directive is omitted, the program sets $\epsilon = 0$.

12. RETAIN SOLUTION.

The solution of the preceding case will be used as initial estimate in the problem to be solved.

13. ADJOINT.

The adjoint equations are solved.

14. START.

Initiates the calculation. First the constants are arranged in a more convenient order, then, after a short hoot, entry is made to the iterative cycle.

15. FLUX.

The values of the space variable and the fluxes at each meshpoint are listed. Division boundaries are indicated by arrows.

16. INTEGRALS

The integrals defined in paragraph 6 are calculated and printed out.

17. WAIT.

Hoot stop.

| Directive | must come before | must come after |
|---|---|---|
| 1. TITLE | | |
| 2. PRINT TITLE | | |
| 3. CONSTANTS | 14 | |
| 4. PATTERN | 14 | 3 |
| 5. Z c etc. | 6 | 1 |
| 6. MESHPOINTS | 7 | 5 |
| 7. DIVISIONS | 14 | 6 |
| 8. BOUNDARY CONDITIONS | 14 | 3, 5 |
| 9. LATERAL LEAKAGE | 14 | 5 |
| 10. LOOP | 14 | 5 |
| 11. ACCURACY | 14 | 5 |
| 12. RETAIN SOLUTION | 14 | 5 |
| 13. ADJOINT | 14 | 5 |
| 14. START | | 5-8 |
| 15. FLUX | | 14 |
| 16. INTEGRALS | | 14 |
| 17. WAIT | | |

Underlined directives are indispensable.

## Appendix 2: Punching and operating instructions.

### Data layout.

A directive should occupy a separate line on the data sheet. Only the first two letters are relevant.

The layout of numbers on the lines after a word is irrelevant except in the case of BOUNDARY CONDITIONS.

Numbers can be punched in either fixed or floating point form. They are terminated by Space, CR LF or brackets.

Every CR must be immediately followed by LF.
Erases are ignored in all positions, in directives, in the title and in numbers.

### Operating instructions.

The program is on a binary tape.

To run in the tape: de-isolate sectors 2 - 479, set key 2, press the Clear Tape and Initial Transfer buttons

When the program is in, the machine comes to a hoot stop. Depressing and releasing key 9 will cause it to read the data tape.

The word WAIT encountered during data input will produce a hoot stop which is passed by depressing and releasing key 9.

Unrecognised directives and some other faults will cause the machine to print out a fault indication and stop. On prepulse it jumps to the WAIT routine ready for more data tape.

If key 0 is set during input the solution of the preceding case will be retained and used as initial estimate in the problem to be solved. The key may be released when the iterative process is started (indicated by a short hoot).

If no ACCURACY directive is given on the data tape, the machine operator has to terminate the iterative process when the desired accuracy is obtained.

(a) One-direction problems.

The keys are examined at the end of every cycle. If key 9 is up the process is terminated and the next directive read from the data tape.

(b) Two-direction problems.

Suppose the radial problem for a cylindrical reactor is being solved. When the estimates of the critical parameter have settled down, put key 9 up. At the end of the current cycle the program will compute the radial buckling and start the iterations for the axial problem. Release key 9. The axial problem is interrupted in the same way and the machine returns to the radial problem. When the radial and axial values of $\nu$ are equal, set key 9 and key 8, and at the end of the current cycle the program will take the next directive from the tape.

If an ACCURACY directive is given but the operator decides to control the process manually, he can run the program with key 7 up. This will set all internal convergence tests out of operation.

LOOP printing is suppressed as long as key 6 is set.

In two-direction problems the successive buckling estimates are printed out if key 5 is set.

In the event of machine breakdown the calculations can be restarted by an Initial Transfer. There are three choices:

- no key set: read directive
- key 0 set : resume the iterative process
- key 3 set : print the fluxes (equivalent to FLUX).

The program comes to a stop which enables the keys to be reset. On prepulse entry is made to the routine required.

Fault print.

A number of obvious blunders are detected by the program. When one of these is encountered, the machine prints out

FAULT n

where n indicates the particular fault, and stops. On prepulse it jumps to the WAIT routine, ready for more data tape.

Fault 1. Word not recognised. Will also appear if the machine reads a number when expecting a directive.

2. Directives in wrong order.

3. Numbers following a directive not accepted
Will also appear if the machine reads a word when expecting a number.

4. Reference to non-existant material index.

5. Capacity exceeded.

6. The conditions for adjoint equations (paragraph 7) are not satisfied.

7. The number of 1's in the pattern symbols must be equal to k - 2G, where k is the number of constants in each medium and G the number of energy groups.

8. The number $\xi$ in a LOOP directive > the outer boundary.

9. $\sigma_g^* = 0$ or $\theta_g = 0$ for all g and in all regions.
In two-direction problems also: two or more regions contain fissionable materials and it must be indicated which constants are to be used for computing the buckling.

Spurious characters, for instance two decimal points in a number or CR followed by a character $\neq$ LF, will cause various loop stops.

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| 12 | Geometric buckling measurements using the pulsed neutron source method. | *N. G. Sjostrand, J. Mednis, T. Nilsson* | 1959 | 12 | 4 |
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