AE-129

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The P_1-approximation for the Distribution of Neutrons from a Pulsed Source in Hydrogen

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Summary

The asymptotic distribution of neutrons from a pulsed, high energy source in an infinite moderator has been obtained earlier [1] in a "diffusion" approximation. In that paper the cross section was assumed to be constant over the whole energy region and the time derivative of the first moment was disregarded. Here, first, an analytic expression is obtained for the density in a P_1-approximation. However, the result is very complicated, and it is shown that an asymptotic solution can be found in a simpler way. By taking into account the low hydrogen scattering cross section at the source energy it follows that the space dependence of the distribution is less than that obtained earlier. The importance of keeping the time derivative of the first moment is further shown in a perturbation approximation.

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LIST OF CONTENTS

Exact solution

The distribution, $N(x, v, t, \mu)$, of neutrons in an infinite moderator of hydrogen gas with a plane source giving a burst of high energy neutrons at time $t = 0$, is given by

$$\frac{\partial N}{\partial t} + v\mu \frac{\partial N}{\partial x} + v\Sigma_s(v)N = \int_v^{v_o} dv' \int_{-1}^{+1} d\mu' \, v'\Sigma(v' \longrightarrow v, \mu_o)N + $$

$$+ \delta(x)\delta(t)\delta(v - v_o) \qquad (1)$$

where

$$\Sigma(v' \longrightarrow v, \mu_o) = 2\Sigma_s(v') \frac{v}{v'^2} \; \delta(\mu_o - \frac{v}{v'}) \qquad (2)$$

The boundary and initial conditions are

$$N(\pm\infty, v, t, \mu) = 0 \text{ and } N(x, v, t, \mu) = 0, \quad t \leq 0 \qquad (3)$$

In order to find a solution of (1) we write

$$N(x, v, t, \mu) = N'\delta(v - v_o) + N'' \qquad (4)$$

In a P_1-approximation in μ, $N''(x, v, t, \mu) = \sum_l \frac{2l + 1}{2} \; P_l(\mu) \, N_l(x, v, t)$, where

$$\frac{\partial N_o}{\partial t} + v \frac{\partial N_1}{\partial x} + v\Sigma_s(v)N_o = \int_v^{v_o} dv' \; v'\Sigma_o(v' \longrightarrow v)N_o + S_o(x, v, t)$$

$$(5)$$

$$\frac{\partial N_1}{\partial t} + \frac{v}{3} \frac{\partial N_o}{\partial x} + v\Sigma_s(v)N_1 = \int_v^{v_o} dv' \; v'\Sigma_1(v' \longrightarrow v)N_1 + S_1(x, v, t)$$

Here, according to [1], the source term is given by

$$S_o(x,v,t) = 2\Sigma_s(v_o)\frac{v}{v_o^2}\,\epsilon(v_o t - |x|)\,\frac{\epsilon(t)}{t}\,e^{-\Sigma_s(v_o)\,v_o t}$$

$$S_1(x,v,t) = 2\Sigma_s(v_o)\frac{v^2}{v_o^4}\,\epsilon(v_o t - |x|)\,\frac{\epsilon(t)}{t^2}\,x\,e^{-\Sigma_s(v_o)\,v_o t} \qquad (6)$$

and

$$\Sigma_o(v'\longrightarrow v) = 2\,\frac{v}{v'^2}\,\Sigma_s(v')$$

$$\Sigma_1(v'\longrightarrow v) = 2\,\frac{v^2}{v'^3}\,\Sigma_s(v') \qquad (7)$$

$$\epsilon(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \qquad (8)$$

The energy variation of $\Sigma_s(v)$ is disregarded, $\Sigma_s(v) = \Sigma_s = 1.33$ cm^{-1}, except for $v = v_o$. Further we approximate the integral term in the second equation by introducing the transport cross section $\Sigma_{tr} = \Sigma_s(1 - \bar{\mu})$, where $\bar{\mu}$ is the average of the cosine of the scattering angle ($\bar{\mu} = 2/3$). The equations (5) have been treated earlier [1] by neglecting the time change of the current i. e. by putting $\partial N_1/\partial t = 0$ and by assuming $\Sigma_s(v_o) = \Sigma_s(v)$. We now want to keep $\partial N_1/\partial t$ but in order to be able at a later stage to regard it as a perturbation we introduce a factor ϵ multiplying it. The solution thus obtained will be assumed to be analytic in ϵ and valid even for $\epsilon = 1$.

First we take only the source term S_o into account, but S_1 will be introduced again later.

We now make a Laplace - and a Fourier-transformation of N_o and N_1 using the parameter k to indicate the Fourier transform with respect to x and the parameter s to indicate the Laplace transform with respect to t, i. e.

$$N_i(x, v, s) = \int_0^{\infty} dt\ e^{-st} N_i(x, v, t), \qquad i = 0.1 \tag{9}$$

$$N_i(k, v, s) = \int_{-\infty}^{+\infty} dx\ e^{ikx} N_i(x, v, s), \qquad i = 0.1 \tag{10}$$

The equations obtained from (5) are

$$(s + v\Sigma_s)\, N_o(k, v, s) - ikv\, N_1(k, v, s) = 2\Sigma_s v \int_v^{v_o} dv' \frac{N_o}{v'} + S_o(k, v, s)$$

$$(11)$$

$$(\epsilon s + v\Sigma_{tr}) N_1(k, v, s) - \frac{ik}{3} v\, N_o(k, v, s) = 0$$

Elimination of N_1 gives

$$\left[s + v\Sigma_s + \frac{k^2 v^2}{3(\epsilon s + v\Sigma_{tr})} \right] N_o(k, v, s) = 2\Sigma_s v \int_v^{v_o} dv' \frac{N_o}{v'} + S_o(k, v, s)$$

$$\ldots\ldots (12)$$

Derivating with respect to v, we get

$$\left[s + v\Sigma_s + \frac{k^2 v^2}{3(\epsilon s + v\Sigma_{tr})} \right] \frac{dN_o}{dv} + \left[2\Sigma_s - \frac{s}{v} + \frac{k^2 v}{3(\epsilon s + v\Sigma_{tr})} - \right.$$

$$\left. - \frac{k^2 v^2 \Sigma_{tr}}{3(\epsilon s + v\Sigma_{tr})^2} \right] N_o = 0 \tag{13}$$

with the boundary condition

$$N_o(k, v_o, s) = \frac{3(\epsilon s + v_o \Sigma_{tr})}{3(s + v_o \Sigma_s)(\epsilon s + v_o \Sigma_{tr}) + k^2 v_o^2} S_o(k, v_o, s) \qquad (14)$$

The solution of (13) is

$$N_o(k, v, s) = \frac{v(\epsilon s + v\Sigma_{tr})}{v_o(\Sigma_s \Sigma_{tr} + k^2/3)} \cdot \frac{(v_o + \alpha s)^A}{(v + \alpha s)^{1+A}} \cdot \frac{(v_o + \beta s)^B}{(v + \beta s)^{1+B}} \cdot$$

$$\cdot S_o(k, v_o, s) = f(s) \cdot S_o(k, v_o, s) \qquad (15)$$

with

$$\begin{matrix}\alpha \\ \beta\end{matrix} = \frac{\epsilon \Sigma_s + \Sigma_{tr} \pm \sqrt{(\epsilon \Sigma_s - \Sigma_{tr})^2 - \epsilon \cdot \frac{4k^2}{3}}}{2(\Sigma_s \Sigma_{tr} + \frac{k^2}{3})} \qquad (16)$$

and

$$\begin{matrix}A \\ B\end{matrix} = \frac{\Sigma_s \Sigma_{tr}}{\Sigma_s \Sigma_{tr} + k^2/3} \mp \frac{\Sigma_s \left[\Sigma_{tr} (\epsilon \Sigma_s - \Sigma_{tr}) + \frac{2k^2 \epsilon}{3}\right]}{(\Sigma_s \Sigma_{tr} + k^2/3) \sqrt{(\epsilon \cdot \Sigma_s - \Sigma_{tr})^2 - \epsilon \, 4k^2/3}} \qquad (17)$$

From (15) we obtain

$$N_o(k, v, t) = \int_0^t d\tau \, f(t - \tau) \, S_o(k, v_o, \tau) \qquad (18)$$

where

$$f(t) = L^{-1}\{f(s)\} \text{ and } S_o(k, v_o, t) = \int_{-\infty}^{+\infty} dx\, e^{ikx} S_o(x, v, t) =$$

$$= \frac{4\Sigma_s(v_o)}{v_o} \frac{e^{-\Sigma_s(v_o) v_o t}}{kt} \sin k v_o t \tag{19}$$

It remains to find the inverse Laplace transform of f(s). We here put $\epsilon = 1$. Introducing $s' = s + v\Sigma_{tr}$ we find

$$f(t) = e^{-v\Sigma_{tr} t} L^{-1}\{f'(s)\} \tag{20}$$

where

$$f'(s) = s^{-1}\left[1 + \frac{v(1-\alpha\Sigma_{tr})}{s\alpha}\right]^{-A-1}\left[1 + \frac{v(1-\beta\Sigma_{tr})}{s\beta}\right]^{-B-1}$$

$$\left[1 + \frac{v_o - \alpha v\Sigma_{tr}}{s\alpha}\right]^{A}\left[1 + \frac{v_o - \beta v\Sigma_{tr}}{s\beta}\right]^{B} \tag{21}$$

According to [2] the inverse laplace transform of f'(s) can be written

$$L^{-1}\{f'(s)\} = \varnothing_2 \left\{A+1,\ B+1,\ -A,\ -B;\ 1;\ -\frac{v(1-\alpha\Sigma_{tr})}{\alpha}t,\ -\frac{v(1-\beta\Sigma_{tr})}{\beta}t,\right.$$

$$\left. -\frac{v_o - \alpha v\Sigma_{tr}}{\alpha}t,\ -\frac{v_o - \beta v\Sigma_{tr}}{\beta}t\right\} \tag{22}$$

where $\varnothing_2$ is a confluent hypergeometric function

$$\emptyset_2 \left\{b_1, \ldots, b_n; c; z_1, \ldots, z_n\right\} = \Sigma \frac{(b_1)_{m_1} \ldots (b_n)_{m_n}}{(c)_{m_1+\ldots+m_n} m_1! \ldots m_n!} \cdot$$

$$\cdot z_1^{m_1} \ldots z_n^{m_n} \qquad (23)$$

$$(a)_o = 1, \ (a)_n = a(a+1) \ldots (a+n-1), \ a = b_k, \ c$$

This function can be simplified by using the rules [3]

$$\emptyset_2 (b_1, \ldots, b_n; \ c; \ z_1, \ldots, z_n) = \emptyset_2 \{b_1, \ldots, b_j - 1, \ldots b_n; \ c; \ z_1, \ldots, z_n\} + \frac{z_j}{c} \emptyset_2 \{b_1, \ldots, b_n; \ c+1; \ z_1, \ldots, z_n\}$$

$$\emptyset_2 \{b_1, \ldots, b_n; \ b_1 + \ldots + b_n; \ z_1, \ldots, z_n\} = e^{z_n} \emptyset_2 \{b_1, \ldots, b_{n-1}; \ b_1 + \ldots + b_n; \ z_1 - z_n, \ldots, z_{n-1} - z_n\} \qquad (24)$$

$$\emptyset_2 \{b_1, \ldots, b_n; \ c; \ z_1, \ldots, z_n\} = e^{z_j} \emptyset_2 \{b_1, \ldots, b_{j-1}, \ c - \Sigma b_j, \ b_{j+1}, \ldots, b_n; \ c; \ z_1 - z_j, \ldots, -z_j, \ldots, z_n - z_j\}$$

Introducing the notations

$$z_1 = - \frac{v(1 - \alpha \Sigma_{tr})}{\alpha}, \quad z_2 = - \frac{v(1 - \beta \Sigma_{tr})}{\beta}$$

$$z_3 = - \frac{v_o - \alpha v \Sigma_{tr})}{\alpha}, \quad z_4 = - \frac{v_o - \beta v \Sigma_{tr}}{\beta} \qquad (25)$$

we see that we can write (22)

$$\phi_2\{A+1,\ B+1,\ -A,\ -B;\ 1;\ z_1 t,\ z_2 t,\ z_3 t,\ z_4 t\} =$$

$$= \phi_2\{A,\ B+1,\ -A,\ -B;\ 1;\ z_1 t,\ z_2 t,\ z_3 t,\ z_4 t\} +$$

$$+ z_1 t \phi\{A+1,\ B+1,\ -A,\ -B;\ 2;\ z_1 t,\ z_2 t,\ z_3 t,\ z_4 t\} =$$

$$= e^{z_1 t}\,\phi_2\{B+1,\ -A,\ -B;\ 1;\ (z_2-z_1)t,\ (z_3-z_1)t,\ (z_4-z_1)t\} +$$

$$+ z_1 t\, e^{z_1 t}\,\phi_2\{B+1,\ -A,\ -B;\ 2;\ (z_2-z_1)t,\ (z_3-z_1)t,\ (z_4-z_1)t\} \qquad (26)$$

According to [3] we can further introduce integral representations for the functions ϕ_2 giving for the last term in (26)

$$\phi_2\{B+1,\ -A,\ -B;\ 2;\ (z_2-z_1)t,\ (z_3-z_1)t,\ (z_4-z_1)t\} =$$

$$= \frac{t^{-1}}{\Gamma(c)\Gamma(c')}\int_0^t d\tau\ \tau^{c-1}\,\phi_2\{B+1,\ -B;\ c;\ (z_2-z_1)\tau,\ (z_4-z_1)\tau\}\cdot$$

$$\cdot\ (t-\tau)^{c'-1}\,\phi_2\{-A;\ c';\ (z_3-z_1)(t-\tau)\} \qquad (27)$$

with the conditions that $c + c' = 2$, $\mathrm{Re}\, c > 0$, $\mathrm{Re}\, c' > 0$. We choose $c = c' = 1$ and get

$$\phi_2\{B+1,\ -A,\ -B;\ 2;\ (z_2-z_1)t,\ (z_3-z_1)t,\ (z_4-z_1)t\} =$$

$$= t^{-1}\int_0^t d\tau\ e^{(z_4-z_1)\tau}\,\phi_2\{B+1;\ 1;\ (z_2-z_4)\tau\}\,\phi_2\{-A;\ 1;\ (z_3-z_1)(t-\tau)\}$$

$$\ldots\ (28)$$

The first term in (26) gives a bit more troubles. Here

$$\phi_2\{B+1,\ -A,\ -B;\ 1;\ (z_2-z_1)t,\ (z_3-z_1)t,\ (z_4-z_1)t\} =$$

$$= e^{(z_3-z_1)t}\ \phi_2\{B,\ A,\ -B;\ 1;\ (z_2-z_3)t,\ (z_1-z_3)t,\ (z_4-z_3)t\} +$$

$$+ (z_2-z_3)t\, e^{(z_2-z_1)t}\ \phi_2\{B+1,\ A,\ -B;\ 2;\ (z_2-z_3)t,\ (z_1-z_3)t,$$

$$(z_4-z_3)t\} \tag{29}$$

and

$$\phi_2\{B,\ A,\ -B;\ 1;\ (z_2-z_3)t,\ (z_1-z_3)t,\ (z_4-z_3)t\} =$$

$$= \frac{1}{\Gamma(c)\,\Gamma(c')} \int_0^t d\tau\, \tau^{c-1}\ \phi_2\{B,\ A;\ c;\ (z_2-z_3)\tau,\ (z_1-z_3)\tau\}\cdot$$

$$\cdot\ (t-\tau)^{c'-1}\ \phi_2\{-B;\ c';\ (z_4-z_3)(t-\tau)\} \tag{30}$$

with $c + c' = 1$ $\mathrm{Re}\, c > 0$, $\mathrm{Re}\, c' > 0$. We now have to regard two cases, namely when $2 > A + B > 1$ and $1 > A + B > 0$, i. e. when $0 < k < \sqrt{3\Sigma_s \Sigma_{tr}}$ and $k > \sqrt{3\Sigma_s\Sigma_{tr}}$. Calculations give with $c = A + B - 1$

$$\phi_2\{B,\ A,\ -B;\ 1;\ (z_2-z_3)t,\ (z_1-z_3)t,\ (z_4-z_3)t\} =$$

$$= \frac{1}{\Gamma(A+B-1)\,\Gamma(2-A-B)} \int_0^t d\tau\, \tau^{A+B-2}\, (t-\tau)^{1-A-B}\, \phi_2\{-B;\ 2-A-B;$$

$$(z_4-z_3)(t-\tau)\}\cdot\ e^{(z_2-z_3)\tau}\Big[\phi_2\{A;\ A+B-1;\ (z_1-z_2)\tau\} +$$

$$+ \frac{(z_2-z_3)\tau}{A+B-1}\ \phi_2\{A;\ B+A;\ (z_1-z_2)\tau\}\Big] \tag{31}$$

for $0 < k < \sqrt{3\Sigma_s\Sigma_{tr}}$ and

$$\phi_2\left\{B,\ A,\ -B;\ 1;\ (z_2-z_3)t,\ (z_1-z_3)t,\ (z_4-z_3)t\right\} =$$

$$= \frac{1}{\Gamma(A+B)\,\Gamma(1-A-B)} \int_0^t d\tau\ \tau^{A+B-1}\,(t-\tau)^{-A-B}\, e^{(z_2-z_3)\tau} \cdot$$

$$\cdot\ \phi_2\left\{A;\ B+A;\ (z_1-z_2)\tau\right\} \cdot \phi_2\left\{-B;\ 1-A-B;\ (z_4-z_3)(t-\tau)\right\}\ , \qquad (32)$$

for $k > \sqrt{3\Sigma_s\Sigma_{tr}}$.

These expressions are not valid for $k = 0$ and $k = \sqrt{3\Sigma_s\Sigma_{tr}}$. However, as we have to integrate over k from 0 to ∞ and as the integrand is finite these two points could be left out. The next step would now be to express the confluent hypergeometric function of one variable, that is now left, in a form that could be fit for computation. If we for simplicity regard the problem of spherically symmetric scattering where $\Sigma_{tr} = \Sigma_s$, we have

$$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\} = \frac{\Sigma_s \pm ik/\sqrt{3}}{\Sigma_s^{\,2} + k^2/3} \qquad (33)$$

$$A = \alpha\Sigma_s,\quad B = \beta\Sigma_s$$

We obtain for the distribution

$$N_o(x,v,t) = \frac{4\Sigma_s(v_o)}{\pi v_o} \int_0^\infty dk \cos kx \int_0^t d\tau\ e^{-v\Sigma_{tr}(t-\tau)} \cdot$$

$$\cdot\ \phi_2\left\{A+1,\ B+1,\ -A,\ -B;\ 1;\ z_1(t-\tau),\ z_2(t-\tau),\ z_3(t-\tau),\ z_4(t-\tau)\right\} \cdot$$

$$\cdot\ \frac{e^{-\Sigma_s(v_o)\,v_o\tau}}{k\,\tau}\ \sin k\,v_o\tau \qquad (34)$$

where

$$\phi_2\left\{A+1,\ B+1,\ -A,\ -B;\ 1;\ z_1t,\ z_2t,\ z_3t,\ z_4t\right\} =$$

$$= \frac{1}{\Gamma(A)\,\Gamma(1-A)\,\Gamma(B)\,\Gamma(1-B)} \int_0^t d\tau \int_0^1 du_1 \int_0^1 du_2 \Big\{ (-\frac{v}{\alpha} + v\Sigma_{tr}) \cdot$$

$$\cdot\ u_1^{B-1}(1-u_1)^{-B} u_2^{-A}(1-u_2)^{A-1} \cdot \left[1 + \frac{(v_o - v)\tau u_1}{\beta B}\right] \cdot$$

$$\cdot \left[1 + \frac{(v_o - v)(t-\tau)(1-u_2)}{\alpha A}\right] \exp\left[(-\frac{v}{\alpha} + v\Sigma_{tr})t + (\frac{v}{\alpha} - \frac{v_o}{\beta})\tau + \right.$$

$$\left. + \frac{v_o - v}{\beta}\tau u_1 + \frac{v - v_o}{\alpha}(t - \tau - \tau') u_2\right] + (\frac{v_o}{\alpha} - \frac{v}{\beta}) v_1^{B-1}(1-u_1)^{-B} \cdot$$

$$\cdot\ u_2^{A-1}(1-u_2)^{-A}\left[1 + \frac{(v_o - v)\tau u_1}{\beta B}\right] \exp\left[(-\frac{v}{\beta} + v\Sigma_s)t + \right.$$

$$\left. + v_o(\frac{1}{\alpha} - \frac{1}{\beta})\tau + \frac{v_o - v}{\beta}\tau u_1 + \frac{(v_o - v)}{\alpha}(t-\tau)\, u_2\right] + \delta_{A+B,\,1+\sigma} \cdot$$

$$\cdot \Big(\tau^{A+B-2}(t-\tau')^{1-A-B} u_1^{-B}(1-u_1)^{-A} u_2^{A-1}(1-u_2)^{B-1} \cdot$$

$$\cdot \left[1 - \frac{1-u_1}{1-A}\right]\left[A+B-1 + v(\frac{1}{\beta} - \frac{1}{\alpha})\tau u_2 + (\frac{v_o}{\alpha} - \frac{v}{\beta})\tau\right] \exp$$

$$\left[(-\frac{v_o}{\alpha} + v\Sigma_{tr})t + (\frac{v_o}{\alpha} - \frac{v}{\beta})\tau + v_o(\frac{1}{\alpha} - \frac{1}{\beta})(t-\tau)u_1 + v(\frac{1}{\beta} - \frac{1}{\alpha})\tau u_2\right]\Big) +$$

$$+ \delta_{A+B,\sigma}\Big(\tau^{A+B-1}(t-\tau)^{-A-B} u_1^{-B}(1-u_1)^{-A} u_2^{A-1}(1-u_1)^{B-1} \cdot$$

$$\cdot \left[1 - A - B + v_o(\frac{1}{\alpha} - \frac{1}{\beta})(t-\tau)(1+u_1)\right] \cdot \exp\left[(-\frac{v_o}{\alpha} + v\Sigma_{tr})t + \right.$$

$$\left. + (\frac{v_o}{\alpha} - \frac{v}{\beta})\tau + v_o(\frac{1}{\alpha} - \frac{1}{\beta})(t-\tau)\, u_1 + v(\frac{1}{\beta} - \frac{1}{\alpha})\tau u_2\right]\Big)\Big\} \qquad (35)$$

Here σ is some number fulfilling $0<\sigma<1$ so that $\delta_{A+B,\sigma} = 1$ when $0 < A+B < 1$. The complexity of the solution even for the simple case (33) is evident. However, we are mainly interested in the distribution for which $v \ll v_o$, and this can be obtained in a simpler way, at least when regarding $\epsilon \frac{\partial N_1}{\partial t}$ as a perturbation.

The asymptotic solution

Of primary interest is the part of the solution (15) that corresponds to $\Sigma_s v_o t \gg 1$ i. e. to $t \gg 0$. This asymptotic solution can be found in the following way. According to (19) most of the contribution to the integral in (18) comes from small τ values. Thus we can use the asymptotic form of f(t), the Laplace invers of f(s) in (15). This is obtained in a well-known way [4]. Let us first regard the simple case of $\epsilon = 0$. Then

$$\alpha = \left(\Sigma_s + \frac{k^2}{3\Sigma_{tr}}\right)^{-1} = \Sigma^{-1}, \quad \beta = 0$$

$$A = \frac{2\Sigma_s}{\Sigma}, \quad B = 0 \tag{36}$$

and

$$f(s) = \frac{v}{v_o} \cdot \frac{(\Sigma v_o + s)^{\frac{2\Sigma_s}{\Sigma}}}{(\Sigma v + s)^{1 + \frac{2\Sigma_s}{\Sigma}}} \tag{37}$$

f(s) is a multiplevalued function of s with singularities at $s = -\Sigma v$ and $-\Sigma v_o$. As $v_o \gg v$ the asymptotic part of the Laplace invers is given by the vicinity of $s = -\Sigma v$, where

$$f(s) \simeq \frac{v}{v_o} \frac{\left[\Sigma \cdot (v_o)\right]^{\frac{2\Sigma_s}{\Sigma}}}{\left[s + \Sigma v\right]^{1 + \frac{2\Sigma_s}{\Sigma}}} \tag{38}$$

giving

$$f(t) = \frac{v}{v_o} \frac{\left[\Sigma v_o t\right]^{\frac{2\Sigma_s}{\Sigma}}}{\Gamma\left|1 + \frac{2\Sigma_s}{\Sigma}\right|} e^{-\Sigma v t} \tag{39}$$

This is just the asymptotic form of the solution given by Koppel [5].

We now try a similar way of getting the distribution when ϵ is a small quantity. Then

$$\sqrt{(\epsilon\Sigma_s - \Sigma_{tr})^2 - \epsilon\frac{4k^2}{3}} = \Sigma_{tr}\left[1 - \epsilon\frac{\Sigma_s\Sigma_{tr} + \frac{2k^2}{3}}{\Sigma_{tr}^{\;2}}\right] \tag{40}$$

and

$$\begin{aligned} \alpha &= \frac{1 - \epsilon\frac{k^2}{3\Sigma_{tr}^{\;2}}}{\Sigma}, \quad \beta = \frac{\epsilon}{\Sigma_{tr}} \\ A &= \frac{2\Sigma_s}{\Sigma} + O(\epsilon^2), \quad B = O(\epsilon^2) \end{aligned} \tag{41}$$

Thus

$$f(s) \simeq \frac{v}{v_o} \frac{\left[1 + \epsilon\frac{k^2}{3\Sigma_{tr}^{\;2}}\right]^{1 + \frac{2\Sigma_s}{\Sigma}} \left[\Sigma v_o\right]^{\frac{2\Sigma_s}{\Sigma}}}{\left[s + v\Sigma(1 + \epsilon\frac{k^2}{3\Sigma_{tr}^{\;2}})\right]^{1 + \frac{2\Sigma_s}{\Sigma}}} \tag{42}$$

and

$$f(t,\epsilon) = \frac{v}{v_o}\left[1 + \epsilon \frac{k^2}{3\Sigma_{tr}^{\ 2}}\right]^{1 + \frac{2\Sigma_s}{\Sigma}} e^{-\Sigma v(1+\epsilon \frac{k^2}{3\Sigma_{tr}^{\ 2}})t} \cdot \frac{\left[\Sigma v_o t\right]^{\frac{2\Sigma_s}{\Sigma}}}{\Gamma\left[1 + \frac{2\Sigma_s}{\Sigma}\right]} \qquad \ldots\ldots (43)$$

Returning now to (5), we try to find a solution when $S_1(x, v, t)$ is given by (6) and $\partial N_1/\partial t$ is a small quantity. Corresponding to (13) we obtain

$$\left[s + v\Sigma_s + \frac{k^2 v^2}{3(\epsilon s + v\Sigma_{tr})}\right] \frac{dN_o(k, v, s)}{dv} + \left[2\Sigma_s - \frac{s}{v} + \frac{k^2 v}{3(\epsilon s + v\Sigma_{tr})} - \right.$$

$$\left. - \frac{k^2 v^2 \Sigma_{tr}}{3(\epsilon s + v\Sigma_{tr})^2}\right] N_o(k, v, s) = ikv \frac{d}{dv}\left(\frac{S_1(k, v, s)}{\epsilon s + v\Sigma_{tr}}\right) \qquad (44)$$

with the boundary value

$$N_o(k, v_o, s) = \frac{3(\epsilon s + v_o\Sigma_{tr})}{3(s + v_o\Sigma_s)(\epsilon s + v_o\Sigma_{tr}) + k^2 v_o^2}\left[S_o(k, v_o, s) + \right.$$

$$\left. + \frac{ik\, v_o\, S_1(k, v_o, s)}{\epsilon s + v_o\Sigma_{tr}}\right] \qquad (45)$$

Solving (44) for small ϵ we get finally for the neutron distribution from a point source (in order to compare with experiments)

$$N_o(r, v, t, \epsilon) = \frac{2\Sigma_o(v_o)\, v}{\pi v_o^2 r} \int_0^\infty dk \sin kr \int_0^t d\tau \cdot$$

$$\cdot\, e^{-v(\Sigma_s + \frac{k^2}{3\Sigma_{tr}})(1+\epsilon\frac{k^2}{3\Sigma_{tr}^2})(t-\tau)}\; e^{-\Sigma_s(v_o)v_o\tau}\,\cdot$$

$$\cdot\,\frac{\left[v_o(\Sigma_s + \frac{k^2}{3\Sigma_{tr}})(1+\epsilon\frac{k^2}{3\Sigma_{tr}^2})(t-\tau)\right]^{\frac{2\Sigma_s}{\Sigma_s+\frac{k^2}{3\Sigma_{tr}}}}}{\Gamma\left[1+\frac{2\Sigma_s}{\Sigma_s+\frac{k^2}{3\Sigma_{tr}}}\right]}\left\{\frac{\sin k v_o\tau}{\tau}\,\cdot\right.$$

$$\cdot\left[1+\epsilon\frac{k^2}{3\Sigma_{tr}^2}\right] - \frac{k}{\Sigma_{tr}}\left[\frac{2\Sigma_s}{3\Sigma_s+\frac{k^2}{3\Sigma_{tr}}} + \epsilon\,\frac{k^2}{3\Sigma_{tr}^2}\right]\cdot$$

$$\left.\cdot\left[\frac{\sin k v_o\tau}{k v_o\tau^2} - \frac{\cos k v_o\tau}{\tau}\right]\right\} \qquad (46)$$

We now assume that this solution should be an analytic function of ϵ and valid even for $\epsilon = 1$.

The distribution (46) with $\epsilon = 0$, has been calculated as a function of t for some r-values at the low In resonance ($v = 1.66 \cdot 10^6$ cm/sec, $v_o = 10^3\, v$, $\Sigma_s = 1.33\ \text{cm}^{-1}$, $\Sigma_s(v_o) = 0.2 \cdot \Sigma_s$, r = 0.5, 10, 15, 20 cm). The full drawn curves in Fig. 1 show the result and Fig. 2 gives the position of the maximum point. Compared to the result obtained in [1] it follows that as a consequence of the low scattering cross section at the source energy the pulse at 1.46 eV will "move out" from the source very rapidly. The broken curves for r = 0,5 and 8 cm show the distribution when $\epsilon = 1$. As follows from (46) it is plausible that the perturbation approach will be invalid when one has to take big k-values into account. In [1] it was shown that big k-values meant large distances from the source. One could thus expect the perturbation approach to be valid only close to the source. Measurements on the space distribution have been made by E. Möller using as a target an In solution contained in a circular cylinder with a diameter of 4 cm and a height of 10 cm. The measured distribution will thus give an average value in a volume at different distances from the source. In Fig. 1, 2 experimentally obtained points are marked with dots and the curve obtained for $r = 5$ has been normalized to the average results obtained for that distance. It is seen that the shape of the curve fits quite well to the points, giving nearly the same maximum time. This is marked with a cross on Fig. 2, and it is seen that it differs noticeably from the result corresponding to $\epsilon = 0$. For $r = 8$ cm the maximum of the curve when $\epsilon = 1$ is seen to fall just a bit below the result for $\epsilon = 0$ and above the experimental result. The importance of keeping the term $\partial N_1/\partial t$ in the equations is thus clear.

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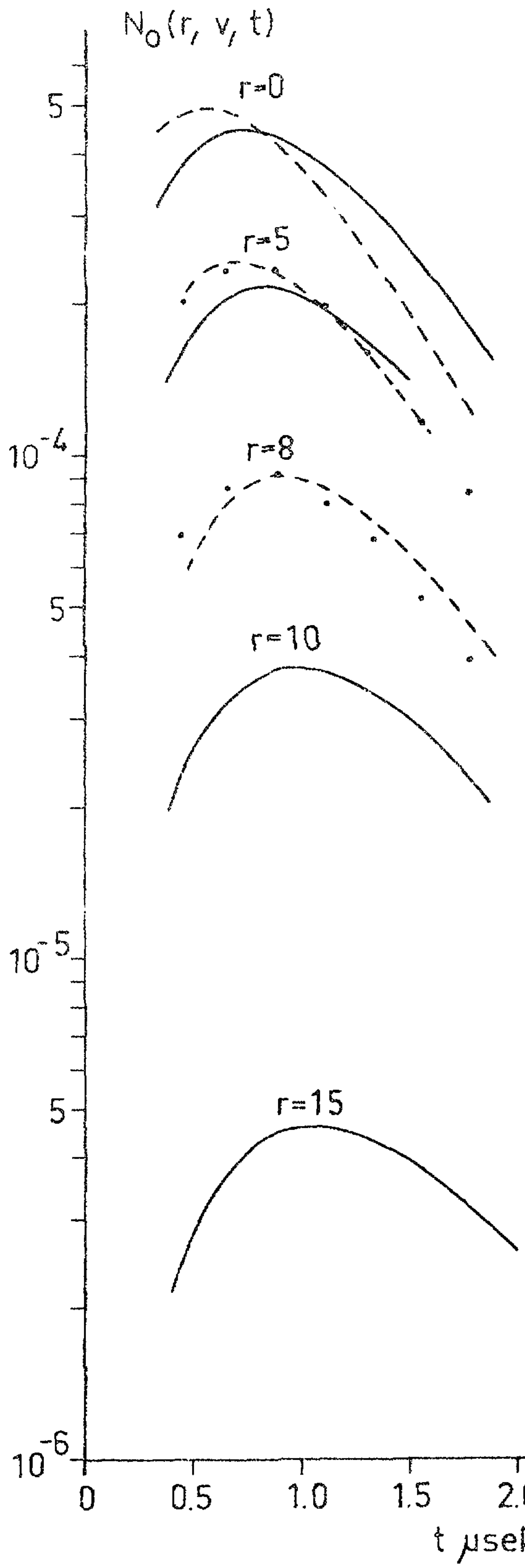
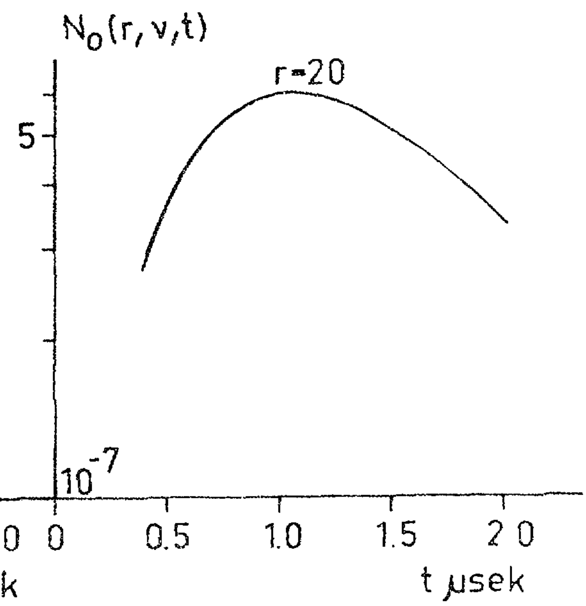
Figure 1.

The asymptotic density (v = 1.66 cm/ corresponding to the In resonance at eV) of neutrons from a high energy pı (1.46 MeV) at r = 0 and t= 0 as a funct of time at the distances r = 0, 5, 8, 1 15 and 20 cm from origo. The solid c ves show the distribution obtained frc equation (5) while disregarding the te $\partial N_1/\partial t$. The broken curves are obtaiı from the same equation regarding ∂N as a perturbation. The dotted curves show the experimental results obtain by Möller.





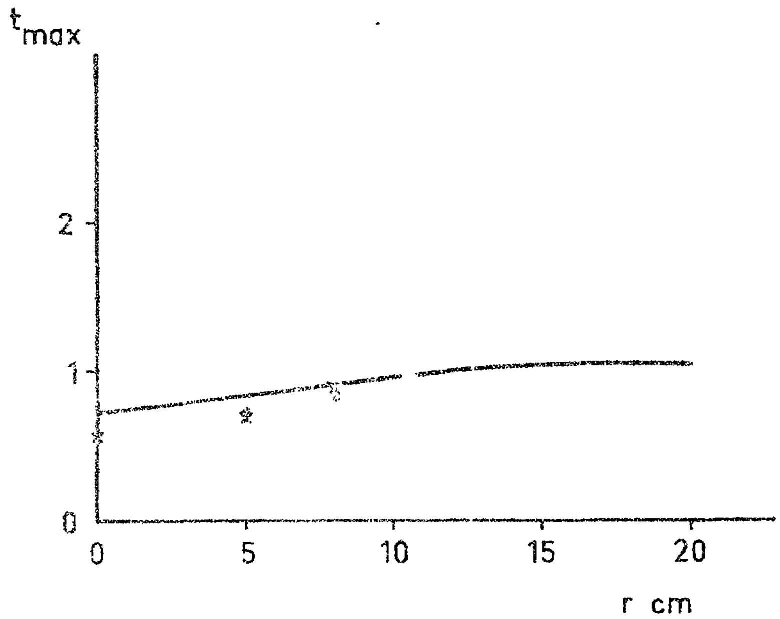


Figure 2.

The maximum point of the curves in Fig. 1 as a function of distance from the source. The solid curve corresponds to the solid curves in Figure 1 and the crosses to the broken curves. The points give the maximum of the experimental curve for r = 5 and 8.

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