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Abstract

The coherent synchrotron radiation (CSR) is one of the main problems limiting the performance of high intensity electron accelerators. The complexity of the physical mechanisms underlying the onset of instabilities due to CSR demands for accurate descriptions, capable of including the large number of features of an actual accelerating device. A code devoted to the analysis of this type of problems should be fast and reliable, conditions that are usually hardly achieved at the same time. In the past, codes based on Lie algebraic techniques, have been very efficient to treat transport problems in accelerators. The extension of these methods to the non-linear case is ideally suited to treat CSR instability problems. We report on the development of a numerical code, based on the solution of the Vlasov equation, with the inclusion of non-linear contribution due to wake field effects. The proposed solution method exploits an algebraic technique, using exponential operators. We show that the integration procedure is capable of reproducing the onset of an instability and the effects associated with bunching mechanisms leading to the growth of the instability itself. In addition, considerations on the threshold of the instability are also developed.

Keywords: *Synchrotron radiation, Instability, Accelerators, Storage-Rings, Free Electron Laser*

Riassunto

La radiazione di sincrotrone coerente (CSR) è uno dei problemi principali che limitano le prestazioni degli acceleratori ad alta intensità di corrente. La complessità dei meccanismi fisici alla base di instabilità dominata da CSR richiede una descrizione accurata, in grado di includere il maggior numero di dettagli.

I codici numerici dedicati all'analisi di tali tipi di problemi devono essere veloci ed affidabili, condizioni che sono difficilmente conciliabili. Nel passato codici numerici dedicati a problemi di trasporto e basati su tecniche Lie algebriche, si sono dimostrati particolarmente efficienti. L'estensione di tali metodi al caso non lineare è uno strumento, in linea di principio, ideale per trarre gli aspetti dinamici della CSR. In questo lavoro si descrive lo sviluppo di un codice, basato sulla soluzione dell'equazione di Vlasov che include termini non lineari dovuti a effetti di wake field.

Il metodo proposto utilizza una tecnica algebrico di integrazione, in grado di riprodurre l'instabilità e gli effetti di bunching a questa associati, che inducono la crescita dell'intensità stessa.

Si discute infine la soglia d'instabilità in termini di parametri fisici associati all'intensità del fascio di elettroni.

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The non-collisional Vlasov or Liouville equation has been solved using an algebraic technique, employing the well established evolution operator method commonly adopted in quantum-mechanics [1]. This method has been exploited to study a variety of phenomena in charged beam dynamics including Free Electron Lasers and it has been extended to the treatment of Fokker-Planck equations [2].

The method of the evolution operator is, in some sense, complementary to other methods employing Lie algebraic tools for the treatment of charged beam transport in accelerators [3].

In the following we will consider evolution equations describing the dynamics of a given charged beam density distribution ρ, of the type

$$\begin{aligned} &\frac{\partial}{\partial s}\rho = H\rho, \\ &\rho\,|_{s=0} = \rho_0 \end{aligned} \qquad (1)$$

where s refers to the propagation coordinate and plays the same role of time in ordinary Liouville equations, while ρ_0 denotes the initial distribution.

The operator H encloses the physical properties of the propagation problem. It may be specified by simple differential operators when describing the e-beam evolution through magnetic lens systems [3], or by integral operators when it accounts for non-local problems associated e.g. with the effects of the wake field on the beam.

The formal solution of (1) can be written as

$$\rho = \exp(Hs)\rho_0 \tag{2}$$

if H is a not explicitly time dependent operator.

In ref. [4] the problem of coherent synchrotron instability (CSR) affecting a beam of N electrons, undergoing a wake field interaction, has been considered and the relevant Vlasov equation has been cast in the form

$$\frac{\partial}{\partial s}\rho = \eta\varepsilon\frac{\partial}{\partial z}\rho + \alpha W(z,s)\frac{\partial}{\partial \varepsilon}\rho \tag{3}$$

with $\varepsilon = E - E_o / E_o$being the energy deviation from the nominal value E_o, η the slip factor, $\alpha = Ne^2 / E_o$

$$W(z,s) = \iint dz'd\varepsilon'\rho(z',\varepsilon',s)W_{||}(z-z') \tag{4}$$

is the wake field potential and $W_{||}(z)$ is the wake function corresponding to the steady state radiation of an ultra relativistic particle in a long magnet [5].

It is evident that we can interpret now H as

$$H = \eta\varepsilon\frac{\partial}{\partial z} - \alpha W(z,s)\frac{\partial}{\partial \varepsilon} \tag{5}$$

and the price to be paid is twofold: we are dealing indeed with an explicitly time-dependent operator and a non linear equation. Let us for the moment neglect the non linearity of the operator and assume that the formal solution of our problem can be cast in the form

$$\begin{aligned} &\rho(z,\varepsilon,s) = \exp(\eta s\varepsilon\frac{\partial}{\partial z} + \alpha\Omega(z,s)\frac{\partial}{\partial \varepsilon})\rho_0(z,\varepsilon), \\ &\Omega(z,s) = \int_0^s W(z,s')ds' , \end{aligned} \tag{6}$$

which implies that we are not including any contribution due to “time ordering” corrections (see below).

With this assumption the exponential operator consists of two, non commuting parts.

We decouple the exponential using the split technique, i.e. by approximating the exponential operator for a small s-interval δs as

$$\begin{aligned}
&\exp(A+B) \cong \exp\left(\frac{1}{2}A\right)\exp(B)\exp\left(\frac{1}{2}A\right) \\
&A = \eta\delta s\varepsilon\frac{\partial}{\partial z}, \\
&B = \alpha\Omega_0(z)\frac{\partial}{\partial\varepsilon} \\
&\Omega_0(z) = \int_0^{\delta s} ds' \iint dz'd\varepsilon'\rho_0(z',\varepsilon',s')W_{||}(z-z')
\end{aligned} \tag{7}$$

where $\Omega_0(z)$ accounts for the effect of the wake due to the initial distribution.

The action of the previous operator on the initial function will be easily calculated by taking into account that

$$\exp\left(\lambda\frac{\partial}{\partial z}\right)\exp\left(\mu\frac{\partial}{\partial\varepsilon}\right)f(z,\varepsilon) = f(z+\lambda,\varepsilon+\mu) \tag{8}$$

thus getting

$$\begin{aligned}
&\rho_1(z,\varepsilon,\delta s) = \rho_0(z_1,\varepsilon_1) \\
&z_1 = z + \eta\delta s\left(\varepsilon + \frac{1}{2}\alpha\Omega_0\left(z + \frac{1}{2}\eta\varepsilon\delta s\right)\right), \\
&\varepsilon_1 = \varepsilon + \alpha\Omega_0\left(z + \frac{1}{2}\eta\varepsilon\delta s\right) .
\end{aligned} \tag{9}$$

The procedure can be iterated, and after n iterations we get

$$\begin{aligned}
&\rho_n(z,\varepsilon,n\delta s) = \rho_{n-1}(z_n,\varepsilon_n,(n-1)\delta s), \\
&z_n = z_{n-1} + \eta\delta s\left(\varepsilon_{n-1} + \frac{1}{2}\alpha\Omega_{n-1}\left(z_{n-1} + \frac{1}{2}\eta\varepsilon_{n-1}\delta s\right)\right), \\
&\varepsilon_n = \varepsilon_{n-1} + \alpha\Omega_{n-1}\left(z_{n-1} + \frac{1}{2}\eta\varepsilon_{n-1}\delta s\right)
\end{aligned} \tag{10}$$

where

$$\Omega_n(z) = \int_{s_{n-1}}^{s_{n-1}+\delta s} ds' \iint dz' d\varepsilon' \rho_n(z',\varepsilon') W_{\|}(z-z') \tag{11}$$

It is also clear that to account for the non linearity due to the dependence of the wake potential on the distribution itself, we have evaluated the distribution modifying the wake at the step $n-1$.

We have omitted in eq. (7) the error of the method given by

$$|E_S| \cong \frac{1}{24}[B+2A,[A,B]] \tag{12}$$

which is easily shown to be of the order δs^3.

Time ordering corrections arise whenever the operator H is explicitly time dependent and such that it does not commute with itself at different times. In the present case we have

$$[H(s),H(s')] \propto \alpha \frac{\partial}{\partial z}(\Omega(z,s) - \Omega(z,s')) \neq 0 \ , \tag{13}$$

so that the correct form for the solution of eq. (1) should be written in a time ordered form as[6]

$$\rho = \left\{ \exp\left(\int_0^s H(s')ds'\right) \right\}_+ \ . \tag{14a}$$

A Magnus expansion can be employed, to explicitly evaluate the time ordering corrections, thus finding [7]

$$\rho = \exp\left(\int_0^s H(s')ds' + C_+(s)\right) \tag{14b}$$

where $C_+(s)$ consists of a sum involving higher order commutators, whose lowest order term is given by

$$\frac{1}{2}\int_0^{\delta s} ds' \int_0^{\delta s'} ds''[H(s''),H(s')] \propto \frac{\alpha}{12}\frac{\partial}{\partial z} W(z,s)\,|_{s=0}\, \delta s^3 \ . \tag{15}$$

Neglecting the time ordering corrections is therefore a consequence of the third order split decoupling, and can no more be exploited if higher order schemes are employed.

The above considerations are however not sufficient to ensure the correctness of the integration, that also requires the smoothness of the distribution function in such a way that

$$\frac{1}{6}\frac{\partial}{\partial z}W_{n-1}(z)\delta s^3\rho_n << \rho_n,$$
$$E\rho_{n-1} << \rho_n \tag{16}$$

A simulation code (TEO [8]) was especially developed to simulate the dynamics of an electron beam by numerical implementation of eq. (10). The initial distribution was sampled on a uniform Cartesian grid. The simulation domain in phase space was monitored at run-time and dynamically expanded in order to follow the distortions induced by the beam dynamics. Throughout the simulation, the resolution of the mesh was kept constant in order to avoid spurious numerical noise due to remapping. The simulations were carried out with an adaptive algorithm for the step size which controlled the accuracy of the computed solution. Also eq. (16) was checked at fixed iteration steps.

In the left side of Fig. (1) we have reported the evolution of the normalized longitudinal distribution $f(z) = \int_{-\infty}^{\infty} \rho(z,\varepsilon)d\varepsilon$ for different values of the longitudinal coordinate s.

It is evident that when the e-beam progresses inside the magnet, the instability develops and a number of peaks appear in the distribution. In the right side of Fig. (1), where no modulation develops, the same evolution is shown without the CSR instability. The situation is better illustrated in Fig. (2) relevant to the phase space distribution at the beginning and at the end of the evolution which shows that the peaks are essentially due to a bunching process induced by the interaction itself (microbunching).

Different benchmarks have been used to check the validity of the present results, in particular comparisons with

a) analytical calculations

b) a particle in cell code (PIC).

Regarding a) we have noted that for a Gaussian distribution the wake potential can be written as (R being the radius of the magnet)

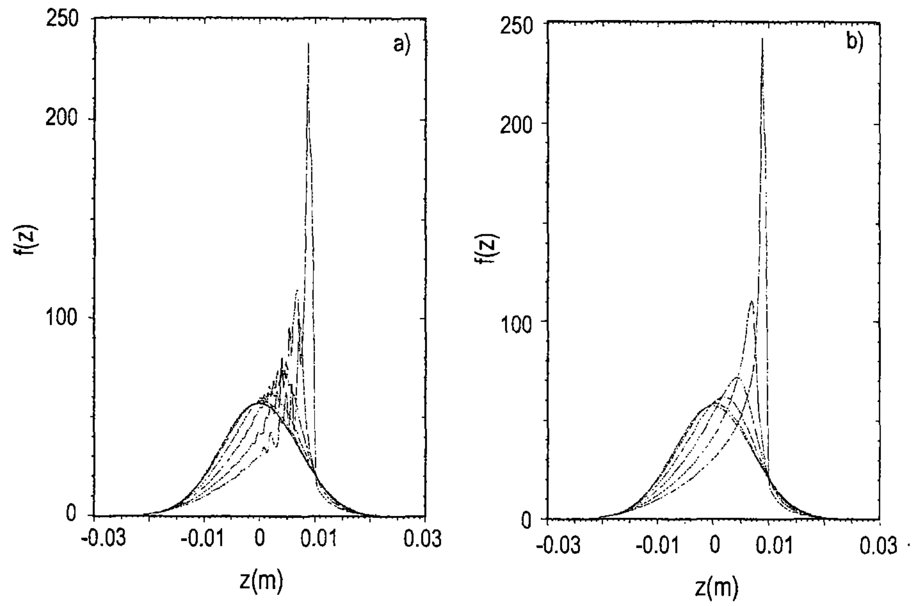

Fig. 1 - Evolution of the normalized longitudinal distribution $f(z)=\int_{-\infty}^{\infty}\rho(z,\varepsilon)d\varepsilon$ *for different values of the longitudinal coordinate* s *with and without CSR instability.*

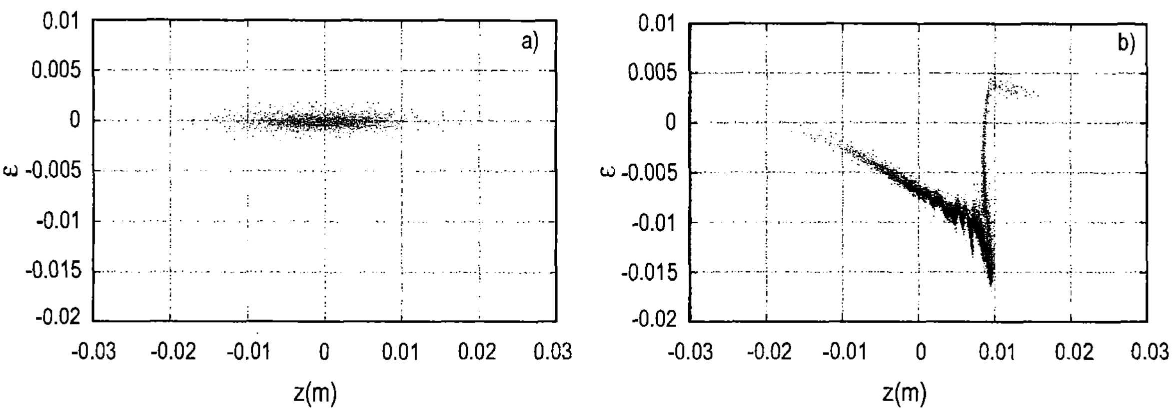

Fig. 2 - Phase space distribution at the beginning (a) and at the end (b) of the evolution.

$$W(z)=-\frac{1}{4\pi\varepsilon_0}\frac{2}{\sqrt{2\pi}\left(\sqrt{3}R\sigma_z^2\right)^{\frac{2}{3}}}F_o\left(\frac{z}{\sigma_z}\right),$$

$$F_o(z)=-\int_{-\infty}^{x}\frac{y\exp(-\frac{y^2}{2})}{(x-y)^{1/3}}dy \qquad (17)$$

which yields a beam energy loss and a correlated induced energy spread per unit length given by

$$\delta E \cong -\frac{0.36}{4\pi\varepsilon_0}\frac{Ne^2}{\left(R\sigma_z^2\right)^{2/3}},$$
$$\sigma_\varepsilon^i \cong \frac{0.246}{4\pi\varepsilon_0}\frac{Ne^2}{\left(R\sigma_z^2\right)^{2/3}} \qquad (18)$$

The above equations are strictly valid for a Gaussian longitudinal distribution. However, as the e-beam progresses inside the magnet, the energy variation due to the wake field modifies the longitudinal distribution through the slip factor. In order to compare eqs. (18) with the results of the simulations, we used a slip factor $\eta = 0$ so that the Gaussian shape could be preserved. The comparisons are shown in Fig. (3) during the evolution inside the magnet, and the agreement is very satisfactory. For the energy spread obtained with eqs. (18), we summed quadratically the correlated wake field induced energy spread and the natural one.

For the benchmark with a particle in cell code, we have adapted a simulation code used to study the microwave instability and the wake field effect in storage rings [9,10]. In this case the longitudinal distribution can be of any shape and we tested several different cases. In Fig. (4) we show the results for a stable case: here the CSR effect is to shorten the bunch, but no microbunching appears. It is worth noticing that, in the PIC code results, a certain level of numerical noise is unavoidable, and this makes it difficult to determine the presence of an eventual weak microbunching instability. Our code, solving the Vlasov equation, gives instead a smoother distribution that allows to determine the instability threshold and to study the evolution of a single coherent bunch mode.

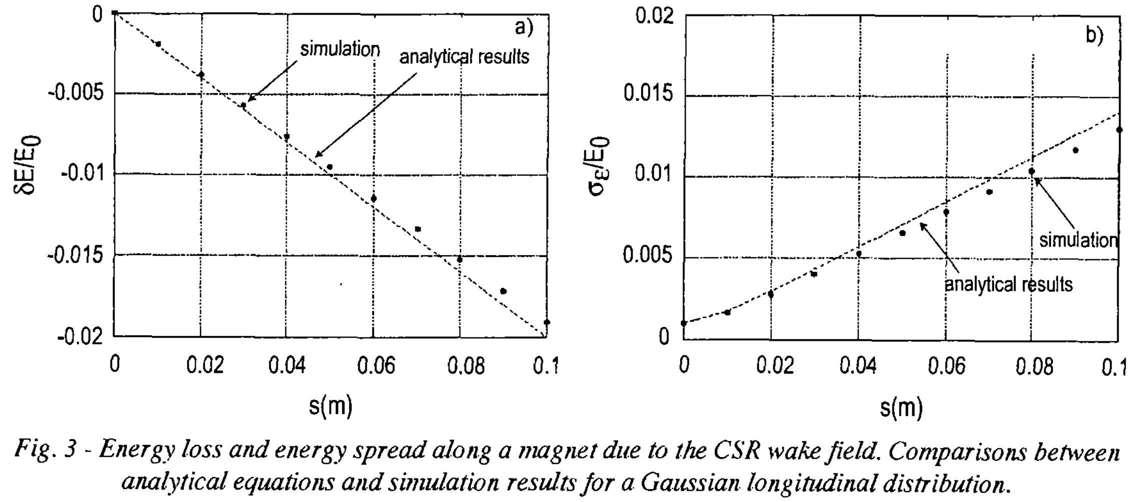

Fig. 3 - Energy loss and energy spread along a magnet due to the CSR wake field. Comparisons between analytical equations and simulation results for a Gaussian longitudinal distribution.

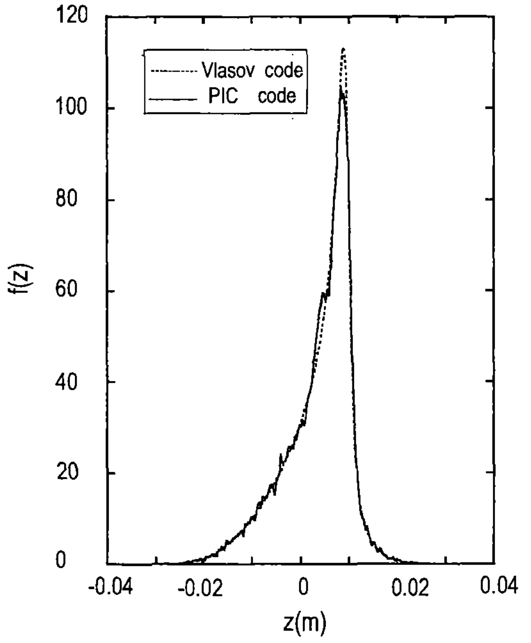

Fig. 4 - Comparison between the longitudinal distribution obtained with the 'Vlasov' code and that obtained with the PIC code in a stable case.

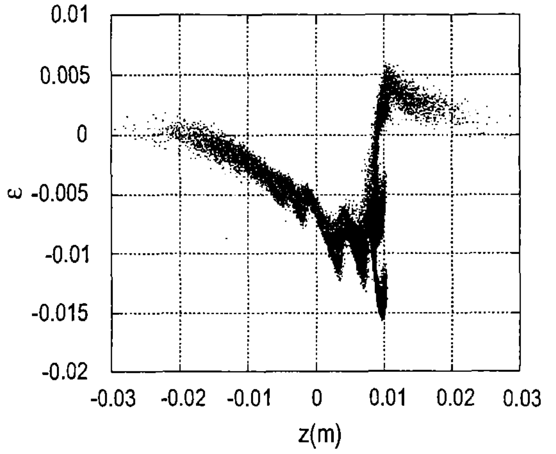

Fig. 5 - Phase space distribution corresponding to Fig. 2 (b) obtained with the PIC code.

Also in the case of strong instability and microbunching effect, the comparisons showed good agreement. In Fig. (5) we present the PIC result in the phase space corresponding to the case presented in Fig. (2), and the microbunching effect is evident.

It is worth stressing why we consider useful the above integration scheme and what may be its advantages over other more conventional methods, like PIC or finite difference integration methods of the Vlasov equation.

The technique we have adopted:

c) preserves the simplecticity of the problem in a natural way and therefore we do not expect particle losses due to numerical artefacts;

d) is, to some extent, analytical, and therefore flexible enough to allow other effects on the beam due to transport elements or to other kind of potentials;

e) gives smoother results than a PIC code, thus allowing accurate study of the instability threshold and of the coherent unstable modes;

f) being of algebraic nature, its link to symbolic beam tracking programs is quite natural;

g) its extension to the Fokker Planck (FP) case is natural: we should redefine the operator in eq.(5) by adding two further terms, accounting for quantum noise and damping, so that the full operator writes

$$
\begin{aligned}
&H = H_L + H_{FP} \\
&H_{FP} = A_1(\varepsilon, s)\frac{\partial^2}{\partial \varepsilon^2} + A_2(\varepsilon, s)\frac{\partial}{\partial \varepsilon}
\end{aligned}
\qquad (19)
$$

where H_L (L stands for Liouville) is the operator given in eq. (5) and A_1, A_2 are functions of ε and possibly of the longitudinal coordinate s. The introduction of these terms does not create any significant problem and we can proceed as in ref. [2] by applying the splitting procedure separately on the L and FP parts.

A final comment on point b): we have undertaken a systematic analysis of the potentials which may counteract the microbunching instability induced by the CSR. The most natural candidate is clearly a Free Electron Laser (FEL) type interaction. The numerical implementation of this problem is straightforward and we have modified the potential function in eq. (3) with the inclusion of a FEL-like potential. When the amplitude of the potential overcomes a given threshold and its wavelength is comparable to that of the CSR instability, the FEL-induced energy spread can control the wake field instability. In Fig. (6) we present a comparison of the normalized longitudinal distribution in three cases: FEL and microbunch instability included, only FEL, only microbunch instability. In all the cases the effect of CSR is included, the FEL amplitude is just at the threshold and the bunch parameters are the same of Fig. (1) in a longitudinal position where the instability starts to appear. The curve with both the effects and that with only the FEL are very close each other. This means that the instability is almost suppressed and the modulation is driven essentially by the FEL that we have chosen with a wavelength twice that of the instability. It is also worth noting that the amplitude of the FEL intensity is such that its potential is about a factor 100 lower than

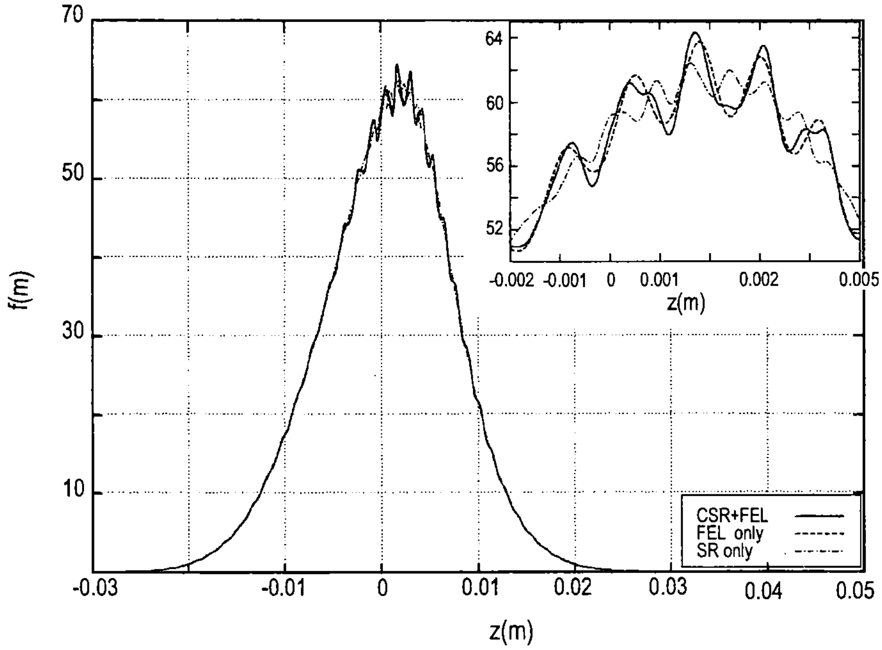

Fig. 6 - Comparison of the normalized longitudinal distribution in three cases: FEL and microbunch instability included, FEL only, microbunch instability only. Inset plot shows an enlargement of the most unstable region.

that induced by the CSR wake field. If we increase the FEL amplitude, naturally the microbunch instability disappears completely and only the FEL modulation remains.

The amount of FEL power necessary to suppress the instability can be easily obtained by noting that the instability is controlled when the FEL induced energy spread is comparable (even lower) with that due to the instability itself (see eq. (18)).

In the small signal regime the FEL induced relative energy spread is [11]

$$\sigma_i \cong \frac{0.387}{N_u}\sqrt{\frac{I}{I_s}} \tag{20}$$

with I, I_s being the laser intensity and the saturation intensity. Comparing the two spreads σ_ε^i and σ_i, we find that the FEL intensity necessary to counteract the instability is

$$I \cong \alpha_{FEL} I_s \ ,$$

$$\alpha_{FEL} = \left(\frac{2}{\pi}\right)^2 \frac{N_u^4 \lambda_u^2 r_0^2}{\gamma^2} \frac{N^2}{\left(R\sigma_z^2\right)^{4/3}} \ , \tag{21}$$

$$I_s\left[\frac{W}{cm^2}\right] = 6.9 \times 10^8 \left(\frac{\gamma}{N_u}\right)^4 \frac{1}{(\lambda_u[cm] k f_b)^2} \ ,$$

where I_s is the FEL saturation intensity, N_u the number of undulator periods, λ_u the undulator wavelength, r_0 the classical electron radius, k the undulator strength and f_b the Bessel factor if the undulator is linearly polarized. The use of typical FEL parameters yields very small value of α_{FEL} which may be estimated around 10^{-5}.

This last result confirms previous investigations, reported in ref. [12] concerning the theory and the experimental evidence of the saw-tooth instability suppression in storage rings by the FEL.

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