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A Parametric Study of a Constant-Mach-Number
MHD Generator with Nuclear Ionization

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Abstract

The influence of electrical and gas dynamical parameters on the length of a linear constant-Mach-number MHD duct has been investigated. The gas has been assumed to be ionized by neutron irradiation in the expansion nozzle preceding the MHD duct. Inside the duct the electron recombination is assumed to be governed by volume recombination. It is found that there exists a distinct domain from which the parameters must be chosen, pressure and Mach number being the most critical ones. If power densities in the order of magnitude 100 MW/m^3 are desired, high magnetic fields and Mach numbers in the supersonic range are needed. The influence of the variation of critical parameters on the channel length is given as a product of simple functions, each containing one parameter.

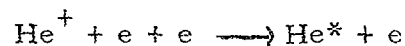
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1. Basic assumptions and derivation of equations

In an earlier paper [1] the concept of nuclear ionization was introduced and some indications of the channel performance were given. In the present work the influence of the parameters involved in this special type of ionization will be more completely presented for the case of He^4 seeded with He^3 , assuming a linear constant-Mach-number generator working between an ideal nozzle and an ideal diffuser. The configuration studied is outlined in Fig. 1a. The gas heated in the nuclear reactor is simultaneously expanded and irradiated in a nozzle 0-1. Electricity is extracted in the adjacent MHD-generator 1-2, followed by a diffuser in which the kinetic energy of the gas is transformed back into thermal energy.

In ref. [1] it was shown that we can assume the electron loss to be dominated by a three body recombination process



characterized by the coefficient α_e depending on the electron temperature T_e as

$$\alpha_e \approx 1.1 \cdot 10^{-8} T_e^{-9/2} \text{ m}^6 \text{ s}^{-1} \quad (1)$$

At the reactor exit the gas temperature is equal to the electron temperature and

$$\alpha_0 \approx 1.1 \cdot 10^{-8} T_0^{-9/2}$$

For an arbitrary temperature T eq. (1) can be transformed into

$$\alpha_e \approx \alpha_0 \left(\frac{T_0}{T} \right)^{9/2} \cdot \left(\frac{T}{T_e} \right)^{9/2} \quad (1a)$$

Equation (1a) shows that the local recombination factor can be expressed as the product of three factors

- i a recombination coefficient given at a certain reference temperature
- ii the ratio between local gas temperature and reference temperature.

iii the ratio between local gas temperature and local electron temperature.

The plasma is assumed to consist of one type of single ionized atoms with the ions closely coupled to the neutral gas. Under steady state conditions the equations of conservation for this mixture give in the absence of electron sources

$$\nabla \cdot (n_n \underline{v}) = \alpha_e n_i n_e^2 \quad (2)$$

$$\nabla \cdot (n_i \underline{v}) = -\alpha_e n_i n_e^2 \quad (3)$$

$$\nabla \cdot \left[n_e (\underline{v} + \underline{v}_e) \right] = -\alpha_e n_i n_e^2 \quad (4)$$

where \underline{v}_e is the drift velocity of the electrons.

By subtracting eq. (3) from eq. (4) and observing the quasi-neutrality of the plasma we obtain the Maxwell equation

$$\nabla \cdot (n_e \underline{v}_e) = 0 \quad (5)$$

and subsequently from eqs. (4) and (1a)

$$\nabla \cdot (n_e \underline{v}) = -\alpha_o \left(\frac{T_o}{T} \right)^{9/2} \cdot \left(\frac{T}{T_e} \right)^{9/2} n_e^3 \quad (6)$$

Equation (6) gives the electron density as a function of the gas velocity \underline{v} and the local gas temperature T if the elevation of the electron temperature T_e above T is known. The calculation of T_e/T is given in ref. [2] where a Maxwellian electron distribution has been assumed. As elastic scattering is the dominating mechanism of energy exchange between electrons and heavy particles, the connection between electron and gas temperature can be deduced from the energy balance

$$\underline{j} \cdot \underline{E}^{\prime} = \frac{2m_e \delta}{m_n} n_e \frac{1}{\tau_{el}} \frac{3}{2} k_B (T_e - T_n) \quad (7)$$

where \underline{E}^{\prime} is the electrical field strength in the moving coordinate system of the electrons. In eq. (7) the electrical power density absorbed by the gas due to ohmic heating is put equal to the average energy lost in elastic collisions per unit time.

This introduces the collision frequency τ_{el}^{-1} . To account for inelastic processes, a correction factor δ is introduced which is close to unity for monoatomic gases but which may be one or more orders of magnitude larger for polyatomic molecules. This limits the use of phenomena associated with elevated electron temperature to noble gases, eventually containing a minor amount of other ingredients.

The number density of heavy particles is

$$n = n_n + n_i \quad (8)$$

Adding eq. (2) and (3) we obtain

$$\nabla \cdot (nv) = 0 \quad (9)$$

It is convenient to introduce the degree of ionization

$$\beta = \frac{n_e}{n} \quad (10)$$

Inserting (10) and (6) and using (9) we get

$$\underline{v} \cdot \nabla \beta = -\alpha_o \left(\frac{T_o}{T} \right)^{9/2} \cdot \left(\frac{T}{T_e} \right)^{9/2} \beta^3 n^2 \quad (11)$$

At this stage the MHD-channel must be specified. We choose the one-dimensional treatment of a split-electrode constant-Mach-number MHD-generator for reasons discussed in [1] and [3]. Additionally, a constant-Mach-number generator facilitates a parametric survey as the crossing of the critical line in the flow phase diagram [4] is avoided.

In solving the flow equations it is convenient to introduce as a variable the total electrical output Q normalized to the stagnation enthalpy times the mass flow at the channel entrance [3]

$$\eta = \frac{Q}{c_p T_{s1} \rho_1 v_1 A_1} = \frac{Q}{c_p T_{s0} \rho_0 v_0 A_0} \quad (13)$$

For the case of the two constraints $k = \text{const.}$, $M = \text{const}$ the flow equations are now proved to be [3]

$$\left(\frac{n}{n_1} \right) = (1 - \eta)^{k-1} \quad (13a)$$

$$\frac{T}{T_1} \approx 1 - \eta \quad (13b)$$

$$\left(\frac{A}{A_1}\right) \approx (1 - \eta)^{-(\kappa-1/2)} \quad (13c)$$

$$\frac{v}{v_1} \approx (1 - \eta)^{1/2} \quad (13d)$$

$$\frac{p}{p_1} \approx (1 - \eta)^\kappa \quad (13e)$$

where

$$\kappa \approx \left\{ 1 + \frac{1}{2}(1 - k)(\gamma - 1)M^2 \right\} \frac{\gamma}{(\gamma - 1)k} \quad (13f)$$

We now want to calculate the length of duct required to extract the normalized electrical power η . The connection between the length x and η can now be deduced from the equation of conservation of energy

$$v\rho \frac{d}{dx} \left(c_p T + \frac{v^2}{2} \right) \approx -\sigma v^2 B^2 k(1 - k) \quad (14)$$

where the decrease in total enthalpy flow per unit length has been put equal to the amount of extracted electrical power per unit volume. In eq. (14) the scalar conductivity σ appears, Hall current being suppressed by segmentation of the electrodes. As in ref. [1] the connection between σ and β is given by an expression derived from classical Langevin theory

$$\sigma = \text{const} \frac{e^2 \beta}{q(2m_e k_B T_e)^{1/2}} \quad (15)$$

where the constant is of the order of unity. In the case of helium, the elastic momentum transfer cross section q is approximately constant and

$$\sigma \approx H \cdot \frac{\beta}{\sqrt{T}} \cdot \sqrt{\frac{T}{T_e}} \quad (16)$$

$$H \approx 9.0 \cdot 10^7 \text{ mho } \text{ }^\circ\text{K}^{1/2} \text{ m}^{-1}$$

H and τ_{el} introduced in eq. (7) are interlinked by

$$\frac{1}{\tau_{el}} = \frac{e^2}{m_e H} n \sqrt{T_e} \quad (16a)$$

Eq. (15) or (16) give the correct conductivity dependence on β and T_{e-} for

$$\frac{E}{n} < 10^{-23} v_m^2$$

and a slight variation of the constant factor H can take care of the E dependence for

$$10^{-23} < \frac{E}{n} < 5 \cdot 10^{-21} v_m^2 \quad [5] \quad (17)$$

This covers the range of variables important for the present purpose.

Using eqs. (16) and (13), eq. (14) can be transformed into

$$\frac{\beta_1}{\beta} (1 - \eta)^{k-1} \left(\frac{T_e/T}{T_{el}/T_1} \right)^{1/2} d\eta = k(1 - k) \frac{dx}{x_1} \quad (18)$$

where x_1 , a characteristic interaction length, is defined by

$$x_1 = \frac{c_p T S I \beta_1 v_1}{\sigma_1 v_1^2 B^2} \quad (19)$$

Using the same equations and taking $\frac{dx}{d\eta}$ from eq. (18), eq. (11) is transformed into

$$\frac{d\beta}{d\eta} = - \frac{\alpha_1 n_1^2}{v_1} \cdot \frac{x_1 \beta_1}{k(1-k)} \left(\frac{T_{el}}{T_1} \right)^{-1/2} \beta^2 (1 - \eta)^{3k-8} \left(\frac{T}{T_e} \right)^4 \quad (20)$$

In order to obtain an expression for T_e/T we use the expression for the ohmic heating of the gas in the segmented channel

$$\underline{j} \cdot \underline{E}' = \sigma v^2 B^2 (k - 1)^2 \quad (21)$$

transforming (7) into a quadratic in T_e/T

$$G \frac{v_1^2 B^2}{P_1} (1 - \eta)^{1-2k} (k - 1)^2 = \frac{T_e}{T} \left(\frac{T_e}{T} - 1 \right) \quad (22)$$

where

$$G = \left(\frac{k_a H}{e} \right)^2 \left(\frac{m_n}{3k_B \delta} \right) \quad (22a)$$

Inserting numerical values one finds

$$G = 0.9 \cdot 10^{-6} \quad (22b)$$

The simultaneous solution of eqs. (18), (20), (22) solves our problem. We rewrite this set by collecting the terms containing the input conditions (reduced to the reactor exit values). In this connection it is convenient to normalize the ionization β to its input value by introducing a new variable

$$b = \frac{\beta}{\beta_1} \left(\frac{T_{e1}}{T_1} \right)^{1/2} \quad (22c)$$

An asterisk will indicate that elevated electron temperature has been assumed at $\eta = +0$, i. e. that the heating of electrons reaches its equilibrium value immediately after electrons have entered the magnetic field at $x = +0$, $\eta = +0$. As this assumption enlarges the characteristic length (eq. (26)) the estimate of the channel length will be conservative. One obtains

$$\frac{dx}{d\eta} = \frac{x_1^*}{k(1-k)} b^{-1} (1-\eta)^{k-1} \left(\frac{T_e}{T} \right)^{1/2} \quad (23)$$

$$\frac{db}{d\eta} = -h_1^* b^2 (1-\eta)^{3k-8} \left(\frac{T_e}{T} \right)^{-4} \quad (24)$$

$$\frac{T_e}{T} = \frac{1}{2} \left\{ 1 + \left[1 + 4GF(1-\eta)^{1-2k} \right]^{1/2} \right\} \quad (25)$$

with the complementary relations

$$x_1^* = x_1 \cdot \left(\frac{T_{e1}}{T_1} \right)^{1/2} \quad (26)$$

$$\frac{T_{el}}{T_1} \approx \left(\frac{T_e}{T} \right) \eta \approx +0 \quad (27)$$

$$h_1^* = h_1 \left(\frac{T_1}{T_{el}} \right)^{1/2} \quad (28)$$

$$h_1 = C_h \cdot P_{oB}^2 (S \cdot a_o^2)^{1/3} T_o^{-3/2} F_h(M) \frac{1}{k(1-k)} \quad (29)$$

$$x_1 = C_x \cdot P_{oB}^2 T_o^{-1} \left(\frac{S}{a_o} \right)^{-1/3} F_x(M) \quad (30)$$

$$F = C_F \cdot T_o (1-k)^2 \cdot P_{oB}^{-2} F_F(M) \quad (31)$$

The functions of M used above are the following

$$F_h = M^{-2} \left(\frac{1 + 1/2(\gamma - 1)M^2}{1 + 1/2(\gamma - 1)M_o^2} \right)^{1/2} \quad (32)$$

$$F_x = M^{-1} \left(\frac{1 + 1/2(\gamma - 1)M_o^2}{1 + 1/2(\gamma - 1)M^2} \right)^{3/2} \quad (33)$$

$$F_F = M^2 \left(\frac{1 + 1/2(\gamma - 1)M^2}{1 + 1/2(\gamma - 1)M_o^2} \right)^4 \quad (34)$$

As $M_o \ll M$ in practice, M_o will be neglected compared to M in the sequel. The constants C_x , C_h , C_F are determined by the properties of the gas and are given by the following relationships:

$$C_x = \frac{m_n}{Hk_a^2} \left(\frac{c_p}{\gamma - 1} \right)^{1/2} \approx 3.51 \cdot 10^{23} \quad (35)$$

$$C_h = \frac{m_h}{(\gamma - 1)Hk_a^2} = 5.97 \cdot 10^{21} \quad (36)$$

$$C_F = c_p(\gamma - 1) = 3.46 \cdot 10^3 \quad (37)$$

As the pressure always appears in the combination p/B we have for the sake of simplicity introduced a reduced pressure

$$p/B = p_B$$

In deriving the expressions (29), (30) the initial ionization at $x = -0$ is needed. This has been discussed in [1], where for the special case of ionization during an expansion in a nozzle in the presence of a constant neutron flux the approximate expression

$$\beta_1 = \frac{1}{n_0} \left(\frac{S}{a_0} \right)^{1/3} \quad (38)$$

has been found.

Any solution of the system (23) - (25) is characterized by a set of parameters

$$\frac{x_1^*}{k(I - k)}, h_1^*, \kappa, GF \quad (39a)$$

related to

$$p_{0B}, T_0, a_0, k, M, S, G \quad (39b)$$

by equations (26) - (37) and (13f).

The constant G has been taken as a parameter as it contains δ whose variation can simulate the influence on the electron temperature of processes other than the balance between electric field heating and losses due to elastic momentum transfer.

2. Variation of parameters

We want now to study how the variation of the "technical" parameters (39b) influences the solution of the channel equations and consider the channel length, being perhaps the most important quantity, as a function of η . This problem can be considerably simplified by limiting the range of parameters. In that aspect a concept of central importance is that the overall thermal efficiency of the whole MHD plant must be kept high. This results in a requirement for high isentropic efficiency of the MHD channel itself [6]. The isentropic efficiency η_{is} is defined as the decrement in stagnation enthalpy of an actual channel due to the extraction of electrical power, divided by the corresponding decrement in enthalpy of an isentropic generator, both expansions starting from the same temperature and working between the same stagnation pressures (Fig. 1b). For constant c_p we obtain

$$\eta_{is} = \frac{T_{1S} - T_{2S}}{T_{1S} - T_{2S}'} = \frac{1 - T_{2S}/T_{1S}}{1 - (p_{2S}/p_{1S})^{(\gamma - 1)/\gamma}} \quad (40)$$

and for the constant-M channel

$$\eta_{is} = \frac{\eta}{1 - (1 - \eta)^{k(\gamma - 1)/\gamma}} \quad (41)$$

which for $\eta \ll 1$ can be approximated by

$$\eta_{is} \sim \eta_p = \frac{1}{k} \cdot \frac{\gamma}{\gamma - 1} \quad (42)$$

or

$$\eta_p = \frac{k}{1 + 1/2(1 - k)(\gamma - 1)M^2} \quad (43)$$

independent of η .

On the other hand for $\eta \sim 1$, implying large temperature and pressure ratios, η_{is} approaches η independently of k and M . For all values of η we find $\eta_i \geq \eta_p$. As η_p can be identified as the polytropic or small stage efficiency this expresses the fact that the polytropic efficiency is below the overall, isentropic efficiency. The relationships between load factor k , Mach number, small stage and overall efficiency, η_p and η_{is} , and expansion number κ as expressed in (41) - (43) are shown in Fig. 2 where $\gamma \approx 5/3$ has been used. The Mach number has been limited to $0 < M < 2.5$ by practical aerodynamic design considerations. Previously, the requirement for high η_{is} was emphasised and our range of primary interest is $\eta_{is} > 0.6$. Furthermore, η , the measure of extracted electrical power is limited by several reasons. If the MHD channel is intended to be a topping device, T_{2S} should be high enough for a modern conventional steam cycle. Additionally (13c) and (13e) show that the channel area and the pressure ratio grow excessively with increasing η . Consequently, a fair estimation is $0 < \eta < 0.4$. This leaves the hatched areas in Fig. 2 as the regions of main interest, and shows that applicable values of k and κ are confined to

$$\begin{aligned} \kappa &< 6 \\ k &> 0.5 \end{aligned}$$

Using $T_0 \approx 1500^\circ\text{K}$, $\gamma \approx 5/3$ and $k \approx 0.8$ we find starting from (31)

$$4GF \sim \frac{0.7M^2}{2} (1 + M^2/3)^4 \gg 1 \quad (44)$$

the inequality being valid for all cases of practical value, as will be shown below. This implies that (25) can approximately be replaced by

$$\frac{T_e}{T} \sim \frac{(GF)^{1/2}}{(1 - \eta)^{\kappa - 1/2}} \quad (45)$$

Inserting in (24) and integrating one finds that

$$\frac{1}{B} \sim (GF)^{-1/4} \left\{ 1 + \frac{h_1(GF)^{-2}}{7\kappa - 9} \left[1 - (1 - \eta)^{7\kappa - 9} \right] \right\} \quad (46)$$

The first term in eq. (46) describes the initial ionization and the bracket shows the change along the channel.

Now the last power term goes quickly to zero for all types of expansions. This means that in order to avoid a decrease in ionization along the channel in excess of an arbitrary chosen factor 2 it is required that

$$\frac{h_1(GF)^{-2}}{7\kappa - 9} < 1 \quad (47)$$

We can rewrite this as

$$\frac{C_H}{(GC_F)^2} \cdot \frac{(S\alpha_o^2)^{1/3}}{7\kappa - 9} \cdot \frac{1}{k(1 - k)} \left[\frac{P_{oB}^{1.5}}{(1 - k)T_o^{7/8} M^{1.5} (1 + M^2/3)^{15/8}} \right]^4 < 1 \quad (48)$$

The left side expression is completely dominated by the term in the square bracket due to its high exponent. This justifies the arbitrary choice of the above mentioned factor 2. Furthermore, in order to evaluate the expression outside the bracket we can use any reasonable numerical values, e.g. for S and α_o the reference values, and $\kappa \approx 5$, $k \approx 0.7$. A further simplification is obtained if T_o is eliminated by inserting its reference value 1500°K . As p_{oB} is proportional to $T_o^{7/12}$, all other parameters kept fixed, a variation of the temperature between 1300°K and 1800°K changes p_{oB} only by about ± 10 per cent. By these considerations (48) is changed into

$$\frac{P_{oB}}{M(1 + M^2/3)^{5/4} (1 - k)^{2/3}} < 1.5 \quad (49)$$

For values of parameters outside the range defined by this inequality b vanishes very quickly. The relation (49) is shown in Fig. 3. The curves $k = \text{const.}$ or $\eta_{is} = \text{const.}$ divide the $p_{oB} - M$ plane into two domains of which the lower one contains the range for acceptable performance of the channel. As (49) imposes a more severe constraint than (44) the latter approximation has been justified. This completes the proof that within certain constraints, given in the expression (48), the normalized initial ionization b is nearly constant and given approximately by the first term of eq. (46).

With this assumption we can integrate (23) and obtain

$$x \sim \frac{x_1}{k(1-k)} (GF)^{1/4} \frac{4}{2k+1} \left[1 - (1-\eta)^{(2k+1)/4} \right] \quad (50)$$

For small η the bracket can be expanded in a binomial series. Taking two terms and transforming x_1 and F into technical parameters we obtain

$$x \sim C_F^{1/4} C_x G^{1/4} p_{oB}^{3/2} T_o^{-3/4} \left(\frac{S}{a_o}\right)^{-1/3} \cdot \frac{1}{k(1-k)^{1/2}} \left[M(1 + M^2/3) \right]^{-1/2} \cdot \eta \cdot f(\eta, \kappa) \quad (51)$$

where

$$f(\eta, \kappa) \approx 1 - \frac{2\kappa - 3}{8} \eta + \dots \quad (52)$$

With the accuracy required in a parametrical survey one can accept

$$f(\eta, \kappa) \sim 1 \quad (53)$$

and consequently eq. (51) shows that the dependence of the channel length on the technical parameters can be reduced to a very convenient form, i. e. the product of simple functions each containing one parameter. For (51) to be valid the parameters must of course be restricted to ranges given by (48) or (49).

Given the exact solution for one channel, the length of others can be estimated using (51).

We finally consider the power density at the channel entrance. For our set of a_o , S , T_o we get from eq. (19) in ref. [1]

$$P_1 \approx 24 \frac{M^2 k(1-k)}{(1+M^2/3)^{1/2}} \cdot \frac{B^2}{p_o} \text{ MW/m}^3 \quad (54)$$

and from (49)

$$P_1 \geq 16 \frac{Mk(1-k)^{1/3} B}{(1+M^2/3)^{7/4}} \text{ MW/m}^3 \quad (55)$$

The equality sign is valid at the highest pressure admissible for a certain Mach number.

3. Conclusions and summary

The influence of electrical and gas dynamical parameters on the length of a linear constant-Mach-number MHD duct has been investigated. The gas has been assumed to be ionized by neutron irradiation in the expansion nozzle preceding the MHD duct. Inside the duct the electron recombination is assumed to be governed by volume recombination depending on electron temperature by the law expressed by eq. (1). The electron temperature has been calculated under the assumption that the dominating process, by which electrons can lose energy, is elastic momentum transfer with neutrals (eq. (7)). It is found that there exists a distinct domain (Fig. 3 or eq. (49)) from which the parameters must be chosen, pressure and Mach number being the most critical ones. For a certain ratio of pressure and magnetic field the lowest admissible Mach number is given. The resulting power density given in eq. (54) can be increased by using higher Mach number and higher magnetic fields. If power densities in the order of magnitude 100 MW/m^3 are desired, high magnetic fields and Mach numbers in the supersonic range are needed. The influence of the variation of critical parameters on the channel length is given as a product of simple functions (eq. (51)), each containing one parameter.

For the purpose of comparison with the approximate solutions discussed above, the system (23) - (25) has been solved with a digital computer and the results have been compared to those obtained by the simple expressions (51) and (52). For all parameters inside the domain where the inequality (48) is valid the channel length is predicted with an accuracy of a few per cent.

The computer solution for some illustrative cases is shown in Fig. 4. $M \approx 1.5$ has been assumed and the channel length is given as a function of the normalized extracted electrical power η . Three sets of curves representing different pressures are shown and each set contains the curves for three different isentropic efficiencies.

In order to illustrate the use of the prescriptions as obtained from Fig. 3, the channel length as a function of p_{oB} for some value of η (e. g. 0.2, see the dotted line in Fig. 4) has been investigated. The points A'' , A' , A and B'' , B' , B for $\eta_{is} \approx 0.7$ and 0.8 , respectively, are obtained. These points follow the $p_{oB}^{3/2}$ law as predicted by eq. (51) with the exception of B where the channel length has increased very much. An inspection of the corresponding points in Fig. 3 shows that A lies above the point P ($p_{oB} \approx 1$, $M \approx 1.5$) and B lies below. This means that P in the case A belongs to the accepted region while in the case B the point P must be rejected.

4. Acknowledgements

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Nomenclature

MKSA-units are used with the exception of the pressure which is given in atm.

General subscripts

e	electron	p	polytrope
el	elastic	S	stagnation value
i	ion	0	exit of reactor \equiv inlet of nozzle
is	isentropic	1	exit of nozzle \equiv inlet of MHD channel
n	neutral	2	exit of MHD channel \equiv inlet of diffuser

Superscript

* indicates correction for elevated electron temperature

Symbols

α	recombination factor
β	degree of ionization
γ	ratio between specific heat at constant pressure and specific heat at constant volume
δ	correction factor accounting for inelastic collisions
η	normalized electrical output or efficiency when appearing with subscript
κ	expansion parameter
ρ	gas density
σ	electrical conductivity
τ	collision time
A	area
B	magnetic field
b	normalized degree of ionization
c_p	specific heat at constant pressure
C_x, C_h, C_F	constants related to parameters given in subscript
E^{\dagger}	electric field in a system moving with gas velocity v
e	electron charge
F_h, F_x, F_F	functions of Mach number

G	constant introduced in connection with elevated electron temperature
H	constant in conductivity formulae
h	dimensionless parameter
j	current density
k	load factor of channel
M	Mach number
m	mass of particle
n	particle density
P	electrical power density
p	pressure
P_B	reduced pressure
Q	total electrical output
q	cross section for elastic momentum transfer
S	source strength of electrons
T	temperature
v	macroscopic velocity (drift velocity in the case of electrons)
x	length of channel
x_1	characteristical interaction length
~	approximately

Constants

γ	5/3
δ	1
c_p	5194 Ws/kg, °K
e	electron charge $1.6 \cdot 10^{-19}$
H	$9.0 \cdot 10^7$ mho °K ^{1/2} m ⁻¹
k_a	Boltzmann's constant $1.37 \cdot 10^{-28}$ atm m ³ /°K
k_B	Boltzmann's constant $1.38 \cdot 10^{-23}$ Ws/°K
m_n	$6.64 \cdot 10^{-27}$ kg for He ⁴

Reference values used in test examples

a_o	$5.6 \cdot 10^{-35}$ m ⁶ s ⁻¹
S	$5.0 \cdot 10^{22}$ m ⁻³ s ⁻¹
T_o	1500 °K

- Fig. 1a. Sketch of MHD-generator with nozzle and diffuser.
- Fig. 1b. Definition of isentropic efficiency.
- Fig. 2. Connection between isentropic generator efficiency η_{is} , p polytrope efficiency η_p , expansion factor κ , load factor k (local efficiency), normalized extracted power η .
- Fig. 3. Useful range of p_{oB} , M for $\eta \approx 0.2$, $T_o \approx 1500$ °K.
- Fig. 4. Channel length as function of η with p and η_{is} as parameters. $M \approx 1.5$.

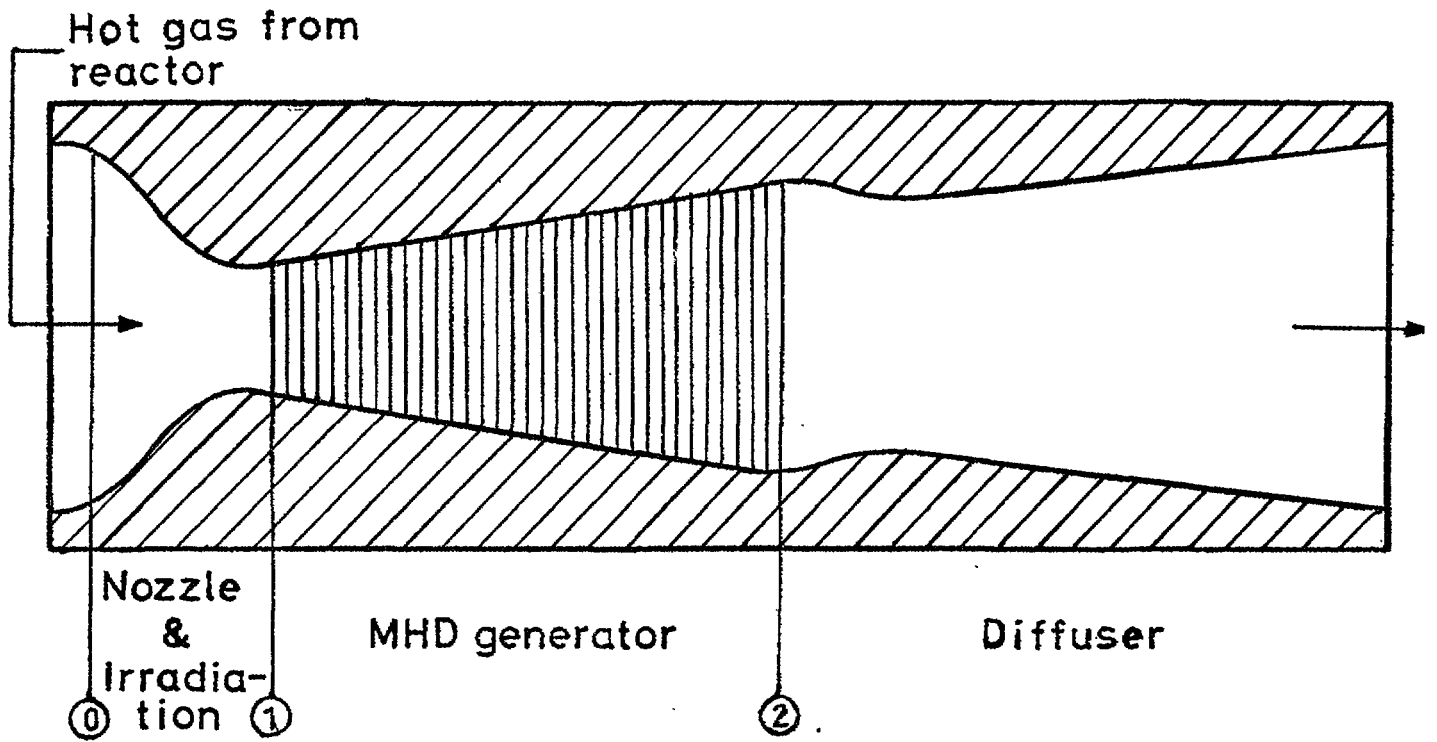


Fig. 1a. Sketch of MHD-generator with nozzle and diffuser.

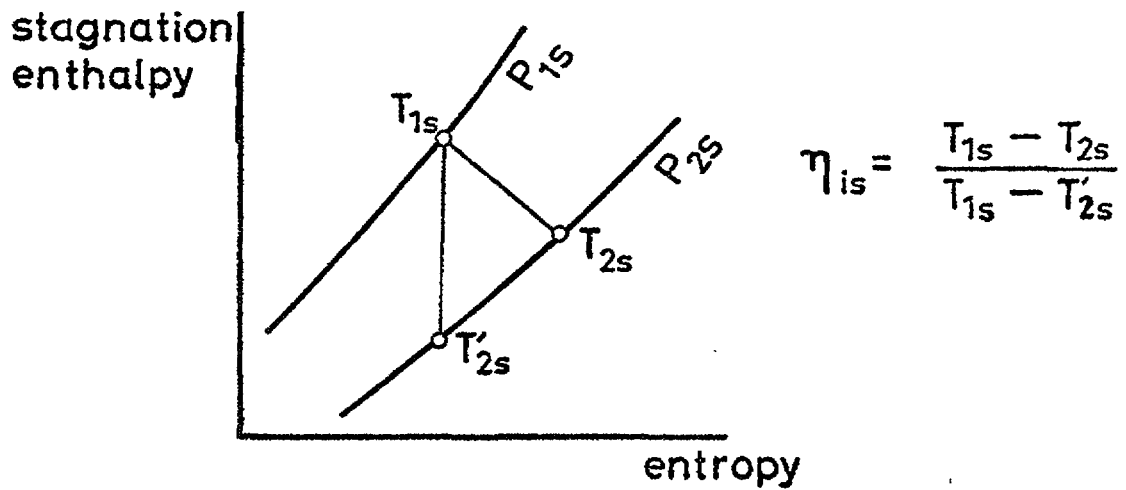


Fig. 1b. Definition of isentropic efficiency.

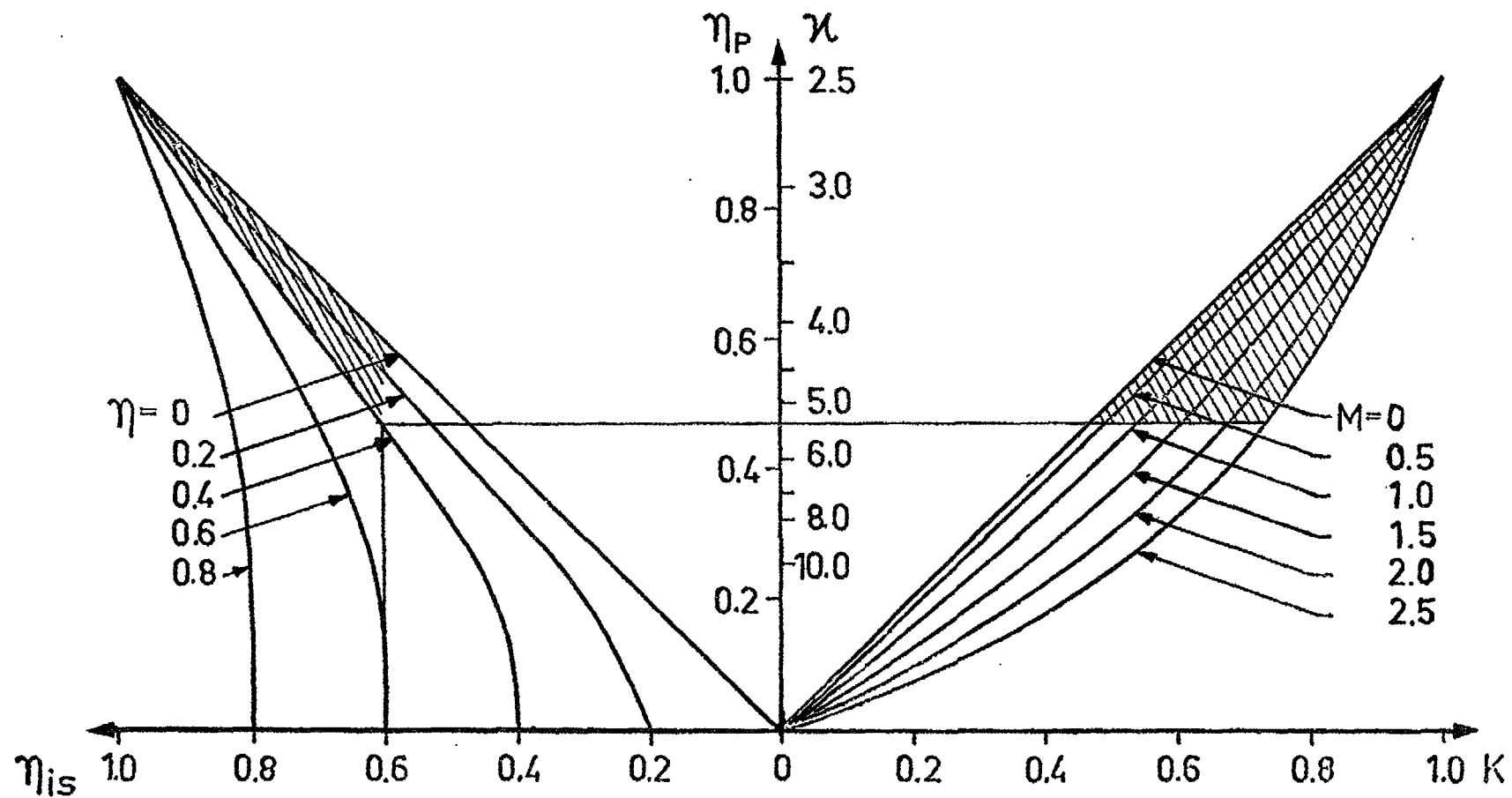


Fig. 2. Connection between isentropic generator efficiency η_{is} , polytropic efficiency η_p , expansion factor κ , load factor k (local efficiency), normalized extracted power η

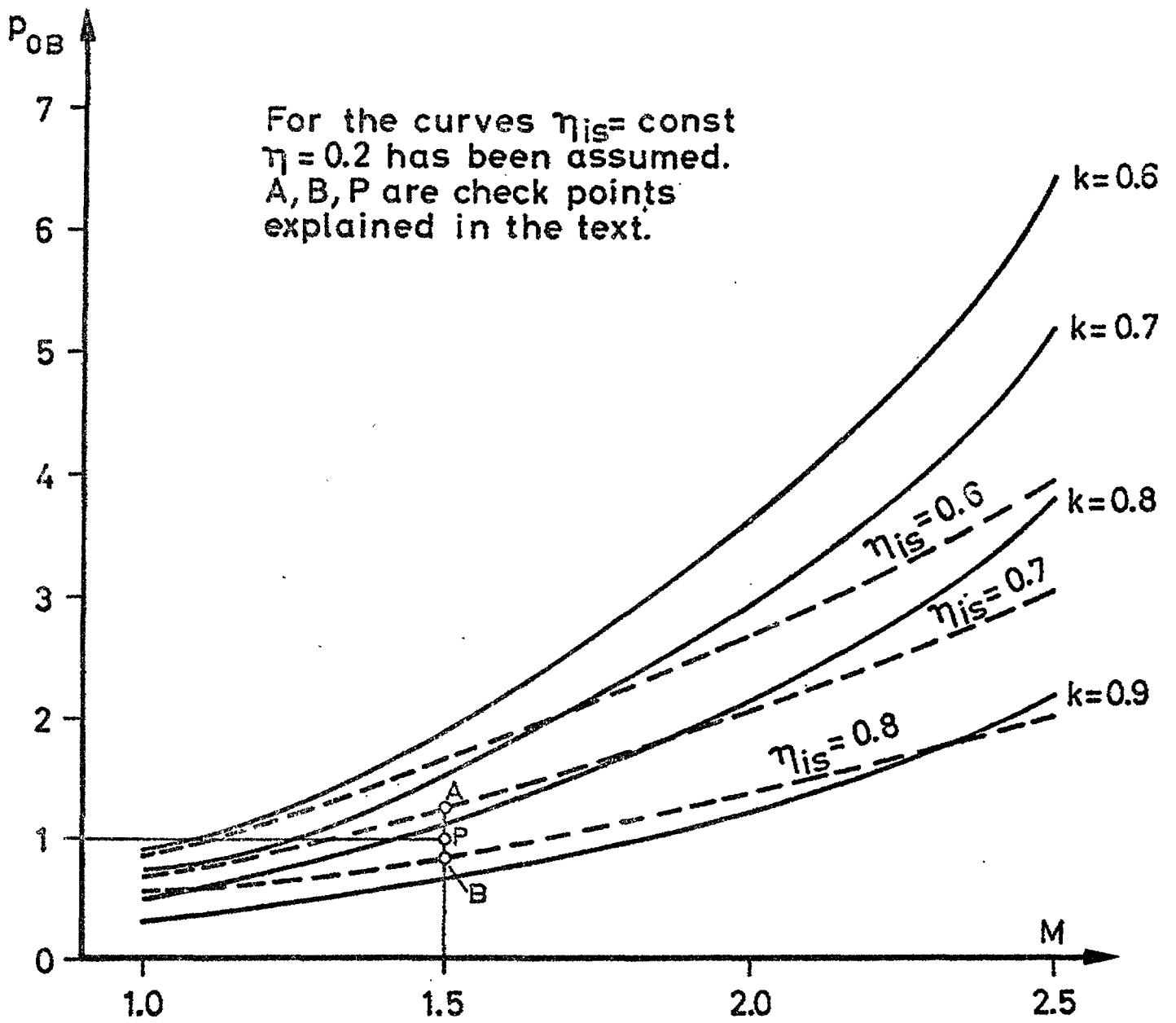


Fig. 3. Useful range of p_{0B} , M for $\eta = 0.2$, $T_0 = 1500^\circ\text{K}$.

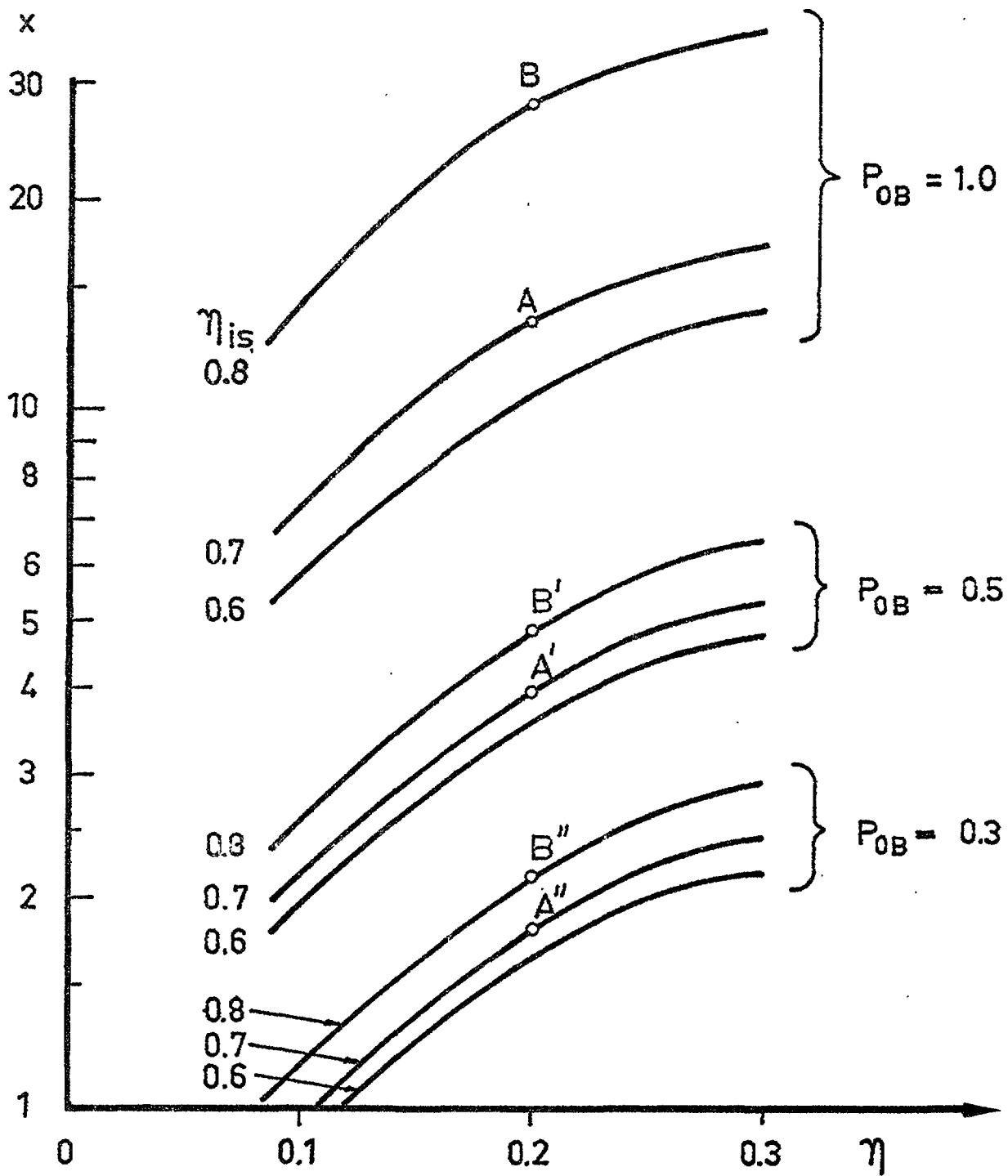


Fig. 4. Channel length as function of η with p and η_{is} as parameters. $M = 1.5$.

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