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Optimal Linear Filters for Pulse Height Measurements in the Presence of Noise

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ABSTRACT

For measurements of nuclear pulse height spectra a linear filter is used between the pulse amplifier and the pulse height recorder so as to improve the signal/noise ratio. The problem of finding the optimal filter is investigated with emphasis on technical realizability. The maximum available signal/noise ratio is theoretically calculated on the basis of all the information which can be found in the output of the pulse amplifier, and on an assumed a priori knowledge of the pulse time of arrival. It is then shown that the maximum available signal/noise ratio can be obtained with practical measurements without any a priori knowledge of pulse time of arrival, and a general description of the optimal linear filter is given. The solution is unique, technically realizable, and based solely on data (noise power spectrum and pulse shape) which can be measured at the output terminals of the pulse amplifier used.

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INTRODUCTION

Measurements of nuclear pulse height spectra are generally carried out by using a linear pulse detector followed by a linear amplifier, a linear filter and a pulse height analyzer. The problem of finding the linear filter which secures the least variance of the measured pulse height is the subject of the present paper. This theme has been of interest since the first pulse amplifier was put into operation, and it appears that the investigations have gone along two distinct lines, a purely theoretical one and a more practical one, which have only few interconnections.

The theoretical line consists of investigations of the properties (transfer function of impulsive response) of optimum filters without paying too much attention to the technical realizability. The main exponent of this line is Wiener [1], who claims that his theoretical solution is technically realizable, although there have been no reports (to the best of our knowledge) on filters which both exhibit "Wiener properties" and work better than filters constructed according to common practice, despite the fact that Wiener's theory [1] has been known for twenty years.

Due to the difficulties in utilizing Wiener's theories in practice for nuclear pulse work technical filters have been designed following a different theoretical basis. Experience has shown that a succession of integrating and differentiating networks does a good job, and optimizations have been carried out by assuming a fixed number of such networks and optimizing the performance by varying each of its time constants. This method has lead to excellent practical results but it is annoying in the respect that it leaves one with the feeling that there might exist other types of technically realizable filters which do a better job.

One difficulty in finding more generally optimized filters lies in properly defining when and how to measure; another is in introducing the condition that the filter must be technically realizable. Here these difficulties are partly overcome by calculating the maximum signal/noise ratio which can be obtained by utilizing all information available at the output terminals of the pulse amplifier, and then finding filters which in practice give this maximum signal/noise ratio when a commonly used method of measurement is applied. This procedure resulted in a family of technically realizable filters which, when connected to a linear amplifier and nuclear detector with given measurable properties, gave the maximum available signal/noise ratio.

1. METHODS AND DEFINITIONS

We will start with a consideration of the output current (voltage) from a linear pulse amplifier which has a linear nuclear pulse detector connected to its input. As an example one could think of a solid state detector plus electronic amplifier.

The output current consists of the sum of two parts, the pulse current and the noise current, and it is assumed that the pulse shape is known. Therefore, the pulse current i_p can be written

$$i_p(t) = pg(t - t_o) \tag{1.1}$$

where p will be referred to as the pulse height, $g(t - t_o)$ is a known function of time, and t_o will be referred to as the time of pulse arrival.

It is further assumed that the pulses are of finite duration; more precisely that $g(t - t_o)$ assumes significant values for some values of t when $T_1 < t - t_o < T_2$ and is equal to zero outside this interval.

The amplifier output current $i_o(t)$ is written

$$i_o(t) = pg(t - t_o) + i_N(t) \tag{1.2}$$

where $i_N(t)$ is a noise current which cannot be predicted in detail, but with some measurable (or calculable) qualities such as an autocorrelation function or frequency power spectrum.

The task is to calculate p with least variance from a knowledge of $i_o(t)$ and some measurable properties of $i_N(t)$. In order to do so we assume that we have a large number of oscillograms of $i_o(t)$ for different time intervals. Each oscillogram contains one pulse with the amplitude p.

The time difference between the start of oscillogram r at τ_r and recorded time is denoted τ, so that the recorded $i_o(t)$ on oscillogram r can be written as

$$i_o(t) = i_o(\tau + \tau_r) = pg(\tau + \tau_r) + i_N(\tau + \tau_r) \quad . \tag{1.3}$$

With the assumption that $\tau_r - t_o$ is equal for all oscillograms, τ_r being known, it is a trivial matter to go from one time origin to another and we will simply write

$$i_o(\tau) = pg(\tau) + i_N(\tau) \ . \tag{1.4}$$

The oscillograms are recorded in a certain time interval 0 through T and the significative values of $g(\tau)$ occur for $0<\tau_1<\tau<\tau_2<T$ Further it is assumed that

$$\overline{i_N(0)\, i_N(\tau_i\,)} = 0 \tag{1.5}$$

and

$$\overline{i_N(\tau_2)\, i_N(T)} = 0 \tag{1.6}$$

where the mean is supposed to be taken over all samples.

The conditions (1.5) and (1.6) can always be granted if the noise has an autocorrelation function of finite length (no dc component), so they do not present any noticeable reduction in the generality of this investigation.

The interval 0 through T is divided in M subintervals $\Delta\tau$ with

$$M\Delta\tau = T \tag{1.7}$$

and the vectors

$$\{i_o\} = \{i_o(0)\ ,\ i_o(\Delta\tau)\ ,\ i_o(2\Delta\tau)\ ,\ \ldots\ i_o(T)\ \} \tag{1.8}$$

$$\{g\ \} = \{g(0)\ ,\ g(\Delta\tau)\ ,\ g(2\Delta\tau)\ \ldots\ g(T)\ \} \tag{1.9}$$

$$\{i_N\} = \{i_N(0)\ ,\ i_N(\Delta\tau)\ ,\ i_N(2\Delta\tau)\ \ldots\ i_N(T)\ \} \tag{1.10}$$

are introduced. $\{i_o\}$ and $\{i_N\}$ are characteristic for the actual sample whereas $\{g\}$ is independent of the sample.

We shall also introduce a noise correlation matrix by defining

$$I_{pq} = I_{qp} = \overline{i_N(p\Delta\tau)\, i_N(q\Delta\tau)} \tag{1.11}$$

where the mean is taken over all samples;
The correlation matrix $\{I\}$ is then given by

$$\{I\} = \left\{ \begin{matrix} I_{00} & I_{01} & I_{02} & \cdots & I_{0M} \\ I_{10} & I_{11} & \cdots & & \\ I_{20} & & & & \\ \vdots & & & & \\ I_{M0} & & & & I_{MM} \end{matrix} \right\} . \tag{1.12}$$

2. MAXIMUM AVAILABLE SIGNAL/NOISE RATIO

From one of the oscillograms described previously p can be determined from each point with known τ. For each integer m, $0 \le m \le M$, p_m is defined by

$$p_m = \frac{i_o(m\Delta\tau)}{g(m\Delta\tau)} = \frac{pg(m\Delta\tau) + i_N(m\Delta\tau)}{g(m\Delta\tau)} ; \tag{2.1}$$

thus p_m is not equal to p, as it has an (unknown) error given by $i_N(m\Delta\tau)/g(m\Delta\tau)$. We will include infinite errors (which occur for $g(m\Delta\tau) = 0$) and thus (2.1) represents $M + 1$ determinations of p and the best value p_o (defined as the value having the least variance) is known [2] to be a weighted mean of all p_m.

The weights are zero for all values of m making $g(m\Delta\tau)$ equal to zero, so we can put the weight number m equal to $h_m\, g(m\Delta\tau)$ and determine h_m so as to make the variance of p_o a minimum.

By defining

$$\{h\} = \{h_o , h_1 , h_2 , \ldots h_M\} \tag{2.2}$$

we get for p_o

$$p_o = \frac{\Sigma h_m g(m\Delta\tau) p_{In}}{\Sigma h_m g(m\Delta\tau)} \qquad (2.3)$$

which by means of (2.1), (2.2), (1.9) and (1.10) can be written

$$p_o = p + \frac{\{h\} \{i_N\}^*}{\{h\} \{g\}^*} \qquad (2.4)$$

The variance ϵ^2 thus is

$$\epsilon^2 = \overline{\left(\frac{\{h\} \{i_N\}^*}{\{h\} \{g\}^*} \right)^2} = \frac{\{h\} \{I\} \{h\}^*}{(\{h\} \{g\}^*)^2} \qquad (2.5)$$

using (1.12).

Now $\{h\}$ shall be determined so as to make ϵ^2 minimum, thus ϵ^2 is differentiated with respect to $\{h\}$ which gives the minimum condition.

$$(\{h\} \{g\}^*) \ \{h\} \{I\} - (\{h\} \{I\} \{h\}^*) \{g\} = 0 , \qquad (2.6)$$

and because $\{h\} \{g\}^*$ must be non zero a quantity q can be defined by

$$q = \frac{\{h\} \{I\} \{h\}^*}{\{h\} \{g\}^*} \qquad (2.7)$$

By means of q, (2.6) is then written as

$$\{h\} \{I\} = q\{g\} \qquad (2.8)$$

If $\{I\}^{-1}$ exists (2.8) can be solved for $\{h\}$ and we get

$$\{h\} = q \{g\} \{I\}^{-1} \qquad (2.9)$$

This solution is valid for any value of q because both p_o and ϵ^2 are independent of the scale factor q. By inserting (2.9) in (2.5) the minimum variance becomes

$$\epsilon^2 = (\{g\} \{I\}^{-1} \{g\}^*)^{-1} \qquad (2.10)$$

In the calculations described information from only a limited number (M) of points is utilized. To utilize all information available we must let $\Delta\tau \to 0$ and $M \to \infty$ keeping $M\Delta\tau$ constant so (2.10) properly should be written

$$\epsilon^2 = \lim_{\Delta\tau \to 0} \left(\{g\} \{I\}^{-1} \{g\}^* \right)^{-1} , \tag{2.11}$$

and the available signal/noise ratio is p/ϵ.

A description of a linear filter with the signal/noise ratio p/ϵ can be based on (2.9) if a meaningful description of

$$\{h\} = \lim_{\Delta\tau \to 0} \{g\} \{I\}^{-1} \tag{2.12}$$

can be found.

In order to do so we assume that a linear filter with the complex, minimum phase transfer function $\nu^{-1}(\omega)$ is connected to the output terminals of the amplifier. The information available at the output of the (intrinsically noise free) filter is unchanged because the original output current can be regained by using an additional filter with the transfer function $\nu(\omega)$ [3].

Let $\nu(\omega)$ be the minimum phase transfer function determined from the noise power frequency spectrum $N(\omega^2)$ by means of

$$\nu\nu^* = N(\omega^2)\, K^{-1} \tag{2.13}$$

where K is a constant making $\nu(\omega)$ dimensionless.

The output current from the filter ν^{-1} contains exactly the same available information as the original one. Therefore the minimum variance ϵ^2 can be found by means of (2.11) if the filter output currents are used instead of the currents from the original amplifier.

After filter ν^{-1} the noise power spectrum $N_\nu(\omega^2)$ is

$$N_\nu = (\nu\nu^*)^{-1}\, N = K . \tag{2.14}$$

The noise is white with the mean square amplitude K per frequency unit. The mean square amplitude in an interval $\Delta\tau$ is, according to Ref. [4], equal to $\frac{1}{2}K(\Delta\tau)^{-1}$, so for the autocorrelation matrix we can write

$$\{I\}_\nu = \frac{1}{2} K (\Delta\tau)^{-1} \{E\} , \tag{2.15}$$

$\{E\}$ being the unit matrix $[(M+1) \cdot (M+1)]$, or

$$\{I\}_\nu^{-1} = 2 \Delta\tau K^{-1} \{E\} . \tag{2.16}$$

The pulse output current from the filter ν^{-1} is denoted by $g_\nu(\tau)$, and by means of (2.9) we get for $\{h\}_\nu$

$$\{h\}_\nu = 2 q \Delta\tau K^{-1} \{g\}_\nu . \tag{2.17}$$

This is valid for any q and hence also for $q = q_o \frac{K}{2\Delta\tau}$, so that

$$\{h\}_\nu = q_o \{g\}_\nu . \tag{2.18}$$

The minimum variance ϵ^2 is found from (2.11)

$$\epsilon^2 = \lim_{\Delta\tau \to 0} \left(2\Delta\tau K^{-1} \sum_{m=0}^{M} [g_\nu(m\Delta\tau)]^2 \right)^{-1}$$

$$= \frac{1}{2} K \left(\int_0^{T_\nu} [g_\nu(\tau)]^2 d\tau \right)^{-1} , \tag{2.19}$$

and p_o is found from (2.4)

$$p_o = \lim_{\Delta\tau \to 0} \frac{\{h\}_\nu \{i_o\}_\nu^*}{\{h\}_\nu \{g\}_\nu^*} = \frac{\int_0^{T_\nu} g_\nu(\tau) i_{o\nu}(\tau) d\tau}{\int_0^{T_\nu} [g_\nu(\tau)]^2 d\tau} \tag{2.20}$$

where the subscript ν denotes that the output from the filter ν^{-1} is considered.

If the filter ν^{-1} is followed by a filter having the impulsive response $g_\nu(T_\nu - \tau)/\int_0^{T_\nu} [g_\nu(\tau)]^2 d\tau$, the output current $i(t)$ from this filter becomes

$$i(t) = \left(\int_0^{T_\nu} [g_\nu(\tau)]^2 \, d\tau \right)^{-1} \int_{-\infty}^{\infty} i_{o\nu}(\tau) g_\nu(t - (T_\nu - \tau)) \, d\tau \qquad (2.21)$$

This current for $t = T_\nu$ is equal to p_o (see (2.20)) because $g_\nu(\tau)$ was assumed to be zero for $\tau < 0$ and for $\tau > T_\nu$.

So far we might draw the conclusion that the very best filter for pulse height measurement is made by first using a filter which transforms the amplifier noise into white noise, and then adding a filter having an impulsive response which is equal to the output pulse from the first filter, only reversed in time.

This conclusion also can be drawn from Wieners's work [1] and a similar conclusion is drawn by Bode and Shannon [3], but apparently no attempts to utilize the conclusion in practice have been successful, and only very few reports on this theme could be found in the literature (unsuccessful experiments are seldom reported). The reasons for the practical failure of such filters are mainly two: first, a filter according to the description above is in general a non minimum phase filter which in its practical appearance is so dependent on the accuracy of its components that slight deviations from their theoretically correct values cause large excess noise power output, and second, the use of such filters demands an a priori knowledge - not accessible from the amplifier output data - of the pulse time of arrival. However, the theory presents a way of calculating the maximum available signal/noise ratio from measurable data and assumed known pulse time and this signal/noise ratio is evidently an upper limit for the signal/noise ratio obtainable by means of practical measurements. As shown in the following chapter this upper limit can be closely approached in practice.

3. MEASURABLE SIGNAL/NOISE RATIO

Let us consider the signal/noise ratio of an experiment for measuring gamma-spectra by means of a Ge(Li) solid state detector plus pulse amplifier, linear filter, and multi channel analyzer. This experiment is shown schematically in Fig. 1, and was in fact the starting point for the present investigation. The original linear filter was an RC-CR circuit optimized according to the theory presented in [5], and the physical conditions were well in accordance with this theory. The detector was known to be able to resolve gamma-lines with a width (at half height) of approximately 3 keV in connection with a better amplifier than the present one, which gave a resolution of 15 keV. Thus the initial detector pulses could be considered as noise free and, still according to [5], it should be possible to increase the signal/noise ratio be a factor $2^{1/2}$ if the optimum RC-CR filter was replaced by the optimum RC-delay line filter. However, all attempts to obtain a larger signal/noise ratio than that obtained with the RC-CR filter failed. Direct measurements of the noise power revealed that the signal/noise ratio obtained from measurement of the gamma-line width obtained by means of the RC-CR filter was nearly twice (1.8) as large as the maximum signal amplitude divided by the root of the mean square noise amplitude. When the optimum RC-delay line filter was used the factor corresponding to 1.8 was 1.3. The signal/noise ratios measured with the two filters were identical within 2 %, with the RC-CR filter perhaps slightly better than the RC-delay line case.

On basis of the directly measured noise power frequency spectrum and the pulse shape the maximum available signal/noise ratio was calculated according to chapter II and it turned out to be equal to the signal/noise ratio determined from the width of the gamma-line when the RC-CR filter was used (within the error of the measurement, appr. 3 %). This result might seem surprising, but, in fact, it was expected because it had been shown already that the variance of the measured signal was considerably smaller ($1.8^2 = 3.2$ times) than the mean square noise amplitude. Thus it was demonstrated that the maximum available signal/noise ratio - under certain circumstances - could be obtained in practice and, surprisingly enough, without any a priori knowledge of the time of pulse arrival.

It would be unreasonable to expect to measure a larger signal/noise ratio than the maximum available one, even with the optimum RC-delay line filter, and the physical explanation that it is not better than the RC-CR filter is that the pulse shape plays an important role; certainly the pulse voltage in the RC-dealy line case is larger than in the RC-CR case, but it dwells a shorter time around its maximum value. For a further investigation of the influence of the pulse shape on the signal/noise ratio a capacitance was connected in parallel to the detector. Theoretically [5] this procedure should result in a larger time constant for the optimum filters. Practical measurements also showed this tendency, but to a much smaller degree than could be expected, and further, the measured signal/noise ratio became smaller than the maximum available one calculated on the basis of the directly measured noise power frequency spectrum and pulse shape. However, if the storage capacitance C (Fig. 1) was increased it could be adjusted such that the maximum available signal/noise ratio was indeed measured when the RC-CR time constant was adjusted to its optimum value according to [5]. It was found empirically that C for optimum performance should have a value such that Δv, the difference between the mean of the actual pulse maxima and the mean steady state voltage of C, was approximately the root of the mean square noise amplitude. This is illustrated in Fig. 2. In practice it might be inconvenient or impossible to adjust the pulse height analyzer in the desired way, however, a simple circuit which is described in the appendix can be used instead.

These results encouraged an attempt to try to find a description of the optimum linear filter in a general formulation from the pulse shape and the noise power frequency spectrum. A stringent mathematical investigation turned out not to be possible, mainly because precise rules for the practical measurement of the pulses could not be found. Attempts to do so indicated, however, that the complex transfer function $\kappa(\omega)$ of the optimal filter could be found as the minimum phase solution to the equation

$$\kappa\kappa^* = \gamma\gamma^* (\nu\nu^*)^{-2} \qquad (3.1)$$

where $\gamma(\omega)$ is the Fourier transform of the pulse shape $g(\tau)$ and $\nu(\omega)\nu^*(\omega)$ is the power frequency spectrum of the amplifier output voltage (including pulses).

This equation is consistent with the empirical results briefly mentioned above. For that particular case κ simply becomes the transfer function of the optimum RC-CR filter when the pulse rate is low. The Fourier transform $\gamma_F(\omega)$ of the filter output pulse becomes

$$\gamma_F = \gamma\kappa \qquad (3.2)$$

and thus by means of (3.1)

$$\gamma_F \gamma_F^* = (\gamma\gamma^*)^2 \; (\nu\nu^*)^{-2} \tag{3.3}$$

From (3.3) it appears that when the pulse rate is high, so that the pulse power frequency spectrum dominates and $\nu\nu^* \to b\gamma\gamma^*$, b being a constant independent of ω, then $\gamma_F \to b^{-1}$ which means that the filter output approach Dirac delta pulses. Thus pile-up errors are also minimized as long as the optimum value of C can actually be obtained; the pulse height analyzer must be so fast that it does not represent a limitation in the resolution.

The validity of (3.1) was empirically verified for different pulse shapes and noise power frequency spectra, obtained by using different resistors in series with the detector (Fig. 1). Also a coaxial cable between the detector and the pulse amplifier as well as a low noise amplifier was tried with positive results so that it was concluded that (3.1) for practical purposes is valid within a large range of both γ and $\nu\nu^*$. An example is shown in the appendix.

The content of this chapter can be summarized in three points:

A. The linear filter which gives maximum signal/noise ratio is a function both of the output voltage of the pulse amplifier (pulse shape, power frequency spectrum) and of the performance of the pulse height analyzer used.

B. If both the filter and the performance of the pulse height analyzer is optimized the maximum available signal/noise ratio can be obtained without an a priori knowledge of the time of arrival of the pulses.

C. The optimal linear filter (with optimized pulse height analyzer) can be described by its amplitude transfer function $|\kappa| = |\gamma| \; |\nu|^{-2}$ and minimum phase conditions.

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APPENDIX

As an example the commonly used cascode amplifier [5] is considered. For the first approximation the noise power spectrum is assumed to be

$$N(\omega^2) = \overbrace{\frac{a^2}{\omega^2}}^{\text{grid current noise}} + \underbrace{b^2}_{\text{shot noise}} \tag{4.1}$$

a^2 and b^2 being constants depending on the input tube. (According to [5] $a^2 = 2eI_g \cdot g_m^2(\Sigma c)^{-2}$ and $b^2 = 2eI_a F^2$, where e is the electron charge, I_g the grid current, g_m the mutual conductance, Σc the sum of capacitances including detector capacitance connected to the grid, I_a the anode current, and F^2 the space charge smoothing factor.)

Making the definition

$$b\, a^{-1} = T_o \tag{4.2}$$

$N(\omega^2)$ can be factorized,

$$N(\omega^2) = \frac{(1 + j\omega T_o)(1 - j\omega T_o)}{(T_o\omega)^2} \cdot b^2 \tag{4.3}$$

and

$$\nu = \frac{1 + j\omega T_o}{j\omega T_o} \quad . \tag{4.4}$$

In the first approximation the pulse can be described as having infinitely short rise time and infinite length* , hence

$$\gamma = \frac{1}{j\omega T_o} \tag{4.5}$$

and the optimum filter transfer function $\kappa = \nu^{-2}\gamma$ is

$$\kappa = \frac{j\omega T_o}{(1 + j\omega T_o)^2} \quad . \tag{4.6}$$

---

* It is not strictly correct to assume infinite pulse length, but in this particular case the assumption happens to be acceptable because ν has a pole for $\omega = 0$.

This results in the commonly used integration and differentiation circuit with equal time constants (= T_o, see ref. [5]).

The considerations above are valid for very low pulse rates only. For higher pulse rates the influence from other pulses on one single measurement becomes significant and (4.1) is not a good approximation to the noise power spectrum which is in this case better described by

$$N(\omega^2) = \frac{a^2}{\omega^2} + b^2 + k\gamma\gamma^* , \qquad (4.7)$$

k being a function of the pulse rate (and the pulse height spectrum). With noise power spectrum given by (4.7) the optimum filter characteristics (4.6) are the same as before except for a reduction of T_o ($T_o^{1} = b\,(a^2 + k^2\, T_o^{-2})^{-\frac{1}{2}}$).

With increasing k (increasing pulse rate), T_o becomes comparable to T_r, the rise time of the pulse, and hence (4.5) is not a good approximation to the pulse shape. The pulse shape is then better described by

$$g(\tau) = p(1 - \exp(-\tau/T_r)) , \qquad (4.8)$$

with

$$\gamma = \frac{1}{j\omega T_r(1 + j\omega T_r)} , \qquad (4.9)$$

which gives rise to a further integration (time constant T_r) in the optimum filter.

To measure the maximum available signal/noise ratio the pulse height analyzer must record a proper mean of the pulse amplitudes in the vicinity of the pulse maximum. This can easily be accomplished by inserting the circuit shown in Fig. 3 with the capacitance adjusted for optimum performance. During the rise and the decay of the pulse the diode resistance is small whereas it is large when the pulse is near its maximum value. Thus a new maximum is formed which is a mean of the original pulse amplitudes in the neighbourhood of the maximum. To utilize the diode characteristic properly the gain of the preceding stages should be so large that the effective noise voltage is a tenth of a volt or more.

By means of the filter with the transfer function $K(\omega)$ and the circuit shown in Fig. 3 the maximum available signal/noise ratio can be obtained within a few per cent.

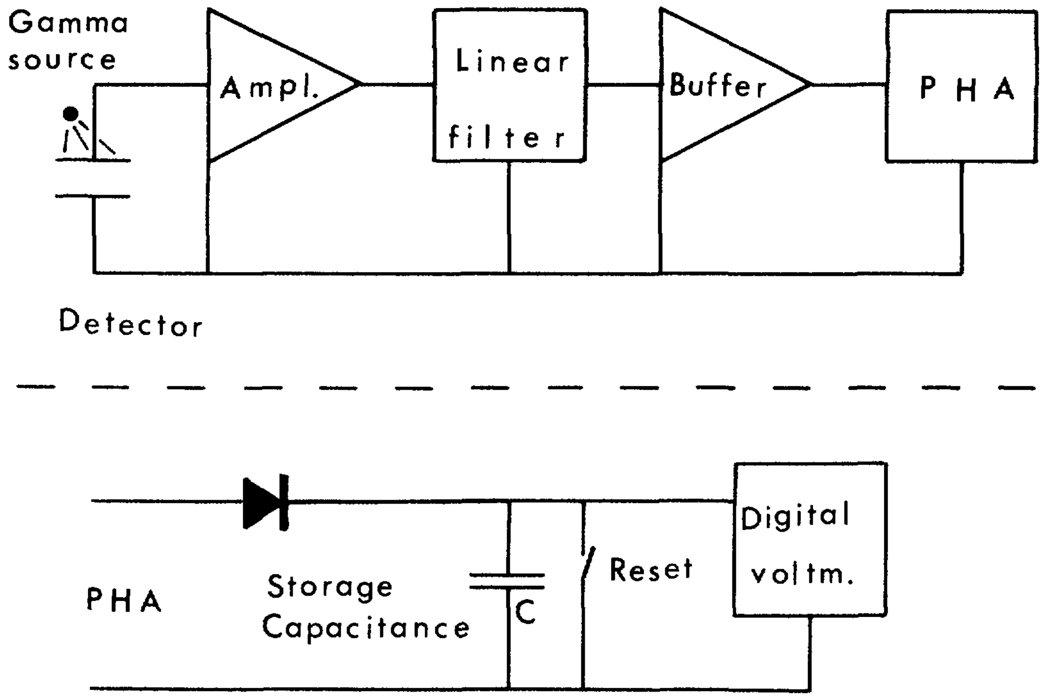

Fig. 1. Principle of pulse height measurement and of PHA = pulse height analyzer.

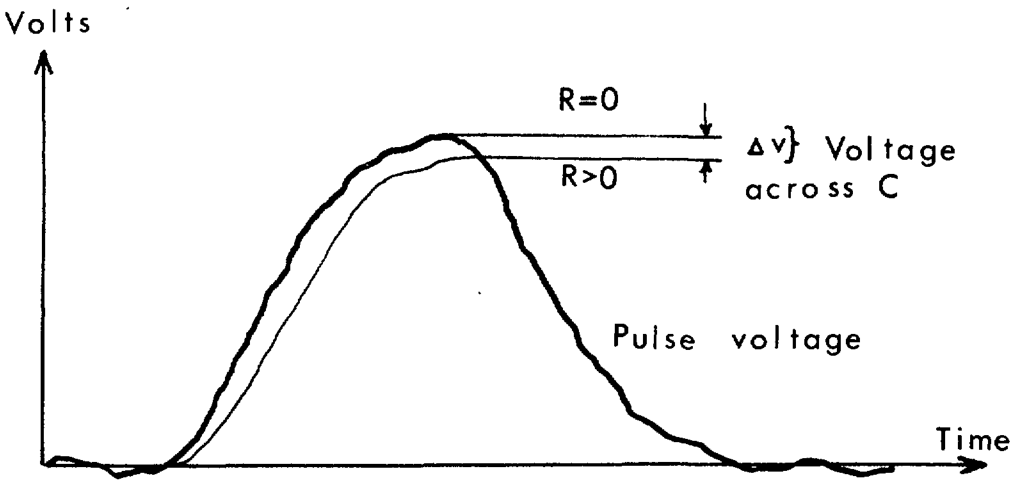

Fig. 2. The voltage across C for two different values of R, R = 0 and R > 0.

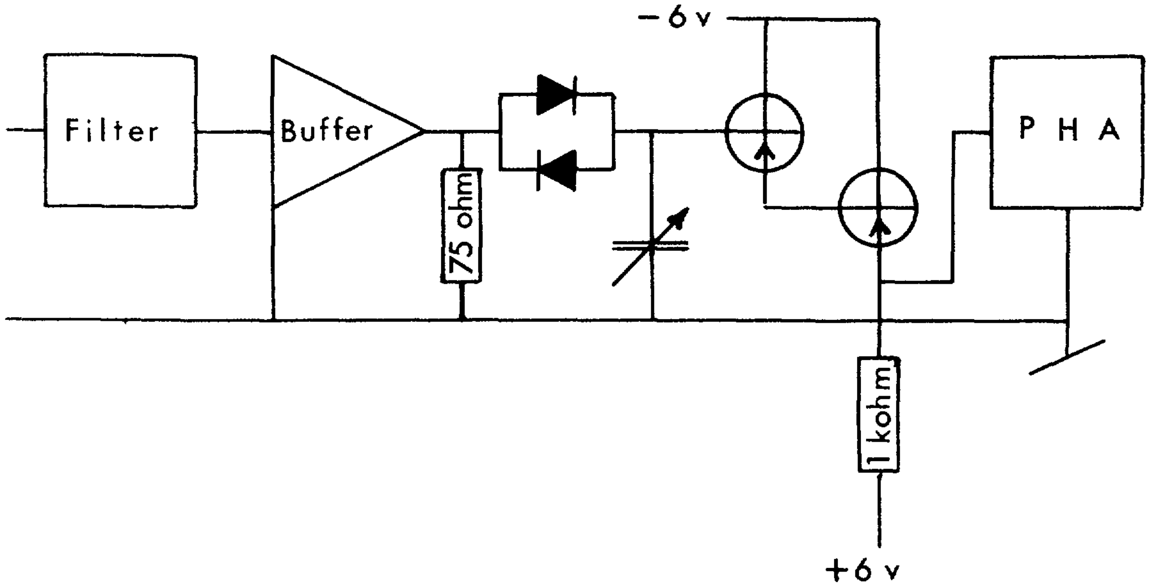

Fig. 3. The circuit between the buffer and the PHA is used to optimize the performance of the pulse height analyzer. Germanium diodes and silicon transistors have been used.

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