Calculation of Power Density with MCNP in TRIGA Reactor

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ABSTRACT

Modern Monte Carlo codes (e.g. MCNP) allow calculation of power density distribution in 3-D geometry assuming detailed geometry without unit-cell homogenization. To normalize MCNP calculation by the steady-state thermal power of a reactor, one must use appropriate scaling factors. The description of the scaling factors is not adequately described in the MCNP manual and requires detailed knowledge of the code model. As the application of MCNP for power density calculation in TRIGA reactors has not been reported in open literature, the procedure of calculating power density with MCNP and its normalization to the power level of a reactor is described in the paper.

1 INTRODUCTION

Monte Carlo computer code, MCNP, is a very powerful and versatile tool for particle transport calculations. It can be used for transport of neutrons, photons and electrons. Transport of neutrons is of special interest for a reactor physicist. MCNP code can be used for calculations of multiplication factor, reaction rates, saturated activities, neutron fluxes and spectra, power peaking factors, reaction rate distributions, shielding etc. Its main advantage is the ability to handle complicated geometries.

MCNP also provides seven standard tally types [1]. All tallies are normalized to one "starting" particle except in KCODE criticality problems, which are normalized to one fission neutron. In order to normalize the result by the thermal power of a system, one must use appropriate scaling factors.

As the procedure of calculating the scaling factors for KCODE calculation is not adequately described in the MCNP manual, it will be described in the paper.

The present paper will focus on standard cell flux (track length estimate of cell flux) F4 tally [1] which is of greatest interest for reactor physics calculations. However the results can be applied also to other neutron tallies.

2 NORMALISATION OF F4 TALLIES IN A CRITICALITY CALCULATION

The multiplication factor is one of the most important properties of a reactor or other system made of fissile material. In MCNP the most common way to calculate the multiplication factor is through the use of KCODE card. It is important to note that all the standard MCNP tallies can be made during a criticality calculation.

Since the MCNP results are normalised to one source neutron, the result has to be properly scaled in order to get absolute comparison to the measured quantities (flux, reaction rate, fission density, etc.). The F4 tally results can be scaled to a desired fission neutron

source (power) level or total neutron pulse strength. The scaling factor can be entered on the FM (tally multiplier) card or can be applied later in data processing.

The neutron birth rate in a fissile system can be calculated from the released energy per unit time i.e. the power of the system. The system producing power P needs $\dfrac{P}{w_f}$ fissions per second, where w_f denotes effective energy released per fission event. Although the value of w_f will vary somewhat with the type of reactor and the detailed core composition, it is typically of the order of 198 MeV for steady state condition. This fission rate produces $\dfrac{P\overline{v}}{w_f}$ neutrons per second, where $\overline{v}$ denotes the average number of neutrons released per fission. (The value of $\overline{v}$ is listed in the MCNP output file in the box containing the final k_{eff} result and represents the value averaged over fissile isotopes and neutron energies). Therefore to normalize an F4 tally by the steady-state thermal power of a critical system, the following scaling factor in units of fission neutrons per unit time should be used

$$S = \frac{P[\text{W}]\overline{v}\left[\dfrac{\text{neutron}}{\text{fission}}\right]}{\left(1.6022 \cdot 10^{-13}\,\dfrac{\text{J}}{\text{MeV}}\right)w_f\left[\dfrac{\text{MeV}}{\text{fission}}\right]} \tag{1}$$

.

The upper scaling factor is appropriate for critical i.e. steady-state power level systems only ($k = 1$). KCODE tallies for subcritical and supercritical systems do not include any multiplication effects because fission is treated as absorption. Therefore one must multiply the equation (1) by $\dfrac{1}{k_{eff}}$ on the right-hand side for subcritical and supercritical systems, respectively. It is important to note that the scaling factor for subcritical and supercritical systems is valid only when one uses neutron source distribution identical in space and energy to the source distribution obtained from the solution of an eigenvalue problem with $k_{eff} \neq 1$.

To conclude, when one wants to scale the calculated F4 tally "flux", Φ_{F4}, one must use the following equation

$$\Phi\left[\frac{\text{neutron}}{\text{cm}^2\text{s}}\right] = \frac{P[\text{W}]\overline{v}\left[\dfrac{\text{neutron}}{\text{fission}}\right]}{\left(1.6022 \cdot 10^{-13}\,\dfrac{\text{J}}{\text{MeV}}\right)w_f\left[\dfrac{\text{MeV}}{\text{fission}}\right]}\frac{1}{k_{eff}}\Phi_{F4}\left[\frac{1}{\text{cm}^2}\right], \tag{2}$$

where Φ denotes the actual total neutron flux in the system.

3   CALCULATION OF POWER DENSITY DISTRIBUTION

3.1   Calculation of power density

The energy released in a nuclear fission reaction is distributed among a variety of reaction products. The majority of the fission energy appears as the kinetic energy of the fission fragments and is deposited essentially at the point of fission. About 97 % of the

recoverable fission energy is deposited directly in the fissile material [2]. In our calculation of the power density distribution we will assume that power density is proportional to fission density. In other word that means that we assume that all of the recoverable fission energy is deposited at the point of fission. We will also assume that there are no temperature feedback effects and that there is only one fissile isotope in the system.

Power density, defined as the energy deposited in the fissile material per unit volume per unit time, can be written as

$$p(\mathbf{r}) = \int_0^\infty w_f(E,\mathbf{r}) \Sigma_f(E,\mathbf{r}) \varphi(E,\mathbf{r}) dE , \qquad (3)$$

where Σ_f and φ denote macroscopic fission cross section and neutron spectrum, respectively. Neutron spectrum $\varphi(E,\mathbf{r})$ is normalized such as

$$\int_0^\infty \varphi(E,\mathbf{r}) dE = \Phi(\mathbf{r}) , \qquad (4)$$

where $\Phi(\mathbf{r})$ is total neutron flux in $cm^{-2}s^{-1}$. The same is valid also for MCNP F4 tally

$$\int_0^\infty \varphi_{F4}(E,\mathbf{r}) dE = \Phi_{F4}(\mathbf{r}) . \qquad (5)$$

Equation (3) represents thermal power density at position $\mathbf{r}$ in the fission system. Hence the total power generated by the fission system is just the integral of the power density over the total volume where $\Sigma_f \neq 0$.

$$P = \int_{V,\, \Sigma_f \neq 0} d^3\mathbf{r} \int_0^\infty w_f(E,\mathbf{r}) \Sigma_f(E,\mathbf{r}) \varphi(E,\mathbf{r}) dE \qquad (6)$$

Assuming that w_f and number density of the fissile material do not depend significantly on the energy and the position in the fissile system, equation (3) can be written as

$$p(\mathbf{r}) = w_f N \int_0^\infty \sigma_f(E,\mathbf{r}) \varphi(E,\mathbf{r}) dE , \qquad (7)$$

where N denotes fissile material atom density and σ_f denotes microscopic total fission cross section. In MCNP the F4 cell flux tally, $\Phi_{F4}(\mathbf{r})$, is averaged over the volume of the sampling cell. Therefore, when we calculate fission density in a certain cell, $_\Delta V_i$ (i runs over all spatial cells of the reactor), we can omit the spatial dependence from the integral in equation (7) and obtain

$$p(\mathbf{r}) = w_f N \int_0^\infty \sigma_f(E) \varphi(E) dE, \quad \mathbf{r} \in {}_\Delta V_i \qquad (8)$$

The integral can be quite easily calculated by using the tally multiplier (FM) card [1], which is used to calculate, for $_\Delta V_i$, any quantity of the form

$$F_R = C \int_{E_{min}}^{E_{max}} \sigma_R(E)\varphi_{F4}(E)dE , \qquad (9)$$

where the constant C is any arbitrary scalar quantity that can be used for normalisation, $\sigma_R(E)$ is microscopic cross section for reaction R taken from MCNP cross-section libraries and E_{min} and E_{max} denote minimum and maximum neutron energy in the system (usually 0 and 20 MeV, respectively). F_R is the quantity for reaction R that is calculated by MCNP and its value can be found in the output file.

Any number of ENDF or special reactions can be used as a multiplier as long as they are present in the MCNP cross-section libraries. If the *C* entry is negative (for type 4 tally only), *C* is replaced by |C| times the atom density of the cell where the tally is made.

Thus for calculating power density we use F4 tally with tally multiplier -6, which is the microscopic total fission cross section, and normalise the result to a desired fission neutron source (power) level or total neutron pulse strength. Using equations (8), (9) and the scaling factor *S* we obtain

$$p(\mathbf{r}) = \frac{P\bar{\nu}M}{w_f} \, w_f N \int_{E_{min}}^{E_{max}} \sigma_f(E)\varphi_{F4}(E)dE, \quad \mathbf{r} \in {}_\Delta V_i \qquad (10)$$

It is interesting that the power density is independent on w_f. Finally we can write power density as

$$p(\mathbf{r}) = P\bar{\nu}NMF_f, \quad \mathbf{r} \in {}_\Delta V_i . \qquad (11)$$

Note that $p(\mathbf{r})$ is a function of $\mathbf{r}$, as it is constant inside each ${}_\Delta V_i$. If we want to obtain "smooth" $p(\mathbf{r})$ distribution, all ${}_\Delta V_i$ must be small.

It is important to note that σ_f in equation (10) is actually macroscopic total fission cross section normalized to one atom of the sampled material. Therefore in case the sampled material contains fissile and non-fissile components (e.g. UO_2, UCO, UZrH, $UO_2 + PuO_2$, etc.) the integral in equation (10) can be simply multiplied by the sampled material atom density, i.e. a mixture of several nuclides, and not the atom density of the fissile material only. If the sampled material contains several fissile isotopes, microscopic total fission cross section, σ_f, is calculated as the weighted average over all fissile isotopes, with regard to their atom fraction. The same is true for $\bar{\nu}$.

3.2   Alternative option for calculation of power density

The alternative option for calculation of power density is much simpler, but it has some limitations. This option is particularly useful when we want to calculate the power density in individual cells or fuel elements. First we calculate fission density (normalised "per atom") i.e. calculate F_f in each cell containing fissile material and obtain

$$\left(F_f\right)_i = \left(\int_{E_{min}}^{E_{max}} \Phi(E)\sigma_f(E)dE\right)_i , \qquad (12)$$

where i denotes the cell index. Since the power produced in one cell is linearly proportional to the number of fissions in that cell, we can calculate the power produced in cell i, P_i, by multiplying the fissile system thermal power, P, by the relative number of fissions in cell i. As the relative number of fissions in cell i is proportional to the product of fission density in cell i and the volume of cell i, we obtain

$$P_i = \frac{(F_f)_i V_i}{\sum_i (F_f)_i V_i} P \, . \tag{13}$$

Using the definition of power density as being the power produced per unit volume we obtain the power density in cell i as

$$p_i = \frac{(F_f)_i}{\sum_i (F_f)_i V_i} P \, . \tag{14}$$

The main limitations of this procedure of fission density calculation are that we have to know the exact volume of each cell and that we have to sample all cells containing fissile material, which is not necessary for the previously described option. Note that V_i can be the volume of the fuel inside the fuel element or very small volume of arbitrary material composition and needs not to contain only fissile material. In both cases p_i corresponds to the average power density in that volume.

3.3   Calculation of power density distribution

The easiest way for calculating the power density distribution can be by using the superimposed mesh tally card, FMESHn (for the present only type 4 tallies are permitted). FMESH card allows the user to define a mesh tally superimposed over the problem geometry. By default, the mesh tally calculates the track length estimate of particle flux, averaged over a mesh cell. If we use the tally multiplier card for fission together with the mesh tally and scale the results to appropriate power level, we can obtain power density distribution. When using tally multiplier card together with superimposed mesh tally card it is recommended to set material number to 0, which causes that the reaction cross sections for the material in which the particle is travelling are used. Thus we do not have to worry in which material particle is travelling.

The FMESH card is extremely powerful and useful method also for calculating flux distributions, power peaking factors, etc. . By using the very fine mesh we can calculate local power peakings and power density distributions near the water channels, which is very difficult if not impossible to do with deterministic methods.

4   POWER DENSITY DISTRIBUTION IN TRIGA MARK II CORE

The superimposed mesh tally feature of the MCNP code was used to calculate detailed power density distribution and power peaking factors of various hypothetical mixed cores of TRIGA Mark II reactor located at Jozef Stefan Institute. The mesh of the mesh tally was so fine that power density distribution was calculated with a resolution of 1mm × 1mm. The disadvantage of such high resolution is the very large number of sampling cells (500×500)

and consequently relatively high relative error of the tally. That is the main reason why we calculated the power density distribution averaged over the fuel height. The results are presented in Figure 1.
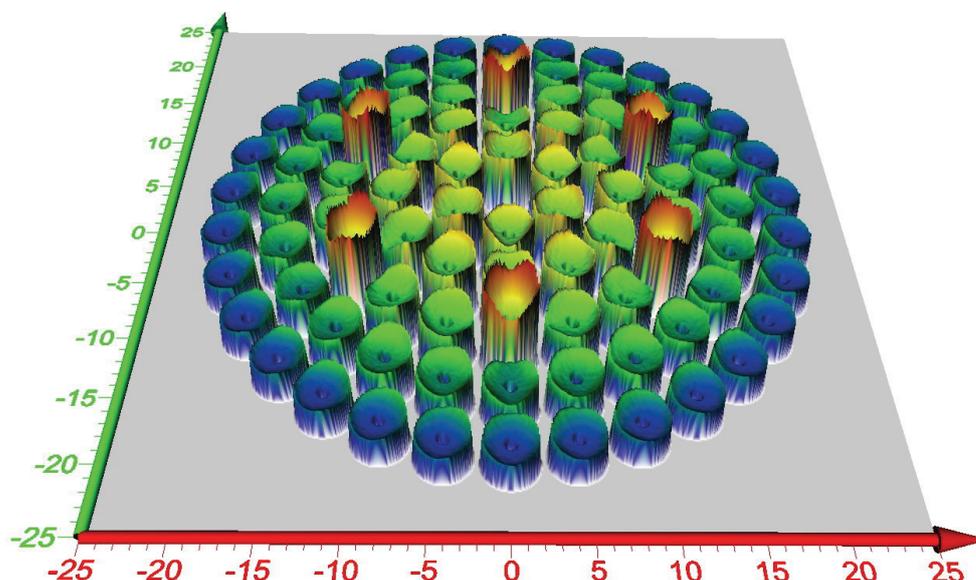

Figure 1: Power density distribution (rel. units) in mixed core with 8.5 w/o standard fuel and 6 LEU fuel elements in the D ring. The numbers on x in y axis are distances from the centre of the core in cm.

5   CONCLUSIONS

We have shown how to scale MCNP tally results to a desired power level of the system. The scaling factors can be derived from the system power, multiplication factor, average number of neutrons released per fission and average energy produced per fission event. When one is familiar with normalizing the results to a certain power level, the MCNP becomes a versatile calculation tool for various reactor physics calculations. One can calculate reaction rates, saturated activities, neutron fluxes and spectra, power peaking factors etc.. The MCNP ability to handle complicated geometries and use of the FMESH card feature of the MCNP code enables one to calculate detailed neutron flux and various reaction rate distributions with a resolution of 1mm×1mm or less (depends on the size of the system, the computer hardware and software). The later feature is especially useful for calculation of power density distribution and local power peaking factors calculation since it can reproduce properly the effects of local power gradients due to small heterogeneities in the core.

REFERENCES

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[2] J. J. Duderstadt, L. J. Hamilton, Nuclear Reactor Analysis, John Wiley & Sons, 1976