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TITULO

SOME COMMENTS ON THE CONCEPT OF ABSORBED DOSE

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Resumen

La principal magnitud física para la evaluación de los efectos inducidos por la radiación ionizante es la dosis absorbida. El ICRU 51 define este concepto como la magnitud $d\bar{\varepsilon}$ dividida por dm, donde $d\bar{\varepsilon}$ es la energía promedio impartida por la radiación ionizante a la materia de masa dm. Sin embargo, nada es dicho acerca de la operación promedio concerniente a la energía impartida ε. No obstante, debido a que ε considera la suma de todos los cambios de la masa en reposo de los núcleos y partículas elementales involucrados en todas las interacciones las cuales ocurren dentro de la masa (es decir, reacciones nucleares y transformaciones de las partículas elementales), la operación de promedio no puede ser realizada con un operador de equilibrio estadístico, sino mas bien, esta tiene que ser definida con un operador estadístico de no-equilibrio, por lo tanto, la dosis absorbida es una función dependiente del tiempo. Adicionalmente, se presenta una discusión para clarificar el equilibrio de radiación y el equilibrio de partículas cargadas dentro del contexto del equilibrio termodinámico.

Abstract

The main physical quantity for the evaluation of the induced effects by ionizing radiation is absorbed dose. ICRU report 51 defines this concept as quantity $d\bar{\varepsilon}$ divided by dm, where $d\bar{\varepsilon}$ is the mean energy imparted by ionizing radiation to matter of mass dm. However, nothing is said about the average operation concerning the stochastic energy imparted ε. Nevertheless, because ε considers the sum of all changes of rest mass of the involved nuclei and elementary particles in all interactions which occur within the mass (i.e. nuclear reactions and transformations of elementary particles), the average operation can not be done with an equilibrium statistical operator, rather, this has to be defined with a non-equilibrium statistical operator, therefore, absorbed dose is a function dependent on time. Furthermore, we present a discussion to clarify the equilibrium radiation and charged particle equilibrium within the context of thermodynamic equilibrium.

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Keywords: Absorbed dose, Cavity theory, Energy imparted, Equilibrium radiation, Charge Particle equilibrium, Nonequilibrium statistical operator.

1.- Introduction

It is very well known the increasing use of the ionizing radiation in several scientific and technological areas of human knowledge. In particular, the physical, chemical and biological effects are magnitudes of interest in the many of applications of these radiations. Nevertheless, a problem emerges by the attempt to quantify those effects. Nonetheless, since the forties it has been clarified that energy imparted is the physical quantity which is better correlated with such effects, [1].

Concretely, the International Commission on Radiation Units and Measurements (ICRU) in their report 51 defines absorbed dose D, in their sec. 1.3.2, [2],"...as the quotient of $d\bar{\varepsilon}$ by dm, where $d\bar{\varepsilon}$ is the mean energy imparted by ionizing radiation to matter of mass dm, thus,

$$D = \frac{d\bar{\varepsilon}}{dm} \qquad \text{...(1)}$$

On the other hand, also the stochastic quantity energy imparted ε is defined at ICRU 51 in sec. 1.3.1 as, "...

$$\varepsilon = R_{in} - R_{out} + \Sigma Q \qquad \text{...(2)}$$

where

R_{in} is the radiant energy incident on the volume, i.e., the sum of the energies (excluding rest energies) of the all those charged and uncharged ionizing particles which enter the volume,

R_{out} is the radiant energy emerging from the volume, i.e. , the sum of the energies (excluding rest energies) of the all those charged and uncharged ionizing particles which leave the volume, and $\sum Q(t)$ is the sum of all changes (decreases : positive sign ; increases : negative sign) of the rest mass energy of nuclei and elementary particles in any interactions which occur in the volume..."

It is possible to observe that given the stochastic nature of ε, this is a magnitude that obeys a time dependent probability function, i.e., $\varepsilon = \varepsilon(t)$. However, nothing is said about the nature of the average that is necessary to carry out in order to obtain $d\bar{\varepsilon}$, the mean energy imparted. Therefore, the objective of this work is to specify the necessary kind of average operation, in order to define appropriately the concept of absorbed dose to do this, we will compare the concepts of Radiation Equilibrium (RE) and Charged Particle Equilibrium (CPE) with thermodynamic equilibrium.

2.- Radiation Equilibrium and Charged Particle Equilibrium vs. Thermodynamic Equilibrium

2.1.Radiation Equilibrium

The RE, according to Attix [3], is defined as the condition in which "... in the non-stochastic limit, for each type and energy of ray entering *v* (interest volume), another identical ray leaves..," condition that is expressed as,

$$(\bar{R}_{in})_u \equiv \left\langle (R_{in})_u \right\rangle = (\bar{R}_{out})_u \equiv \left\langle (R_{out})_u \right\rangle \quad \text{... (3a)}$$

and,

$$(\bar{R}_{in})_c \equiv \left\langle (R_{in})_c \right\rangle = (\bar{R}_{out})_c \equiv \left\langle (R_{out})_c \right\rangle \quad \text{... (3b)}$$

here, the bar and $\langle\ \rangle$ symbols indicate averaging operation with respect an ensemble, where the nature of which be clarified in section 2.3. Additionally, $(R_{in})_u$ is the radiant energy of uncharged particles entering *v*, i.e., the sum of the energies (excluding rest energies) of the all those uncharged ionizing particles which enter the volume *v*. $(R_{out})_u$ is the radiant energy of uncharged particles leaving from *v*, i.e., except those which originated from radiative losses of kinetic energy by charged particles while are in *v*. $(R_{in})_c$ is the radiant energy of the charged particles entering *v*, and

$(R_{out})_c$ is the radiant energy of the charged particles leaving from v.

Therefore, according to the Eqs. (2) and (3), the mean energy imparted is expressed by,

$$\bar{\varepsilon}(t) \equiv \langle \varepsilon(t) \rangle = \langle \sum Q(t) \rangle \qquad \ldots(4)$$

It means that under RE conditions the expectation value of the energy imparted to the matter in v is equal to the mean energy emitted by the radioactive material in v, excluding that given to neutrinos. But, $\sum Q$ is the sum of all nuclear reactions and transformations of elementary particles which occur in v, situation that implies that $Q = Q(t)$, i.e. it is a time dependent physical magnitude. In other words, one could not calculate their average with respect to an ensemble corresponding to state of thermodynamic equilibrium, because such statistical operator that characterizes an equilibrium ensemble can not depends on the time, [4, 5].

Therefore, the point that here has to be explained is: What kind of ensemble is necessary to have in order to get the average of the energy imparted?. The answer, is that this should be a non-equilibrium statistical operator, [6]. With the purpose to clarify the nature of the non-equilibrium operator proposed, we analyze the CPE in the context of the definition of the absorbed dose.

2.2 Charged Particle Equilibrium

Attix establishes that, [2]: ." .CPE exist for the *v* (volume) if each charged particle of a given type and energy leaving *v* is replaced by an identical particle of the same energy entering, in terms of expectation values.." Condition that is written with aid of equation (3a) for the radiant energy flux and of the following expression for flux of charge,

$$\langle \dot{q}_{in} \rangle = \langle \dot{q}_{out} \rangle \qquad \ldots(5)$$

However, the conditions of the Eqs. (3a) and (5) correspond to a stationary state, where although there is no explicit dependence on time, there exist constant flows of charge and energy. On the other hand, a state of thermodynamic equilibrium is characterized by its time independent, and by the non existence of fluxes of any physical magnitude [4, 5]. Therefore, in conditions of CPE the Eq. (2) is expressed as,

$$\langle \varepsilon \rangle = \langle (R_{in})_u \rangle - \langle (R_{out})_u \rangle + \langle \Sigma Q \rangle \qquad \ldots(6)$$

Now, thinking about the case in which $\sum Q = 0$, the mean imparted energy in the volume of interest depends only of the net flux of radiant energy of the uncharged particles, which are deposited in the volume. As is already known, this radiant energy flux, comprises a stationary flux, which renders a secondary field of charge radiation, and this radiation is measured as a current I. This current is used to calculate the absorbed dose rate in Cavity theory [3]. Theory in which we explicitly have that,

$$\dot{D}(t) \propto \langle \dot{\varepsilon}(t) \rangle \propto \frac{d}{dm}\frac{dq(t)}{dt} = I = cte \qquad \ldots(7)$$

In the most general case in which $\sum Q(t) \neq 0$, the rate of absorbed dose is an explicit function of the time,

$$\dot{D}(t) \propto \langle \dot{\varepsilon}(t) \rangle \propto \frac{dI(t)}{dm} \neq cte, \qquad \ldots(8)$$

a differential equation that renders the following equality,

$$D(t) \propto \langle \varepsilon(t) \rangle \propto \int \frac{dI(t)}{dm} \cdot dt = f(t) \qquad \ldots(9)$$

where $f(t)$ is a non lineal time dependent function.

Summarizing, the mean value of energy imparted in Eqs. (4) and (9) is an explicit function of the time, because $\sum Q(t) \neq 0$, even in the case there is a stationary associate to the total radiant energy flux (RE), or when this stationary state appears relate with CPE. In both cases emerges an implicit time dependence in the definition of D.

2.3 Non-equilibrium Ensemble

It is very well known that in equilibrium statistical mechanics, to carry out to average a magnitude of interest the ensembles used are: microcanonical, canonical and grand canonical, i.e.,

$$\overline{A} \equiv \langle A \rangle \equiv \int A \cdot \rho_{eq}(\Gamma) \cdot d\Gamma \qquad \text{...(10)}$$

In the case of the expectation value for a physical magnitude dependent on time, from statistical mechanics of non-equilibrium, [5], we have formally,

$$\overline{A}(r,t) \equiv \langle A(r,t) \rangle \equiv \int A(\Gamma) \cdot \rho_{noneq}(\Gamma,t) \cdot d\Gamma \qquad \text{...(11)}$$

where ρ_{noneq} is a non-equilibrium statistical operator which is a function of time. However, outside of the local equilibrium the functional form of such operator is unknown.

In fact, this is the main problem to be solved in this discipline, i.e., to characterize and to quantify non-equilibrium states.

Concretely, in the case of the definition of the absorbed dose, the ensemble with which it is necessary to carry out the averaging operation is a non-equilibrium one. On one hand, because term $\sum Q(t)$ evaluates nuclear reactions and transformations of elementary particles that occur inside the volume of interest. On the other hand, and for the other side, because still in the case that there are not nuclear reactions or transformations of elementary particles ($\sum Q = 0$), the measurement of the absorbed dose is carried out at most in conditions of RE or of CPE, which are stationary states defined in terms of the flux of the radiant energy. But it is very well known, that stationary states are non-equilibrium states, [5,7].

Indeed, ρ_{noneq} is in any particular case very difficult determine. The late argument is in fact the reason that propels us to resort to numerical techniques (for example, Monte Carlo simulation or numerical transport theory, [8]), in order to be able to determine in particular problems the absorbed dose in situations of interaction radiation matter. Or to take advantages to the experimental techniques based on the cavity theory , where are used stationary states by means of conditions CPE or transitory CPE, in order to eliminate dependence on time in D (r, t).

Finally, in the literature, objections have already appeared against the concept of absorbed dose [9], however, the arguments put forward these authors are questionable in several aspects.

Firstly, the definition of the absorbed dose does not violate the law of conservation of the energy, but these authors arrive to this conclusion because they do not make a correct balance of energy in $\sum Q(t)$ for the considered reaction (n, γ). Concretely, it is true that there would be a change in the number of mass by one in the rest mass in rest in the nucleus (≈-932 MeV), but also there would be a change in the rest mass the neutron, that is absorbed, and therefore it disappears (≈+940 MeV). The gamma radiation that is emitted corresponds to the difference between both rest masses.

Secondly, it is not necessary to make use of other physical magnitudes besides energy imparted, like for example the internal energy or the rate flux of non-ionizing radiant energy in order to define D (r, t). It is enough only to specify that averaging operation, in order to obtain the absorbed dose. Average that should be carried out with a non-equilibrium statistical operator, from which explicitly is obtained the absorbed dose as a time dependent function.

Thirdly, the difference between the energy imparted and the energy absorbed is not the non-ionizing mean energy , which abandons the volume of interest. More accurately, this difference is the term $\sum Q(t)$, which has been very well discussed by Carlsson *et al*, [10,11,12,13].

4.- Conclusions

Summing up, it is enough to specify that aforementioned averaging operation in the definition of the absorbed dose has to be performed employing a non-equilibrium statistical operator. This point enables us to clarify the emerging confusions by considering absorbed dose as time dependent function.

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