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TRANSPORT IN CONSTRICTED QUANTUM HALL SYSTEMS: BEYOND THE KANE-FISHER PARADIGM

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Abstract

A simple model of edge transport in a constricted quantum Hall system with a lowered local filling factor is studied. The current backscattered from the constriction is explained from a matching of the properties of the edge-current excitations in the constriction (ν_2) and bulk (ν_1) regions. We develop a hydrodynamic theory for bosonic edge modes inspired by this model, stressing the importance of boundary conditions in elucidating the nature of current transport. By invoking a generalised quasiparticle-quasihole symmetry of the quantum Hall circuit system, we find that a competition between two tunneling process determines the fate of the low-bias transmission conductance. A novel generalisation of the Kane-Fisher quantum impurity model is found, describing transitions from a weak-coupling theory at partial transmission to strong-coupling theories for perfect transmission and reflection as well as a new symmetry dictated fixed point. These results provide satisfactory explanations for recent experimental results at filling-factors of 1/3 and 1.

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I. INTRODUCTION: WHAT DO THE EXPERIMENTS SAY?

The quantum Hall effects are essentially the low temperature physics of a disordered 2DEG placed in a strong perpendicular magnetic field [1]. For particular values of the external magnetic field, the incompressible Hall fluid in the bulk and gapless current carrying edge excitations lead to a vanishing longitudinal resistance and a quantised Hall resistance. Electronic correlations are crucial for the fractional quantum Hall effect, a gapped ground state with fractionally charged quasiparticle excitations [2] which were observed in shot-noise measurements [3]. Local quasiparticle tunneling between the oppositely directed current carrying edge states of a Hall bar is known theoretically to be a singular perturbation, with the strong-coupling behaviour that of two effectively disconnected quantum Hall bubbles [4]. Recent experiments studying transport through gated constrictions in quantum Hall systems at integer as well as fractional filling factors [5, 6] have, however, shown the need for a deeper understanding of inter-edge tunneling. A signature of departure from the traditional quantum Hall scenario can be observed in these experiments from the following. Imposing a finite bias at a pair of local split-gates (V_G) causes a backscattered current across the bulk, leading to a finite, edge-bias (V) independent, longitudinal resistance drop. A fractional Hall conductance (g) is simultaneously measured across the constriction at finite V corresponding to a filling factor, ν_2, lower than that in the bulk (ν_1). Further, for large V_G, g is observed to dip sharply and vanish with a power-law dependence on V as $|V| \to 0$. A comparison with the theory of Fendley etal. [7] for inter-edge Laughlin quasiparticle tunneling suggests strongly that the constriction transmission is governed by ν_2. This is particularly unexpected for an integer quantum Hall system [5].

A particularly intriguing observation is that of the evolution of the transmission conductance g as V_G is varied in the limit of vanishing V. While g shows a zero-bias minimum at sufficiently large V_G, decreasing V_G leads first to a bias-independent transmission at a particular value of the gate-voltage $V_G = V_G^*$ and then to an enhanced zero-bias transmission for yet lower values of V_G. This behaviour of the zero-bias g is also observed across a wide range of temperatures. The bias-independence of g at a certain V_G^* and its enhancement at $V_G < V_G^*$ are quite unexpected from the conventional theoretical viewpoint [4]. Qualitatively similar results were found for the integer cases of $\nu_1 = 1$ [5] as well as for the fractional cases of $\nu_1 = 1/3, 2/5$ and $3/7$ [5, 6], allowing for the possibility of a common explanation. Current theoretical efforts [8, 9, 10] have, however, been unable to provide any simple explanations. The scenario proposed in ref.[8] involves the complications of stripe states arising from longer range interactions. However, it fails to present any mechanism explaining the surprising evolution of g with V_G. The same is also true of proposals of line junctions [9] and the effects of inter-edge interactions on quasiparticle tunneling [10]. Thus, bearing in mind that the theory of refs.[4, 7] matches the experiments in only a very restricted parameter regime, the lack of a clear theoretical understanding represents an important problem to be addressed here.

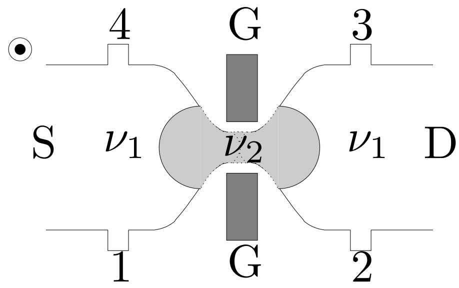

FIG. 1: A schematic diagram of a QH bar at a bulk filling fraction ν_1 and with a gate-voltage (G) controlled split-gate constriction which lowers the filling fraction in the constriction region to ν_2. S and D show the source and drain ends of the Hall bar while 1 to 4 signify the current/voltage terminals.

II. THE THEORETICAL CHALLENGES FACED

We now reflect on the theoretical challenges involved in building a model appropriate to the experiments. First, the two scenarios of ballistic transport studied in ref.[4] are perfect transmission (see Fig.(3) given below) and perfect reflection (see Fig.(4) given below) at the constriction respectively; inter-edge tunneling of quasiparticles and electrons is then treated perturbatively in the two scenario respectively. In this way, Kane and Fisher were able to map these two transport scenarios onto that of electron backscattering from a weak impurity in a Tomonaga-Luttinger liquid (TLL) and electron tunneling between two TLLs respectively [4]. They concluded that the first scenario was an unstable fixed point (in the RG sense) with regards to inter-edge quasiparticle tunneling, while the second scenario was a stable fixed point; the RG flow was, therefore, unidirectional from the former to the latter. This was later confirmed in the thermodynamic Bethe Ansatz-based exact solution to the problem given by Fendley, Ludwig and Saleur [7].

In treating a scenario of intermediate ballistic transmission, however, we first need a model describing partial transmission and reflection *without* resorting to perturbative inter-edge tunneling processes. The treatment of inter-edge tunneling will then lead to a non-trivial generalisation of the quantum impurity problem of refs.[4, 7]. Second, the most probable effect of a split-gate system is to create a smooth and long constriction potential, depleting the local electronic density (and hence lowering the local filling factor) locally from its value in the bulk. Indeed, this led Roddaro and co-workers [5] to conjecture on the likelihood of a small region in the neighbourhood of the constriction with a reduced filling factor ($\nu_2 < \nu_1$) being the cause of their puzzling results (see fig.(1)). The conjecture, however, remained unsubstantiated by the formal analysis of a concrete theoretical model. Thus, their explanations for the $\nu = 1$ system remained suggestive at best and no attempt at unifying the observations at both integer and fractional values of ν could be made. It is the aim of this work to develop a simple model to deal with this lacunae. In doing so, we devote our attention solely to short-ranged electronic correlations which cause the formation of chiral Tomonaga-Luttinger liquid (TLL) edge states (without the intervention of any stripe states [8] arising from longer range interactions).

III. RESULTS FROM A LANDAUER-BUTTIKER FORMULATION

We begin by deriving some results for ballistic edge transport through a constriction region with reduced filling fraction ν_2 in a Hall bar geometry (see fig.(1)) from a few simple considerations. First, we estimate the spatial extent, L_{con}, of the ν_2 region. Noting that the transport data taken at a temperature of 50mK does not appear to show any interference effects arising from coherence across the entire constriction [5], L_{con} can be safely assumed to be longer than the thermal length $L_T = hv/k_B T$ (where v is the edge velocity). For a typical $v = 10^3\text{ms}^{-1}$ [11] and $T = 50$mK, $L_T \sim 1\mu$m. Clearly, $L_{con}(> 1\mu\text{m}) >>$ magnetic length $l_B(\sim 100\text{Å})$, justifying our assumption of the mesoscopic nature of the ν_2 region. Now, we make two reasonable assumptions in framing the model. First, that the voltage (Hall) bias between the edges of the sample is unaffected by the local application of a gate-voltage at a constriction, as long as the bulk of the system is in an incompressible quantum Hall state (ν_1). Second, that the two-terminal conductance measured across the constriction is determined by the lowered filling-fraction of the quantum Hall ground state in the constriction (ν_2).

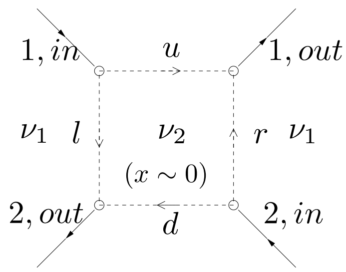

FIG. 2: A schematic diagram of the "constriction" system given by the dashed box around the region $x \sim 0$ and symbolised by the filling fraction ν_2 lower than that of the bulk, ν_1. The four chiral fields approaching and leaving this region are shown by the arrows marked as $1, in$, $1, out$, $2, in$ and $2, out$. The dashed horizontal and vertical lines at the junction represent the edge states which are transmitted (u, d) and reflected (l, r) at the constriction respectively.

Then, for a current I injected from the source, we know that $I = g_b V_{42}$ where $g_b = \nu_1 e^2/h$ is the bulk Hall conductance and V_{42} is the source-drain edge-bias. The second assumption gives the current transmitted ballistically through the constriction as $I_{tr} = g_c V_{42}$, where $g_c = \nu_2 e^2/h$ is the two-terminal conductance measured across the constriction. From the first assumption, we find the transmitted current as $I_{tr} = (g_c/g_b)I = (\nu_2/\nu_1)I$. Kirchoff's law then gives the current reflected at the constriction as $I_{ref} = I - I_{tr} = (1 - \nu_2/\nu_1)I$. The minimum value of the reflected conductance is then $g^{back} = I_{ref}/V_{42} = (1 - \nu_2/\nu_1)G_b = (\nu_1 - \nu_2)e^2/h$, and quantised at an effective filling factor for the reflected current as $\nu_{ref} = \nu_1 - \nu_2$ (in units of e^2/h). Further, the "background", edge-bias independent, value of the longitudinal resistance drop across the constriction is

$$R^{BG} = (V_4 - V_3)/I = (1 - \nu_2/\nu_1) g_b^{-1} \ . \tag{1}$$

The current backscattered from the constriction is presumably carried in a gapless region lying in-between the bulk and constriction regions.

IV. RESULTS FROM A WEN-TYPE HYDRODYNAMIC EDGE STATE MODEL

By relying on the same assumptions, we now present a hydrodynamic model of gapless, current carrying, chiral edge density-wave excitations [12] describing ballistic transport through the transmitting and reflecting edges at the constriction (shown schematically in fig.(2)). As this is a free field theory in the bosonic edge fields, quasiparticle tunneling arises from the exponentiation of these fields. Focusing on the immediate relevance of the results of this model to the experiments, we keep mathematical details for elsewhere [13]. We assume the constriction region length $2a$ to lie in the range $l_B << 2a << L$, where L is the total system size and l_B is the magnetic length; the external arms $(1in, \ldots, 2out)$ meet the internal ones $(u, \ldots, l)$ at the four corners of the constriction. From our assumptions, ν_1 governs the properties of the four outer arms while ν_2 that of the upper and lower (transmitted) arms of the circuit at the constriction. The effective filling factor for the right and left (reflected) arms of the circuit (ν_{ref}) is treated as a parameter to be determined. Ballistic transport in the various arms of the circuit shown in fig.(2) is given by a Hamiltonian H describing the energy cost for edge-density distortions [12] $H = H^{ext} + H^{int}$ where

$$H^{ext} = \frac{\pi v}{\nu_1} [\int_{-L}^{-a} dx \ (\rho_{1in}^2 + \rho_{2out}^2) + \int_a^L dx \ (\rho_{2in}^2 + \rho_{1out}^2)] \tag{2}$$

and H^{int} has the same form as H^{ext} but with the densities $(\rho_u, \ldots, \rho_l)$ and filling factors ν_2 and ν_{ref} placed appropriately. The velocity v of the edge-excitations is taken to be the same for all arms; instead, we focus on the effects of a changing filling factor. The densities ρ are, as usual, represented in terms of bosonic fields ϕ describing the edge displacement, e.g, $\rho_{1in} = 1/2\pi\partial_x\phi^{1in}$, $\rho_{2out} = -1/2\pi\partial_x\phi^{2out}$ [12]. The commutation relations satisfied by these fields are familiar, e.g.,

$$[\phi^{1in}(x), \partial_x\phi^{1in}(x')] = i\pi\nu_1\delta(x - x') \tag{3}$$

and so on for the other fields. The equations of motion found from H describe the ballistic motion of chiral edge-density waves, e.g, $(\partial_t + v\partial_x)\rho^{1in}(x,t) = 0$, $\rho^{1in}(x,t) \equiv \rho^{1in}(x - vt)$ etc.

A. The importance of boundary conditions

The H given above, however, needs to be supplemented with matching conditions at the corners of the constriction for a complete description. From the form of H, it is clear that we need two matching conditions at each corner; a reasonable choice is one defined on the fields and one on their spatial derivatives. We choose, for instance, at the top-left corner

$$\begin{aligned}\phi^{1in}(x=-a) &= \phi^{u}(x=-a)+\phi^{l}(y=-a)\\ \partial_x\phi^{1in}(x=-a) &= \partial_x\phi^{u}(x=-a)+\partial_y\phi^{l}(y=-a)\end{aligned} \tag{4}$$

where x and y are the spatial coordinates describing the $(1in, u)$ and l arms respectively. The equation of continuity leads to the familiar form for the current operator, e.g, $j^{1in} = 1/2\pi\partial_t\phi^{1in}$ etc. Thus, current conservation at every corner is easily seen from the first matching condition. The reflecting chiral edge modes thus convey a finite "backscattered" current across the sample. In this way, we formally establish the intermediate ballistic transmission scenario as observed in the experiments. Charge density fluctuations at each corner are described by the matching condition on $\partial_x\phi$. This matching condition is a statement of the conservation of net charge density at each corner. In this way, the two matching conditions together establish the continuity of current and charge density at every corner of the junction system.

Using eqs.(4), we compute the commutation relation $[\phi^l, \partial_y\phi^l]_{y\to -a} = ([\phi^{1in}, \partial_x\phi^{1in}] - [\phi^u, \partial_x\phi^u])_{x\to -a}$, which leads once again to $\nu_{ref} = \nu_1 - \nu_2$. In another check, from the standard Kubo formulation [14], we reproduce $g_{1in,1out} = \nu_2$ and $g_{1in,2out} = \nu_1 - \nu_2$. It is also worth noting that the cases of a perfect Hall bar ($\nu_2 = \nu_1$) and two Hall bubbles separated by vacuum ($\nu_2 = 0$) can be modelled as special limiting cases of the matching conditions (eqs.(4)) given earlier. This is easily seen as follows. For $\nu_1 = \nu_2$, the commutation relation of the reflecting edge states vanishes, killing its dynamics. This can also be understood within a hydrodynamic prescription [12], where a vanishing effective filling factor (the amplitude of the Kac-Moody commutation relation, eq.(3)) leads to a diverging energy cost for edge charge density fluctuations; the dynamics of the bosonic field characterising such fluctuations is thus completely damped. Thus, the reflecting edge states carry no current, while the transmitting edge states perfectly transmit all incoming current into the outgoing arms on the *opposite* side of the constriction. The matching conditions eqs.(4) at the four corners are then reduced to

$$\begin{aligned}\phi^{1,in}(x=-a) &= \phi^{u}(x=-a)\ ,\ \phi^{u}(a)=\phi^{1out}(x=a),\\ \phi^{2,in}(x=a) &= \phi^{d}(x=a)\ ,\ \phi^{d}(-a)=\phi^{2out}(x=-a),\\ \partial_x\phi^{1,in}(-a) &= \partial_x\phi^{u}(-a)\ ,\ \partial_x\phi^{u}(a)=\partial_x\phi^{1out}(a),\\ \partial_x\phi^{2,in}(a) &= \partial_x\phi^{d}(a)\ ,\ \partial_x\phi^{d}(-a)=\partial_x\phi^{2out}(-a).\end{aligned} \tag{5}$$

These identifications of the fields and their spatial derivatives lead to the continuity conditions which underpin the hydrodynamic theory of Wen [4, 12] for the case of the two infinite chiral edges (say, upper and lower) of a Hall bar (with filling factor ν_1), and eq.(3) then reproduces the well-known Kac-Moody commutation relation everywhere along the edges. This is shown in Fig.(3) below.

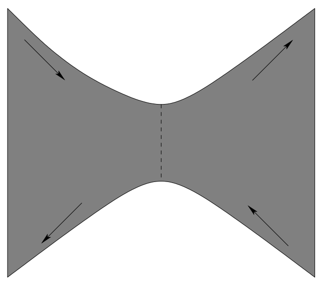

FIG. 3: A schematic diagram of the quantum Hall bar system with a "constriction" which promotes quasiparticle tunneling between two points on oppositely directed edges of the system (dashed line). The upper and lower edges are continuous everywhere and therefore have boundary conditions on the field ϕ and its spatial derivative $\partial_x\phi$ as given above in equns.(5).

Similarly, for the case of $\nu_2 = 0$, the commutation relation for the transmitting edge states vanishes, killing its dynamics: they carry no current, while the reflecting edge states perfectly convey all incoming current into the outgoing arms on the *same* side of the constriction. Thus, the matching conditions eqs.(4) at the four corners are reduced to

$$\begin{aligned}
\phi^{1,in}(x=-a) &= \phi^{l}(y=-a)\ ,\ \phi^{l}(a) = \phi^{2out}(x=-a),\\
\phi^{2,in}(x=a) &= \phi^{r}(y=a)\ ,\ \phi^{r}(-a) = \phi^{1out}(x=a),\\
\partial_x\phi^{1,in}(-a) &= \partial_y\phi^{l}(-a)\ ,\ \partial_y\phi^{l}(a) = \partial_x\phi^{2out}(-a),\\
\partial_x\phi^{2,in}(a) &= \partial_y\phi^{r}(a)\ ,\ \partial_y\phi^{r}(-a) = \partial_x\phi^{1out}(a).
\end{aligned} \tag{6}$$

Again, these identifications of the fields and their spatial derivatives lead to the continuity conditions which underpin the hydrodynamic theory of Wen [4, 12] for the case of the infinite chiral edges (say, left and right) of two distinct Hall bubbles (each with filling factor ν_1) separated by vacuum, and again reproduce the familiar Kac-Moody commutation relations everywhere along the edges. This is shown in Fig.(3) below. We have, thus, been able to construct a family of free

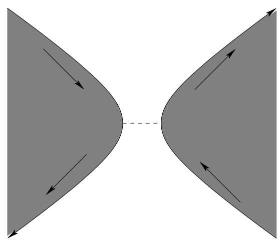

FIG. 4: A schematic diagram of a system of two quantum Hall droplets separated by vacuum and with a "constriction" which promotes electron tunneling between two points on adjaecent (and oppositely directed) edges of the system (dashed line). The right and left edges are continuous everywhere and therefore have boundary conditions on the field ϕ and its spatial derivative $\partial_x\phi$ as given above in equns.(6).

theories describing ballistic transport through the constriction at intermediate transmission, with those of complete transmission and reflection representing two special cases. This is the key step in generalising the quantum impurity model of refs.[4, 7] and represents a significant advance.

B. Quasiparticle-quasihole symmetry and its role in inter-edge tunneling

Now to the role of quasiparticle (qp) tunneling in determining low-energy transport. First, it is clear that local qp tunneling processes between the (u, d) arms will be dictated by the constriction filling factor ν_2. We can write such a process, located deep within the constriction, as $\lambda_1 \cos(\phi^u(x = 0) - \phi^d(x = 0))$. From ref.[4], we know that the RG equation for the qp tunnel coupling λ_1 is given by $d\lambda_1/dl = (1 - \nu_2)\lambda_1$. With $\nu_2 < 1$, λ_1 will grow under the RG flow to strong coupling. There is, however, also a local tunneling process between the (l, r) arms to consider. It is revealed by the quasiparticle-quasihole (qp-qh) symmetry of the completely filled effective lowest Landau level of the qps (i.e., the ground state of the Hall fluid) in the bulk which is protected by a gap larger than all other energy scales in the problem [15]. In fig.(5), we present an extension of the electron-hole symmetry of the $(\nu_1 = 1, \nu_2)$ constriction geometry [5] to a general (ν_1, ν_2) system by employing the notion of a relative filling factor (obtained by dividing throughout by the bulk ν_1).

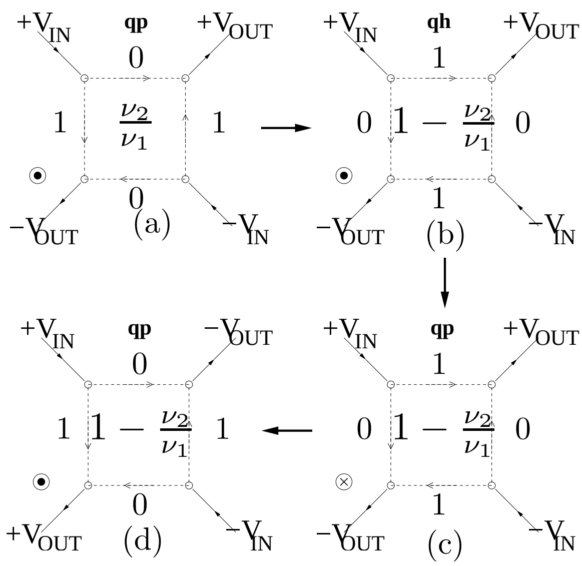

FIG. 5: The quasiparticle-quasihole (qp-qh) symmetry of the (ν_1, ν_2) constriction geometry in terms of the relative filling factor. The source-drain bias $2V_{in}$ is applied to the two incoming arms while $\pm V_{out}$ are the equilibration potentials of the two outgoing arms. See text for a detailed description of the transformations linking (a) to (d).

The QH system shown in fig.(2) has a relative filling factor of unity in the bulk and ν_2/ν_1 inside the constriction (fig.(5a)). Now, a partially filled effective lowest Landau level of qps with relative filling ν_2/ν_1 can equivalently be studied in terms of a partially filled effective Landau level of qhs with relative filling $1 - \nu_2/\nu_1$ over a completely filled effective Landau level of qps. Thus, a qp-qh conjugation transformation (fig.(5b)) maps onto a description in terms of qhs. Noting that qhs are time-reversed qps, the qh system is mapped onto one of qps with the direction of the external magnetic field reversed (fig.(5c)). A final rotation of 180° about the axis of the two outgoing arms (fig.(5d)) returns us to a circuit of qps with the same geometry. In the process, however, the relative filling of the constriction has changed from ν_2/ν_1 to $1-\nu_2/\nu_1$. Further, the transmitted and reflected outgoing arms (defined with respect to the source-drain bias) of the original circuit have been interchanged. Importantly, this qp-qh transformation also reveals the existence of a qh-tunneling process between the two reflected current arms $(l,r):\ \lambda_2\cos(\phi^l(y=0)-\phi^r(y=0))$, with the RG equation for λ_2

$$d\lambda_2/dl = (1-(1-\nu_2/\nu_1))\lambda_2 = \nu_2\lambda_2/\nu_1 \tag{7}$$

Again, with $\nu_2 < \nu_1$ and $(\nu_1,\nu_2) > 0$, λ_2 will also grow under the RG flow to strong coupling.

However, while λ_1 reduces the constriction transmission, λ_2 increases it. A comparison of the two RG equations reveals that for a critical value of $\nu_2^* = \nu_1/(1+\nu_1)$, both couplings grow equally quickly and the qp-qh symmetry of the system fixes the constriction transmission t at its weak-coupling value of $t(\nu_2^*) = g_{1in,1out}/\nu_1 = 1/(1+\nu_1)$. The critical $(\nu_2^*, t(\nu_2^*))$ values obtained for $\nu_1 = 1, 1/3$ are $(1/2, 1/2)$ and $(1/4, 3/4)$ respectively; these match exactly the critical filling factor and associated bias-independent transmission values obtained in ref.[5]. Further, for $\nu_2 < \nu_2^*$ ($\nu_2 > \nu_2^*$), the coupling λ_1 (λ_2) will grow to strong coupling faster than the coupling λ_2 (λ_1), thereby causing a dip (peak) in the constriction transmission at low energies (bias/temperature). This is in conformity with the zero-bias evolution of t with V_G discussed earlier [5, 6]. This is also reflected in $g_{1in,1out} \to 0$ (ν_1) and $g_{1in,2out} \to \nu_1$ (0) in the strong coupling limit of $\nu_2 < \nu_2^*$ ($\nu_2 > \nu_2^*$). These results are summarised in the RG phase diagram for our model (fig.(6)). The origin represents the family of weak-coupling fixed point theories at partial transmission described earlier, while the RG flows are to the familiar fixed point theories [4, 7] of complete reflection ($\nu_2 < \nu_2^*$), complete transmission ($\nu_2 > \nu_2^*$) and to a new symmetry dictated fixed point theory on the diagonal ($\nu_2 = \nu_2^*$). This underlines the novel generalisation we have achieved of the quantum impurity problem of refs.[4, 7].

V. CONCLUSIONS AND OUTLOOK

In conclusion, we have demonstrated that the puzzling results obtained in the experiments [5, 6] can be understood from a simple model for the constriction with a filling factor lower than that of the bulk. The model presents a formal realisation of the conjecture of a quantum Hall constriction circuit with a filling factor lower than that of the bulk. Ballistic transport in the presence of a finite backscattered current (and associated longitudinal resistance) is understood as a consequence of current conservation and matching of the properties of the edge excitations in the bulk and constriction regions. An important formal advancement is thus made in constructing a

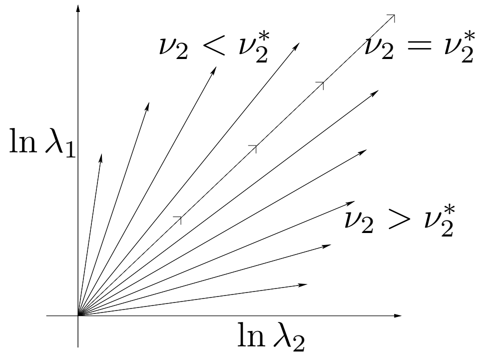

FIG. 6: The RG phase diagram for the model as a plot of the function $\ln\lambda_1/\ln\lambda_2 = \nu_1(1/\nu_2 - 1)$. All RG flows lead away from the weak-coupling unstable fixed point at the origin. Properties of the dashed critical line and regions above ($\nu_2 < \nu_2*$) and below ($\nu_2 > \nu_2^*$) are explained in the text.

chiral TLL model for the constriction circuit and a generalisation of the quantum impurity model studied in refs.[4, 7] developed.

By invoking a qp-qh symmetry of this model for a quantum Hall constriction system, we explain the experimentally observed evolution of the low-bias constriction transmission t and the critical ν_2^* in several quantum Hall systems ($\nu_1 = 1/3, 1$) as arising from the competition between a qp and a qh tunneling process in determining the conductances at strong coupling. The results for the qp-qh symmetric ν_2^* and transmission $t(\nu_2^*)$ serve as predictions to be tested at other ν_1. A consequence of note is that our model offers insight towards a study of the effects of partial constriction transmission on the shot-noise properties of quantum Hall edge states, an issue where experiments and theory have remained unreconciled [6].

This study has also revealed the existence of novel gapless edge states that lie in between gapped quantum Hall fluids with differing filling factors. In our study, such states carried the reflected current between the two edges of the Hall bar, making the scenario essentially one of intermediate transmission. While the phenomenological hydrodynamic edge state model developed in the present work, and containing these novel edge states, meets with considerable success in explaining the various puzzles presented by the experiments [5, 6], it will be even more satisfying to explore the emergence of such a model from that of a theory containing the bulk degrees of freedom as well. Such an investigation can be carried out by starting from a Chern-Simons Ginzburg-Landau type theory [16] of a quantum Hall with a spatially dependent filling factor, and will be the focus of a future work.

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