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A CONSTRICTED QUANTUM HALL SYSTEM AS A BEAM-SPLITTER: UNDERSTANDING BALLISTIC TRANSPORT ON THE EDGE

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Abstract

We study transport in a model of a quantum Hall edge system with a gate-voltage controlled constriction. A finite backscattered current at finite edge-bias is explained from a Landauer-Buttiker analysis as arising from the splitting of edge current caused by the difference in the filling fractions of the bulk (ν_1) and constriction (ν_2) quantum Hall fluid regions. We develop a hydrodynamic theory for bosonic edge modes inspired by this model. The constriction region splits the incident long-wavelength chiral edge density-wave excitations among the transmitting and reflecting edge states encircling it. These findings provide satisfactory explanations for several puzzling recent experimental results. These results are confirmed by computing various correlators and chiral linear conductances of the system. In this way, our results find excellent agreement with some of the recent puzzling experimental results for the cases of $\nu_1 = 1/3,\ 1$.

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I. INTRODUCTION

Despite being a subject of intense experimental and theoretical interest, much is yet to be learnt of the combined effects of electron correlations and impurities on the transport properties of low-dimensional strongly correlated systems. The availability of several non-perturbative theoretical methods for studying the physics of systems in one spatial dimension has, however, allowed for considerable progress to be made for such systems[1]. A physical one-dimensional system ideal for studying these issues are fractional quantum Hall edges (FQHE)[2,3]. Considerable experimental advances have been made in exploring the physics of the edge states[4] and in confirming many of the theoretical predictions made of the remarkable properties of these systems[5]. Several recent experiments have, however, pointed out the need to develop a deeper theoretical understanding of inter-edge quasi-particle tunneling phenomena in FQHE systems with gate-voltage controlled constrictions[6,7,8,9]. These experiments serve as the primary motivation for the models proposed in this work.

The phenomenological description of ballistic transport on the chiral edges relies on the following scenario. For no backscattering coupling between the two edges of opposite chirality at, say, $x \sim 0$, we have a system of two chiral 1D systems which are continuous at $x = 0$. This can be seen by consulting Figure (1) given below for the case of the fields $(\phi_{1,in}, \phi_{1out})$ and $(\phi_{2in}, \phi_{2out})$ being continuous. Upon introducing a small RG-relevant inter-edge tunnel coupling, we are left at strong backscattering coupling with a system in which the earlier edges are now discontinuous across $x = 0$; they have, in fact, now become reconnected in a different configuration, with the fields $(\phi_{1,in}, \phi_{2out})$ and $(\phi_{2in}, \phi_{1out})$ now being continuous (as can be seen in Figure (1) below). This means that, in order to describe ballistic transport intermediate between these two which is characterised by a finite backscattering of current, one must consider the possibility of the fields describing the chiral edge excitations as being discontinuous across $x = 0$. In doing so, it appears necessary to rely on ideas non-perturbative in nature. Insights on these issues were gained recently in Ref.([10]), in the form of a new model for the constriction geometry in quantum Hall system which, while being simple in essence, is clearly beyond the paradigm of the quantum point contact. We aim here to develop the ideas presented in that work, exploring more fully the consequences of such a constriction system.

As will be discussed in the next section, several recent experiments on inter-edge tunneling in FQHE systems show that it is possible to use the voltage of a split-gate constriction to tune the inter-edge transmission to values intermediate to those in the two scenarios described above. Further, they reveal a very interesting evolution of the transmission through the constriction with decreasing inter-edge bias. This will lead us to formulate a simple phenomenological model for the split-gate constriction region. We will then perform a Landauer-Buttiker analysis and compute the conductances of the model. The results of this analysis will be seen to point to some interesting conclusions for transport in the presence of a constriction. It is now well established that the

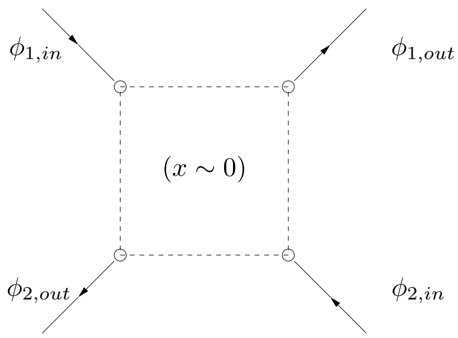

FIG. 1: A schematic diagram of the "boundary" in our system given by the dashed box around the region symbolised by $(x \sim 0)$. The four chiral fields approaching and leaving this region are shown by the arrows marked as $\phi_{1,in}$, $\phi_{1,out}$, ϕ_{2in} and $\phi_{2,out}$. The dashed horizontal and vertical lines at the junction represent quasiparticle transmission in various directions.

low-energy theory for the dynamics of the gapless long-wavelength excitations on the edges of a FQH system are described by a hydrodynamic continuum chiral TLL theory[2] of propagating density disturbances which are bosonic in nature. Adhering to the spirit of such a hydrodynamic description, we formulate a continuum model for the constricted quantum Hall edge system in section III. We complete the study in section V by computing several chiral correlators and conductances of the hydrodynamic circuit model for ballistic edge transport. We then summarise by presenting a brief comparison of the results of our model with those obtained from recent experiments in section IV. We also outline some open directions.

II. MODEL FOR A SPLIT-GATE CONSTRICTION

We now propose a simple, phenomenological model for a split-gate constriction created in a quantum Hall system. A schematic diagram of an experimental setup of a FQH bar with a gate-voltage controlled constriction is shown below in Figure (2).

As is indicated in Figure (2), the constriction is created electrostatically in a two-dimensional electron gas (2DEG) quantum Hall system at filling-fraction ν_1 by the electronegative gating of metallic split-gates. An important effect of the split-gate constriction is to bring the two counter-propagating edges of the Hall fluid in close proximity, allowing for the possibility for quasiparticles to tunnel between them. As discussed earlier, this has been a major focus in the study of the physics of FQHE systems. However, an often neglected effect of the split-gates is that the electric field induced by them reduces the 2DEG density (and hence the filling-fraction of the Hall fluid) in the narrow constriction region; the inter-particle correlations in the constriction are thus likely to increase in strength. We can, therefore, expect the filling-fraction of the FQH fluid in the constriction, ν_2, to be a function of ν_1 as well as the gate-voltage V_g, i.e., $\nu_2 \equiv \nu_2(\nu_1, V_g)$, in such

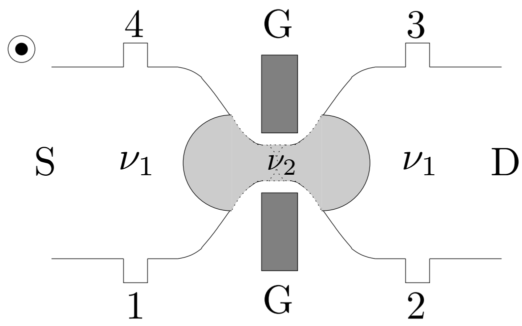

FIG. 2: A schematic diagram of a FQH bar with a gate-voltage (G) controlled split-gate constriction which lowers the electronic density in the constriction region as well as brings the top and bottom edges of the Hall fluid in close proximity, allowing for tunneling to take place between the opposite edges. S and D signify the source and drain ends of the Hall bar while the numbers 1 to 4 signify the current/voltage lead connections. The external magnetic field points out of the plane of the paper.

a way that (i) $\nu_2 = \nu_1$ for $V_g = 0$ (i.e., no constriction) and (ii) $\nu_2 < \nu_1$ for $V_g < 0$ (i.e., with a constriction). While the filling-factor ν_1 (for ν_1^{-1} being an odd integer, such that we have only single edge states) can be related to the strength of the inter-edge density-density interactions, g_{edge}, in the bulk of the FQH system[3,11,12]

$$\nu_1 = (1 + g_{edge})^{-1/2} , \tag{1}$$

no such simple relation exists, at present, for the filling-factor in the constriction, ν_2. Clearly, this will need a greater understanding of the role of the gate-voltage V_g in creating the constriction.

A. Surprises from the experiments

We now turn to a discussion of the several puzzling results observed in experiments on transport through split-gate constrictions in integer[6] and fractional[7,8] quantum Hall systems and outline the several intriguing results observed therein. Working with an experimental setup as shown in Fig.(2), a finite dc bias between the two edges coming towards the constriction V_c is imposed through the source (S) terminal while the drain (D) terminal as well as terminals 1 and 2 are kept grounded.

(i) A current I is incident on the constriction from the upper-left edge and is partially transmitted with the transmitted current finally being collected at the terminal 3. The reflected current is collected at terminal 1 and gives rise to a bias-independent longitudinal differential resistance across the constriction at large bias V_c.

(ii) The two-terminal differential conductance $G(V_c)$ is measured at temperatures as low as $250\text{m}K < eV_c$ and gives the transmission coefficient of the constriction $0 \leq t(V_c)$ $(= G(V_c)/G_0) \leq 1$ (where $G_0 = \nu_1 e^2/h$ is the Hall conductance of the bulk; $\nu_1 = 1$ in Ref.[6] and $\nu = 1/3$ in Ref.[7,8]).

At sufficiently large values of the gate-voltage V_g and large bias V_c, $t(V_c)$ is observed to saturate with $|V_c|$ at a value less than unity. Further, $t(V_c)$ is observed to dip sharply and vanish with a power-law dependence on V_c as $|V_c| \to 0$. A comparison with the theory of inter-edge Laughlin quasiparticle tunneling developed by Fendley etal.[13] suggests strongly that the constriction transmission is governed by the local filling-factor of the Hall fluid in the constriction, even though this region is likely to be small in extent. This is unexpected for the case of the bulk being in an integer quantum Hall state[6] where edge transport is understood in terms of noninteracting electron charge carriers.
(iii) A particularly intriguing observation is that of the evolution of the constriction transmission $t(V_c)$ at very low temperatures (e.g., 50mK) as the split-gate voltage V_g is varied in the limit of vanishing inter-edge bias V_c. While $t(V_c)$ shows a zero-bias minimum at sufficiently large V_g, decreasing V_g leads to a bias-independent transmission at a particular value of V_g and then to an enhanced zero-bias transmission for yet lower values of V_g. The same behaviour of the zero-bias transmission is also observed by holding the gate-voltage V_g fixed and lowering the temperature from 700mK to 50mK. For the case of $\nu_1 = 1$, the bias-independent transmission is observed at a value of $t^* = 1/2$ [6] while for $\nu_1 = 1/3$, it is observed at $t^* = 3/4$ [7,8]. A similar enhancement of the zero-bias transmission at sufficiently weak gate-voltages was also reported for the case of bulk filling-fractions $\nu_1 = 2/5$ and $3/7$[8]. The bias-independence as well as the enhancement of the transmission $t(V_c)$ is quite unexpected from the viewpoint of the theoretical framework of edge tunneling described earlier.
(iv) The constriction transmission for a bulk $\nu = 2$ system displayed two dip-to-peak evolutions, with bias independent behaviours observed at $t^* = 1/4, 1/2$ and $3/4$ [6]. This appears to indicate the independent effects of the two edge modes in the $\nu = 2$ system.
(v) Varying considerably the size and shape of the metallic gates (which form the constriction region) did not appear to affect the dip-to-peak nature of the evolution of the constriction transmission with the strength of the gate voltage [14].

Let us now consider the probable effects of a split-gate voltage constriction. Clearly, other than promoting the tunneling of quasiparticles between oppositely directed edges (due simply to enhanced wavefunction overlap due to the proximity of the edges), the more noteworthy effect is likely to be the creation of a smooth and long constriction potential, which depletes the local electronic density (and hence lowers the local filling factor) locally from its value in the bulk. Indeed, this led Roddaro and co-workers[6] to conjecture on the possibility of a small region in the neighbourhood of the constriction with a reduced filling factor ($\nu_2 < \nu_1$) as the cause of their puzzling results (see fig.(2)). This conjecture, however, remained unsubstantiated by the formal analysis of a concrete theoretical model. Thus, their explanations for the $\nu = 1$ system remained suggestive at best and no attempts at unifying the observations at both integer and fractional values of ν were made. Thus, the pressing questions that remain to be answered are as follows. What drives the gate-voltage tuned insulator-metal transition at vanishing edge-bias in the constriction system (as evidenced by the dip-to-peak evolution with decreasing strength of the gate voltage)? Can purely local interedge quasiparticle tunneling processes, which need an interplay of impurity scattering and electronic correlations [15], be the sole cause? Is there a symmetry governing the edge-bias independent response of the constriction transmission at a critical value of the constriction filling factor (as seen by tuning the gate voltage)? If the system is indeed critical at this point, what does its gapless theory look like?

At the same time, earlier theoretical efforts[16,17,18] were unable to provide any simple explanations of these experimental observations. Most notably, the scenario proposed in ref.[16] involved the complications of stripe states arising from longer range interactions. However, it failed to present any mechanism in explaining the evolution of g with V_G. The same is also true of proposals of line junctions[17] as well as the effects of inter-edge interactions on quasiparticle tunneling[18]. Thus, keeping in mind that the theory of refs.[13,15] matches the experiments in only a very restricted parameter regime, the lack of a clear theoretical understanding remained an important problem to be addressed. The creation of a model with an effort towards explaining the puzzles was, therefore, the main motivation of an earlier work [10]. In what follows, this model is first formulated and then analysed in detail.

B. Landauer-Buttiker analysis of transport

Inspired by these experimental findings, we build, in the remainder of this section, a simple phenomenological model of a FQH split-gate constriction with a reduced local filling-fraction. In this way, we aim to provide a qualitative understanding of some of the observations discussed above. Furthermore, certain elements of this simple model will then be employed as input parameters in a more sophisticated theory involving bosonic edge excitations in subsequent sections in providing explanations of some of the other, more puzzling, experimental results. The analysis of this model will be carried out in two ways. The first will involve an explicit calculation of the various Landauer-Buttiker conductances of the measurement geometry. In the second analysis, we will show how the results of the explicit calculation can be derived more simply by making two assumptions of the system at hand.

We begin by performing a Landauer-Buttiker analysis of the edge circuit [19]. This is shown below in Fig.(3). The central feature of our model is the region of lowered filling factor (ν_2) assumed to be created by the split gate constriction gates. Let us now estimate the spatial extent, L_{con}, of the ν_2 region. This can be done by noting that the transport data taken at a temperature of 50mK does not appear to show any interference effects arising from coherence across the entire constriction[6]. Thus, L_{con} can be safely assumed to be longer than the thermal length $L_T = hv/k_BT$ (where v is the edge velocity). For a typical $v = 10^3\text{ms}^{-1}$ [20] and $T = 50$mK, $L_T \sim 1\mu$m. Clearly, $L_{con}(> 1\mu\text{m}) >>$ magnetic length $l_B(\sim 100\text{Å})$, justifying our assumption of the mesoscopic nature of the ν_2 region.

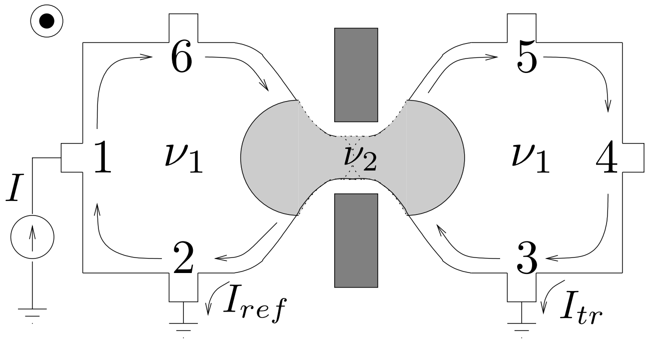

FIG. 3: A schematic diagram of a QH bar constriction circuit with the filling factors of the bulk and constriction regions being ν_1 and ν_2 respectively. The numbers 1 to 6 signify the various current and voltage terminals. The current I is sent into the circuit at terminal 1 by the current source, while the current transmitted through (I_{tr}) and the current backscattered from (T_{ref}) the constriction are received at terminals 3 and 2 respectively. The external magnetic field points out of the plane of the paper.

In a Landauer-Buttiker analysis [21], the net currents flowing in the various arms are assumed to satisfy linear relations with the applied voltages (valid for small values of the voltages), with the proportionality factors being the various transmission coefficients for the quantum system. Solving the various linear relations between the currents and voltages gives us the various conductances of the system. It is helpful to use the fact that the net current for voltage probes is zero, and that we have the freedom to set the voltage of one of the terminals to zero (as currents are related to applied voltage differences). Thus, in Fig.(3), we set $V_4 = 0$, and since terminals $4, 5$ and 6 are voltage probes, $I_4 = 0 = I_5 = I_6$. When put together with the fact that terminals 2 and 3 are grounded, i.e., $V_2 = 0 = V_3$, this allows us to exclude the current-voltage relation for terminal 4 altogether (i.e., remove one row from the transmission matrix linking the currents and voltages). Thus, we can write the current-voltage relations in matrix form as

$$\mathbf{I} = \bar{\mathbf{T}}\ \mathbf{V} \tag{2}$$

where the current and voltage column vectors are $\mathbf{I} = (I_1, I_2, I_3, I_5, I_6)$ and $\mathbf{V} = (V_1, V_2, V_3, V_5, V_6)$ respectively and the $\bar{\mathbf{T}}$ transmission matrix is given by

$$\bar{\mathbf{T}} = \begin{pmatrix} \nu_1 & -\nu_1 & 0 & 0 & 0 \\ 0 & \nu_1 & -\nu_2 & 0 & -\nu_{ref} \\ 0 & 0 & \nu_1 & -\nu_1 & 0 \\ 0 & 0 & -\nu_{ref} & \nu_1 & -\nu_2 \\ -\nu_1 & 0 & 0 & 0 & \nu_1 \end{pmatrix},$$

where ν_{ref} is the transmission coefficient for the current backscattered from the constriction. We now solve these linear relations. Measuring all voltages with respect to terminal 4 (which we have set to zero), we can see that as $I_6 = 0$, we find $V_6 = V_1$. Further, from $I_5 = 0$, we get

$$V_5 = \frac{\nu_2}{\nu_1} V_6 = \frac{\nu_2}{\nu_1} V_1 \ . \tag{3}$$

The current leaving the circuit at terminal 3 is given by $I_3 = -I_{tr} = -\nu_1 V_5$ (where I_{tr} is the current transmitted through the constriction region from terminal 6 to terminal 5). This gives us

$$I_{tr} = \nu_1 \frac{\nu_2}{\nu_1} V_1 = \nu_2 V_1 \ . \tag{4}$$

In a similar manner, we can compute the current leaving the circuit at terminal 2 (which, with terminal 3 being grounded, consists entirely of the current backscattered from the constriction) as $I_2 = -I_{ref} = -\nu_{ref} V_6$. Then, from overall current conservation in our circuit, the total injected current is given by $I_1 = \nu_1 V_1 = I_{tr} + I_{ref}$, which gives us

$$I_{ref} = (\nu_1 - \nu_2) V_1 \ . \tag{5}$$

This leads us to $\nu_{ref} = \nu_1 - \nu_2$. This expression for ν_{ref} can also be found very simply by noting that the constraint of unitarity for the transmission matrix means that the sum of the elements in every row (or every column) must add up to zero [21]. We can now also compute the conductance (in units of e^2/h) due to the current backscattered from the constriction as

$$G^{back} = \frac{I_{ref}}{V_1} = \nu_1 - \nu_2 \ . \tag{6}$$

This also gives us the "background" value of the resistance drop across the constriction as

$$R^{BG} = \frac{V_6 - V_5}{I_1 - I_5} = \frac{G^{back}}{\nu_1^2} \ . \tag{7}$$

Having carried out the Landauer-Buttiker analysis, we now show how all of the results obtained therein can be rederived through a simple analysis of the circuit which relies on essentially two assumptions on the nature of the system at hand and the conservation of current [10]. This will allow us to reflect on the simplicity and efficiency of the assumptions. Thus, let us begin by stating the assumptions made and show how they lead in a straightforward way to simple relations for several physical quantities measured in the experiment. These are:

(i) the voltage bias between the two edges of the sample (i.e., the Hall voltage for the system being in a quantum Hall state) is not affected by the local application of a gate-voltage at a constriction as long as the bulk of the system is in an incompressible quantum Hall state with filling-fraction ν_1,

(ii) the two-terminal conductance measured across the constriction is determined by the current transmitted through it, which in turn is governed by the filling-fraction of the Hall fluid in the constriction, ν_2. This needs the breakup of the current coming towards the constriction to take place at the boundary and constriction Hall fluid regions (which is sufficiently far away from the center of the constriction region).

Thus, by denoting the current injected into the system from the source terminal as I, we know that $I = G_b V_{63}$ where $G_b = \nu_1 e^2/h$ is the bulk Hall conductance and V_{63} is the edge-bias. From assumption (ii), denoting the current transmitted through the constriction as I_{tr}, it is clear that $I_{tr} = G_c V_{63}$, where $G_c = \nu_2 e^2/h$ is the two-terminal conductance measured across the constriction. Putting these two relations together using assumption (i), we obtain the transmitted current I_{tr} in terms of the incoming current I as

$$I_{tr} = \frac{G_c}{G_b} I = \frac{\nu_2}{\nu_1} I \ . \tag{8}$$

Thus, we see that our assumptions give us a very simple relation for the the splitting-ratio γ for the currents at the constriction (which is simply related to the transmission coefficient of the constriction discussed above for no inter-edge tunneling) as being $\gamma = \nu_2/\nu_1$. Now, from Kirchoff's law for current conservation, we get the current reflected at the constriction $I_{ref} = I - I_{tr} = (1 - \nu_2/\nu_1)I$. This then gives the minimum value of the backscattering conductance as

$$G^{back} = I_{ref}/V_{63} = (1 - \nu_2/\nu_1) G_b = (\nu_1 - \nu_2)\frac{e^2}{h} \ . \tag{9}$$

G_{back} is simply related to the reflection coefficient of the constriction for no inter-edge tunneling, and shows that the *effective* filling fraction governing G_{back} is $\nu_{ref} = \nu_1 - \nu_2$. Now, with the current at terminal 5, I_5, being the transmitted current I_{tr}, we get $I_5 = G_b V_5 = I_{tr} = G_c V_6$ (since $V_3 = 0$), giving $V_5 = (G_c/G_b)V_6$. We then find the "background" value of the longitudinal resistance drop across the constriction to be

$$R^{BG} = \frac{V_6 - V_5}{I_1 - I_5} = (1 - \frac{\nu_2}{\nu_1}) G_b^{-1} \tag{10}$$

which arises from the partial reflection and transmission of the incoming edge current due to the mismatch of the filling-fraction in the bulk and constriction regions. The experimentally obtained value for R^{BG} is, in fact, used by the authors of Refs.[6,7] to determine the value of the constriction filling-factor ν_2 from eq.(10). Further, we can see that G^{back} and R^{BG} are simply related by $G^{back} = G_b^2 R^{BG}$. More generally, the differential longitudinal drop across the constriction dV_{65}/dI is related to (and also experimentally determined in[7]) the differential backscattering conductance dI_{ref}/dV_{63} by the simple relation, as seen earlier

$$\frac{dI_{ref}}{dV_{63}} = G_b^2 \frac{dV_{65}}{dI} \ . \tag{11}$$

Further, we also check that the Hall conductances measured on the two sides of the constriction are determined by ν_1 alone

$$\frac{I_{tr}}{V_{53}} = G_b = \frac{I}{V_{62}} \ . \tag{12}$$

Thus, we see that by allowing for the constriction region to have a reduced filling-fraction (ν_2) than that of the bulk (ν_1) and making the two assumptions stated above, we are able (i) to find a simple expression for the splitting-ratio γ of the current incident on the constriction (or, the zeroth constriction transmission coefficient) as well as (ii) find an expression for the longitudinal resistance drop across the constriction which arises from the breakup of the current.

At the heart of these results lies the fact that a constriction region with a reduced filling-fraction necessitates the transfer of charge from the incoming edge to the opposite outgoing edge via the incompressible bulk. Put another way, it becomes imperative to consider the non-conservation of edge current in studying transport across such a constriction. This is characterised by the presence of a current reflected at the boundary of the bulk and constriction regions in the model setup above. While charge dissipation away from the edge can be modeled in terms of quasiparticle tunneling at multiple point-contact junctions[22,23], such a mechanism appears to be incompatible with the experimental finding of an edge bias-independent current reflected from the constriction region. The existence of a narrow gapless region of Hall fluid lying in between the incompressible bulk and constriction Hall fluid regions may well provide an answer: such a gapless region would act as a channel for the current reflected from the constriction region. It is, therefore, tempting to speculate on the possibility of a non-perturbative physical mechanism[24] of a chiral Tomonaga-Luttinger liquid undergoing charge dissipation along a short stretch of its length while in contact with a bath (the gapless region) as being the microscopic origin for the phenomenological model described above.

While there are ways of studying the electrostatic effects of a gate-voltage controlled constriction on the incompressible quantum Hall fluid[12,16], we have instead chosen a particularly simple and tractable path for modeling the edge structure which involves very few details pertaining to the bulk. The electrostatic calculations of Ref.[16] explore the possibility of edge reconstruction within the constriction region, i.e., long-range interactions between electrons in the quantum Hall ground state giving rise to a set of compressible and incompressible stripes at the edge[25]. In this work, however, we consider only short ranged electron correlations, which cause the formation of the chiral TLL state without any intervening stripe states[2]. Further, we neglect the possibility of the formation of line-junction nonchiral TLLs across the vacuum regions in the shadow areas of the metallic gates[17], focusing instead on the transmitted and reflected edge states arising from the nature of the Hall fluid inside the constriction. Thus, we devote our attention to short-ranged electronic correlations which cause the formation of chiral Tomonaga-Luttinger liquid (TLL) edge states (without the intervention of any stripe states [16] arising from longer range interactions).

As we will see in the following sections, such a model of a constriction in a quantum Hall sample allows for considerable progress to be made in developing a (quadratic) effective field theory for the ballistic transport of current in terms of propagating chiral edge density-wave excitations. Interesting consequences for quasiparticle tunneling will then be shown to result from the exponentiation of these quadratic fields, in particular, giving rise to the competition between two RG relevant quasiparticle tunneling operators which determine the fate of the low-bias transmission and reflection conductances through the constriction. In this way, we will show how our model is able to provide a qualitative understanding of the puzzling findings of the experiments mentioned above in a unified manner. While it appears difficult at first to formulate a continuum model describing a scenario of intermediate ballistic transmission of current through such a constriction by a quadratic bosonic field theory similar to that of Wen[2], we find that considerable progress can be made by understanding the role of matching (or boundary) conditions in such a theory. In this way, we are able to set up in the following section a very general Hamiltonian, as well as action, formalism describing transport through such a constriction system.

III. CONTINUUM THEORY FOR THE CONSTRICTION SYSTEM

In this section, we develop a continuum theory for the model of the constriction system presented above. However, for the sake of clarity and continuity, we begin by presenting the basic ingredients of Wen's continuum theory for the infinitely long chiral-Tomonaga Luttinger liquid [2].

A. Continuum theory for infinite chiral TLL

Wen's hydrodynamic formulation describes the excitations of such a system in terms of chiral bosonic density wave modes. The Hamiltonian (and the action) is quadratic in the bosonic field $\phi(x,\tau)$ (where τ is the Euclidean time) and has two parameters: the edge velocity v and the filling fraction ν. This is shown below in Fig.(4).

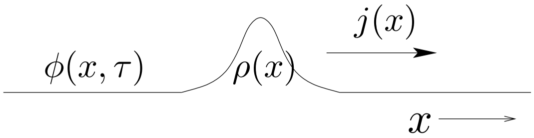

FIG. 4: A schematic diagram for the infinitely long right-moving quantum Hall edge state. The edge displacement is given by the bosonic field $\phi(x,\tau)$ while the edge density and current are given by $\rho(x,\tau) \sim \partial_x\phi$ and $j(x,\tau) \sim \partial_\tau\phi$ respectively.

The energy cost for density distortions of the edge of the quantum Hall system were shown by Wen to lead to a Hamiltonian (for, say, the right-moving edge of a Hall bar)

$$H = \frac{v}{4\pi\nu}\int_{-\infty}^{\infty} dx(\partial_x\phi_R(x,\tau))^2 \ . \tag{13}$$

The equal-time (Kac-Moody) commutation relation for the bosonic field ϕ_R is given by

$$[\phi_R(x), \partial_x\phi_R(x')] = i\pi\nu\delta(x-x') \ , \tag{14}$$

which makes $\partial_x\phi_R$ the momentum canonically conjugate to ϕ_R. The edge density distortion is given by $\rho(x) = \partial_x\phi_R(x)/(2\pi)$ and the Hamilton equation of motion gives

$$i\partial_\tau\rho_R = i[H,\rho_R] = -v\partial_x\rho_R(x,\tau)\ . \tag{15}$$

This gives us that the density $\rho_R(x,\tau) = \rho_R(x+iv\tau)$. Further, from the equation of continuity

$$i\partial_\tau\rho + \partial_x j = 0\ , \tag{16}$$

we find the current density as $j_R = -i\partial_\tau\phi_R/(2\pi)$. Fourier transforming the equation of motion gives us the expected linear dispersion relation for the edge density waves as $\omega = vk$. From the commutation relations, we obtain the Legendre transformation for the Hamiltonian $H(\phi_R)$. This leads to the Euclidean action for the chiral (right moving) TLL as

$$S_R = \frac{1}{4\pi\nu}\int_0^\beta d\tau\int_{-\infty}^{\infty} dx\partial_x\phi_R(i\partial_\tau + v\partial_x)\phi_R(x,\tau)\ . \tag{17}$$

The Hamiltonian for the left-moving edge density wave is the same as that given above by for $\phi_R \to \phi_L$, but the density $\rho_L = -\partial_x\phi_L/(2\pi)$. As the equal-time commutation relation $[\phi_L(x),\partial_x\phi_L(x')] = -i\pi\nu\delta(x-x')$, the action for the left moving edge chiral TLL has a Legendre transformation term $-i\partial_\tau\phi_L\partial_x\phi_L$.

B. Continuum theory for the constriction edge model

We now formulate a continuum theory for the constriction edge model discussed in section II along the lines of the Wen hydrodynamic description described just above. The aim will, therefore, be to develop a quadratic theory in bosonic fields in an edge model consisting of chiral, current carrying gapless edge-density wave excitations describing ballistic transport through the transmitting and reflecting edges states surrounding the constriction region. This is shown in Fig.(5) below. As discussed earlier, such a model is critically needed in order to describe the experimentally observed scenario of intermediate ballistic transmission through the constriction [6]. We take the spatial extent of the constriction region $2a$ to lie in the range $l_B << 2a << L$, where L is the total system size and l_B is the magnetic length; the external arms $(1in,\ldots,2out)$ meet the internal ones $(u,\ldots,l)$ at the four corners of the constriction. From our earlier discussions, it is also evident that ν_1 governs the properties of the four outer arms while ν_2 that of the upper and lower (transmitted) arms of the circuit at the constriction. The effective filling factor for the right and left (reflected) arms of the circuit (ν_{ref}) is treated as a parameter to be determined. We focus in this work on the effects of a changing filling fraction, keeping the edge velocity v the same everywhere.

We will now set forth the Hamiltonian formulation of the model. This approach will elucidate the importance of matching (or boundary) conditions in providing a correct and consistent description of the dynamics of the system[10]. The energy cost for chiral density-wave excitations

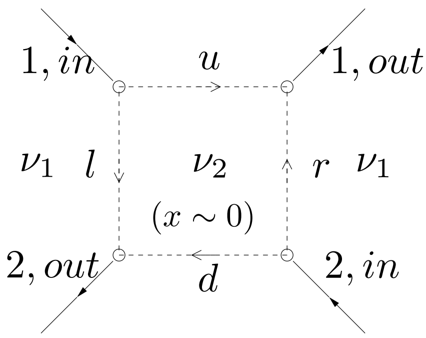

FIG. 5: A schematic diagram of the "constriction" system given by the dashed box around the region $x \sim 0$ and symbolised by the filling fraction ν_2 lower than that of the bulk, ν_1. The four chiral fields approaching and leaving this region are shown by the arrows marked as $1, in$, $1, out$, $2, in$ and $2, out$. The dashed horizontal and vertical lines at the junction represent the edge states which are transmitted (u, d) and reflected (l, r) at the constriction respectively.

that describe ballistic transport in the various arms of the circuit shown in fig.(5) is given by a Hamiltonian $H = H^{ext} + H^{int}$ where

$$\begin{aligned} H^{ext} &= \frac{\pi v}{\nu_1}[\int_{-L}^{-a} dx\ (\rho_{1in}^2 + \rho_{2out}^2) + \int_a^L dx\ (\rho_{2in}^2 + \rho_{1out}^2)]\ , \\ H^{int} &= \frac{\pi v}{\nu_2}\int_{-a}^{a} dx\ (\rho_u^2 + \rho_d^2) + \frac{\pi v}{\nu_{ref}}\int_{-a}^{a} dy\ (\rho_r^2 + \rho_l^2)\ . \end{aligned} \tag{18}$$

The densities ρ are, as usual, represented in terms of bosonic fields ϕ describing the edge displacement[2]

$$\begin{aligned} \rho_{1in} &= 1/2\pi\partial_x\phi^{1in}, \rho_{1out} = 1/2\pi\partial_x\phi^{1out} \\ \rho_{2in} &= -1/2\pi\partial_x\phi^{2in}, \rho_{2out} = -1/2\pi\partial_x\phi^{2out} \\ \rho_u &= 1/2\pi\partial_x\phi^u, \rho_d = -1/2\pi\partial_x\phi^d \\ \rho_l &= 1/2\pi\partial_y\phi^l, \rho_r = -1/2\pi\partial_y\phi^r\ . \end{aligned} \tag{19}$$

The commutation relations satisfied by these fields are familiar

$$\begin{aligned} [\phi^{1in}(x), \partial_x\phi^{1in}(x')] &= i\pi\nu_1\delta(x - x') \\ &= -[\phi^{2out}(x), \partial_x\phi^{2out}(x')], \end{aligned}$$

$$\begin{aligned} [\phi^{1out}(x), \partial_x\phi^{1out}(x')] &= i\pi\nu_1\delta(x - x') \\ &= -[\phi^{2in}(x), \partial_x\phi^{2in}(x')], \end{aligned}$$

$$\begin{aligned} [\phi^{u}(x), \partial_x\phi^{u}(x')] &= i\pi\nu_2\delta(x - x') \\ &= -[\phi^{d}(x), \partial_x\phi^{d}(x')], \end{aligned}$$

$$
\begin{aligned}
[\phi^l(y), \partial_y \phi^l(y')] &= i\pi\nu_{ref}\delta(y-y') \\
&= -[\phi^r(y), \partial_y \phi^r(y')] \ .
\end{aligned} \tag{20}
$$

Further, the Hamiltonian equations of motion derived from H again describe the ballistic transport of chiral edge density waves

$$
\begin{aligned}
(\partial_t - v\partial_x)\rho^{1in}(x,t) &= 0 = (\partial_t - v\partial_x)\rho^{1out}(x,t) \\
(\partial_t + v\partial_x)\rho^{2in}(x,t) &= 0 = (\partial_t + v\partial_x)\rho^{2out}(x,t) \\
(\partial_t - v\partial_x)\rho^{u}(x,t) &= 0 = (\partial_t + v\partial_x)\rho^{d}(x,t) \\
(\partial_t - v\partial_y)\rho^{l}(y,t) &= 0 = (\partial_t + v\partial_y)\rho^{r}(y,t) \ .
\end{aligned} \tag{21}
$$

The H given above, however, needs to be supplemented with matching conditions at the corners of the constriction for a complete description. From the form of H, it is clear that we need two matching conditions at each corner; a reasonable choice is one defined on the fields and one on their spatial derivatives. We choose, for instance, at the top-left corner

$$
\begin{aligned}
\phi^{1in}(x=-a) &= \phi^u(x=-a) + \phi^l(y=-a) \\
\partial_x\phi^{1in}(x=-a) &= \partial_x\phi^u(x=-a) + \partial_y\phi^l(y=-a)
\end{aligned} \tag{22}
$$

where x and y are the spatial coordinates describing the $(1in, u)$ and l arms respectively. Similarly, we choose the following matching conditions at the other three corners as

$$
\begin{aligned}
\phi^{1out}(x=a) &= \phi^u(x=a) + \phi^r(y=-a) \\
\partial_x\phi^{1out}(x=a) &= \partial_x\phi^u(x=a) + \partial_y\phi^r(y=-a) \\
\phi^{2in}(x=a) &= \phi^d(x=a) + \phi^r(y=a) \\
\partial_x\phi^{2in}(x=a) &= \partial_x\phi^d(x=a) + \partial_y\phi^r(y=a) \\
\phi^{2out}(x=-a) &= \phi^d(x=-a) + \phi^l(y=a) \\
\partial_x\phi^{2out}(x=-a) &= \partial_x\phi^d(x=-a) + \partial_y\phi^l(y=a) \ .
\end{aligned} \tag{23}
$$

The equation of continuity leads to the familiar form for the current operator $j^\alpha = -i\partial_\tau\phi^\alpha/(2\pi)$, where $\alpha = (1in, 1out, \ldots, l, r)$. Thus, we can easily see that current conservation at every corner arises from the matching conditions on the bosonic fields ϕ. While the transmitting chiral edge modes convey a finite current across the constriction, the reflecting chiral edge modes convey a finite "backscattered" current across the sample. In this way, we formally establish the intermediate ballistic transmission scenario as observed in the experiments. Charge density fluctuations at each corner are described by the matching conditions on $\partial_x\phi$. This matching condition is a statement of the conservation of net charge density at each corner. In this way, the two sets of matching conditions together establish the continuity of current and charge density at every corner of the junction system.

Using eqs.(22), we compute the commutation relation

$$[\phi^l, \partial_y \phi^l]_{y\to -a} = ([\phi^{1in}, \partial_x \phi^{1in}] - [\phi^u, \partial_x \phi^u])_{x\to -a} \ , \tag{24}$$

giving us $\nu_{ref} = \nu_1 - \nu_2$. The commutation relation for $\phi^r(y \to a)$ similarly yields $\nu_{ref} = \nu_1 - \nu_2$ once again. This is in conformity with our result for ν_{ref} from the Landauer-Buttiker calculation. We now demonstrate explicitly that the cases of a perfect Hall bar ($\nu_2 = \nu_1$) and two Hall bubbles separated by vacuum ($\nu_2 = 0$) can be modeled as special limiting cases of the matching conditions (eqs.(22)) given earlier. For $\nu_1 = \nu_2$, the commutation relation of the reflecting edge states vanishes, killing its dynamics. This can also be understood within a hydrodynamic prescription[2], where a vanishing effective filling factor (the amplitude of the Kac-Moody commutation relation, eq.(20)) leads to a diverging energy cost for edge charge density fluctuations; the dynamics of the bosonic field characterising such fluctuations is thus completely damped. Thus, the reflecting edge states carry no current, while the transmitting edge states perfectly transmit all incoming current into the outgoing arms on the *opposite* side of the constriction. The matching conditions eqs.(22) at the four corners are then reduced to

$$\begin{aligned}
\phi^{1,in}(x=-a) &= \phi^u(x=-a) \ , \ \phi^u(a) = \phi^{1out}(x=a), \\
\phi^{2,in}(x=a) &= \phi^d(x=a) \ , \ \phi^d(-a) = \phi^{2out}(x=-a), \\
\partial_x\phi^{1,in}(-a) &= \partial_x\phi^u(-a) \ , \ \partial_x\phi^u(a) = \partial_x\phi^{1out}(a), \\
\partial_x\phi^{2,in}(a) &= \partial_x\phi^d(a) \ , \ \partial_x\phi^d(-a) = \partial_x\phi^{2out}(-a).
\end{aligned} \tag{25}$$

These identifications of the fields and their spatial derivatives lead to the continuity conditions which underpin the hydrodynamic theory of Wen[2,15] for the case of the two infinite chiral edges (say, upper and lower) of a Hall bar (with filling factor ν_1), and eq.(20) then reproduces the well-known Kac-Moody commutation relation everywhere along the edges. This is shown in Fig.(6) below.

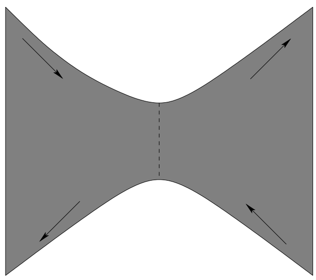

FIG. 6: A schematic diagram of the quantum Hall bar system with a "constriction" which promotes quasiparticle tunneling between two points on oppositely directed edges of the system (dashed line). The upper and lower edges are continuous everywhere and therefore have boundary conditions on the field ϕ and its spatial derivative $\partial_x\phi$ as given above in equns.(25).

Similarly, for the case of $\nu_2 = 0$, the commutation relation for the transmitting edge states vanishes, killing its dynamics: they carry no current, while the reflecting edge states perfectly convey all incoming current into the outgoing arms on the *same* side of the constriction. Thus, the matching conditions eqs.(22) at the four corners are reduced to

$$\begin{aligned}
\phi^{1,in}(x=-a) &= \phi^{l}(y=-a)\ ,\ \phi^{l}(a) = \phi^{2out}(x=-a),\\
\phi^{2,in}(x=a) &= \phi^{r}(y=a)\ ,\ \phi^{r}(-a) = \phi^{1out}(x=a),\\
\partial_x\phi^{1,in}(-a) &= \partial_y\phi^{l}(-a)\ ,\ \partial_y\phi^{l}(a) = \partial_x\phi^{2out}(-a),\\
\partial_x\phi^{2,in}(a) &= \partial_y\phi^{r}(a)\ ,\ \partial_y\phi^{r}(-a) = \partial_x\phi^{1out}(a).
\end{aligned} \tag{26}$$

Again, these identifications of the fields and their spatial derivatives lead to the continuity conditions which underpin the hydrodynamic theory of Wen[2,15] for the case of the infinite chiral edges (say, left and right) of two distinct Hall bubbles (each with filling factor ν_1) separated by vacuum, and again reproduce the familiar Kac-Moody commutation relations everywhere along the edges. This is shown in Fig.(6) below. We have, in this way, constructed a family of free

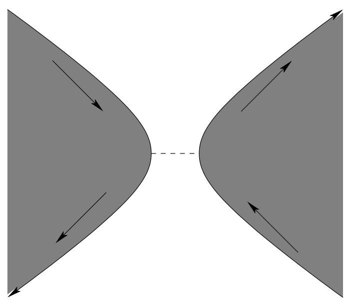

FIG. 7: A schematic diagram of a system of two quantum Hall droplets separated by vacuum and with a "constriction" which promotes electron tunneling between two points on adjacent (and oppositely directed) edges of the system (dashed line). The right and left edges are continuous everywhere and therefore have boundary conditions on the field ϕ and its spatial derivative $\partial_x\phi$ as given above in equns.(26).

theories describing ballistic transport through the constriction at intermediate transmission, with those of complete transmission and reflection representing two special cases. This represents an importance advance in generalising the quantum impurity model of refs.[13,15].

IV. CORRELATORS AND CONDUCTANCES OF THE CONSTRICTION MODEL

In this section, we present computations of various density-density correlators of the fields in the constriction model for the three cases of ballistic transport (i.e., no interedge tunneling). We then employ these correlators in a Kubo formulation to compute the chiral linear dc conductances of the system. In this way, we will confirm the physical picture developed in the last section.

In all that follows, we switch from the Euclidean time τ to Matsubara frequencies $\bar{\omega}_n$. This will also be seen to facilitate the computation of the the linear dc conductances. Thus, we begin by computing certain density-density correlators, e.g., $\langle[\partial_x\phi^{1in}_{\bar{\omega}_n}(x),\partial_x\phi^{1out}_{-\bar{\omega}_n}(x')]\rangle$, for the free theory S given earlier (i.e., S in the absence of all interedge tunneling processes)

$$\begin{aligned}\langle[\partial_x\phi^{1in}_{\bar{\omega}_n}(x),\partial_x\phi^{1out}_{-\bar{\omega}_n}(x')]\rangle &= \langle[(\partial_x\phi^u_{-a}+\partial_y\phi^l_{-a})(\partial_x\phi^u_a+\partial_y\phi^r_{-a})]\rangle e^{-|\bar{\omega}_n(x'-x-2a)/v|}\\ &= \langle[\partial_x\phi^u_{-a},\partial_x\phi^u_{-a}]\rangle e^{-|\bar{\omega}_n(x'-x)/v|}\\ &= \frac{2\pi\nu_2}{v^2}|\bar{\omega}_n|e^{-|\bar{\omega}_n(x'-x)/v|}\ ,\end{aligned}\tag{27}$$

where we have used the commutation relations for the various fields and the fact that all transport on the various edges is ballistic and described by the solutions to the chiral equations of motion for the edge density waves given earlier. Further, for the sake of notational brevity, we suppressed the $\bar{\omega}_n$ frequencies in all subscripts on the right hand side, keeping only the spatial dependence in the subscripts in the correlator expressions. The $e^{-|\bar{\omega}_n(x'-x)/v|}$ factor is the expected phase (easily seen upon performing an analytic continuation to real frequencies ω) associated with the ballistic transport between the points x and x'. As we will soon see when deriving the expressions for the linear dc conductances, this phase factor vanishes upon taking the limit of vanishing frequencies, while the filling factor dependence is crucial.

In the same way, we find the other density-density correlators between the fields outside the constriction as

$$\begin{aligned}\langle[\partial_x\phi^{1in}_{\bar{\omega}_n}(x),\partial_x\phi^{2out}_{-\bar{\omega}_n}(x')]\rangle &= -\frac{2\pi(\nu_1-\nu_2)}{v^2}|\bar{\omega}_n|e^{-|\bar{\omega}_n(x'-x)/v|},\\ \langle[\partial_x\phi^{2in}_{\bar{\omega}_n}(x),\partial_x\phi^{2out}_{-\bar{\omega}_n}(x')]\rangle &= -\frac{2\pi\nu_2}{v^2}|\bar{\omega}_n|e^{-|\bar{\omega}_n(x'-x)/v|}\ ,\\ \langle[\partial_x\phi^{2in}_{\bar{\omega}_n}(x),\partial_x\phi^{1out}_{-\bar{\omega}_n}(x')]\rangle &= \frac{2\pi(\nu_1-\nu_2)}{v^2}|\bar{\omega}_n|e^{-|\bar{\omega}_n(x'-x)/v|}.\end{aligned}\tag{28}$$

As we will now see, by using the fact that the charge and current densities for a chiral edge bosonic field ϕ are simply related to another another, these density-density correlators can be employed in computing several 2-terminal chiral linear (dc) conductances. These conductances can be derived from a linear response type Kubo formulation [15,26,27], yielding relations linking them to the correlators computed above in the form of retarded response functions (obtained upon performing an analytic continuation from Matsubara frequencies $\bar{\omega}_n$ to real frequencies ω)

$$g_{\alpha\beta}(x,x') = \lim_{\omega\to 0}(-1)^{[\tilde{\alpha}+\tilde{\beta}]}\frac{e^2v^2}{2\pi h\omega}\times\langle[\partial_x\phi^{\alpha}_{\omega}(x),\partial_x\phi^{\beta}_{-\omega}(x')]\rangle\ ,\tag{29}$$

where $(\alpha,\beta)=(1in,\ldots,2out)$ are the terminal indices, $(\tilde{\alpha},\tilde{\beta})$ are the terminal numbers (i.e., 1 and 2) associated with these terminal indices and $[\tilde{\alpha}+\tilde{\beta}]$ is a number modulo 2 such that the factor $(-1)^{[\tilde{\alpha}+\tilde{\beta}]}$ restores the direction of net current flow from source to drain as positive. From the expressions for the correlators given earlier, it is clear that in the dc limit $\omega\to 0$, the linear conductances no longer depend on the spatial coordinates of x and x'; this is a consequence of the fact that transport along the edges is ballistic and equilibriation takes place only in the reservoirs [27]. We can now simply use the various correlators computed above in calculating the various 2-terminal chiral linear conductances of the system. Thus, our calculations of the chiral linear conductances $g_{1in,1out}$ and $g_{1in,2out}$ (in units of e^2/h), representing the transmission and reflection through the constriction for the case of ballistic transport give

$$g_{1in,1out} = \nu_2 \ , \ g_{1in,2out} = \nu_1 - \nu_2 \ . \tag{30}$$

The other two conductances $g_{2in,1out}$ and $g_{2in,2out}$ can be computed in precisely the same manner. The physical picture of weak coupling ballistic transport presented earlier is, in this way, immediately confirmed from the expressions given above.

V. SUMMARY AND OUTLOOK

We begin this section by bringing together all our results in order to compare them with the findings of the experiments [6,7,8,14]. In our phenomenological model for edge state transport in the presence of a gate-voltage controlled constriction, we have chosen to model the constriction by a mesoscopic region of lowered electronic density (and hence, filling fraction ν_2) in comparison to that in the bulk (of filling fraction ν_1). The reduced transmission through the constriction at high edge-bias (i.e., partial transmission ballistic transport) is explained in terms of the transmitting edge states of the constriction, whose properties are governed solely by the constriction quantum Hall fluid. Further, the experimentally observed current backscattered from the constriction (and received in a terminal on the opposite side of the Hall bar) is explained in terms of the existence of a gapless edge state lying in between the bulk and constriction quantum Hall fluids (and whose properties are governed by both the bulk as well as the constriction fluids). This matches the experimental observations for the cases of $\nu_1 = 1/3, 1$ [6,7] and $\nu_1 = 2/5, 3/7$ [8] quite well.

To summarise, we have, in this work, introduced a model which describes intermediate conductance scenarios in the problem of tunneling in 1D chiral systems by constructing a model for a constricted region (i.e., with a lower filling fraction, ν_2, than that of the Hall fluid in the bulk, ν_1). A Landauer-Buttiker analysis of ballistic transport reveals that the constriction acts as a junction for the chiral density waves incident on it by splitting them into currents on the transmitting and reflecting arms of the constriction region. An edge state model is then formulated in terms of a hydrodynamic theory of long-wavelength, low-energy chiral density-wave excitations. Specifically, we are able to describe the dynamics of the constriction junction in terms of two pairs of edge fields, (ϕ^u, ϕ^d) and (ϕ^l, ϕ^r), whose properties are governed by the effective filling fractions ν_2 and $\nu_1 - \nu_2$ respectively. The constriction is connected to two incoming- and two outgoing-chiral modes. constriction, The conductances $g_{1in,1out}$ and $g_{1in,2out}$ computed from this model for the case of ballistic transport are found to match qualitatively the experimental findings of Refs.[6,7].

Given the success the phenomenological edge model proposed in this work meets in providing explanations for the various puzzling experimental observations, we now turn to a discussion of certain aspects of the model. First, the model relies essentially on treating the filling fractions of the quantum Hall ground state in the bulk (ν_1) and constriction(ν_2) regions as the two parameters of the model. While such a model is sensible for the case of when both (ν_1, ν_2) take values from among the special fractions representing incompressible quantum Hall ground states, how far can we trust it for the case of when the quantum Hall ground states in the bulk and constriction regions are compressible? The answer could lie in a work by Levitov, Shytov and Halperin [28] provides a generalisation of the chiral TLL edge state to the case of a compressible quantum Hall state in the bulk via a composite-fermion Chern-Simons formulation. Thus, it should be possible to derive the edge model for the constriction system proposed here for a general quantum Hall ground state by proceeding along similar lines.

This study has also revealed the existence of novel gapless edge states that lie in between gapped quantum Hall fluids with differing filling factors. In our study, such states carried the reflected current between the two edges of the Hall bar, making the scenario essentially one of intermediate transmission. While the phenomenological hydrodynamic edge state model developed in the present work, and containing these novel edge states, meets with considerable success in explaining the various puzzles presented by the experiments[6,8], it will be even more satisfying to explore the emergence of such a model from that of a theory containing the bulk degrees of freedom as well. Such an investigation can be carried out by starting from a Chern-Simons Ginzburg-Landau type theory[29] of a quantum Hall with a spatially dependent filling factor, and will be the focus of a future work. Finally, we note that it remains a challenge to be able to develop a microscopic understanding of the dependence of the constriction filling-fraction ν_2 proposed in our model on the gate-voltage V_g. Accomplishing this will allow us to make detailed quantitative comparisons with available experimental data as well as propose future experiments.

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