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**ON THE THERMAL STABILITY FOR A MODEL
REACTIVE FLOW WITH VISCOUS DISSIPATION**

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Abstract

We study the thermal stability of a reactive flow of a third - grade fluid with viscous heating and chemical reaction between two horizontal flat plates, where the top is moving with a uniform speed and the bottom plate is fixed in the presence of an imposed pressure gradient. This study is a natural continuation of earlier work on rectilinear shear flows. The governing equations are non - dimensionalized and the resulting system of equations are not coupled. An approximate explicit solution is found for the flow velocity using homotopy - perturbation technique and the range of validity is determined. After the velocity is known, the heat transport may be analyzed. It is found that the temperature solution depends on the non - Newtonian material parameter of the fluid, Λ , viscous heating parameter, Γ , and an exponent, m . Attention is focused upon the disappearance of criticality of the solution set $\{\beta, \delta, \theta_{max}\}$ for various values of Λ , Γ and m , and the numerical computations are presented graphically to show salient features of the solution set.

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Introduction

Considerable attention has been given to generalized Couette flow, in part because it is a patterned example for diverse phenomena and industrial applications. Generalized Couette flow occurs when a fluid is placed between two long parallel flat plates, where the top plate is moving in its own plane with a constant speed and the bottom plate is fixed in the presence of externally imposed pressure gradient [1]. This flow corresponds to an ideal and limiting case of the flow between concentric rotating cylinders (see, [2] and [3]). There are many practical applications in [4] and further references cited there - in.

For unidirectional Newtonian fluid with first - order reaction, Adler [5] studied criticality for steady developed reactive flow between symmetrically heated parallel walls while Zaturaska [6] investigated criticality for the reactive plane Couette flow. Studies by Shohnhiwa and Zaturaska [7] and [8] examined transitional values for each of the reactive flows in the aforementioned under physically reasonable assumptions. We should mention that the generalized Couette flow was not investigated, in part because the resulting equation for the velocity field is linear and solution to the problem is a superposition of the simple Couette flow over the plane Poiseuille flow.

The shearing motion for channel flow using other constitutive relations, has been studied by many authors ([4, 9, 10, 11, 12, 13, 14]).

In particular, the steady state equation of motion of incompressible Newtonian fluid and the third - grade fluids are both second - order ordinary differential equations. The marked difference between the case of the Navier Stokes theory and that for fluids of third - grade is that ignoring the nonlinearity in the case of the third - grade fluids, reduces to Newtonian fluids. Thus, in this special case, the third - grade fluid is a generalized Newtonian fluid. The aforementioned third - grade fluid motion has been analyzed by [9, 15, 16, 17, 18, 19, 20]. It can be easily showed that if the pressure gradient is dropped from the third - grade flow, the profile for the simple Couette flow is recovered. However, since the third - grade fluid is nonlinear in the velocity and the principle of superposition being not applicable, the generalized Couette device should be treated as an individual problem.

Third - grade fluid with the generation of chemical reaction heat inside the slab has recently been investigated. In [21] we have shown for the plane Poiseuille flow that the nonlinear effects from the velocity and temperature fields introduced decaying for the transitional values of the dimensionless central temperature. Criticality and transition for a steady reactive simple Couette flow of a viscous fluid have also been obtained in [22] using numerical method. In fact, what makes one flow situation different from another is the boundary conditions, rheological properties of the the type of flow and physicochemical parameters, such as exothermicity and reactivity.

The goal of this paper is therefore to investigate the system of equations for a reactive viscous flow of an incompressible, homogeneous fluid of third - grade in a generalized Couette

device. We provide an analytical framework using a homotopy - perturbation technique in place of perturbation method in evaluating the velocity field. We will use this velocity field in the temperature equation and then study the thermal stability. Comparative evaluation of the non - Newtonian material parameter in this study allow us to confirm that the phenomenon cannot be neglected.

The Physical and Mathematical model

The derivation presented here is due to Szeri and Rajagopal [16]. The geometry consists of two parallel plates that are infinite in the \bar{x} - direction, located in the $\bar{y} = -\bar{y}_0$ and $\bar{y} = \bar{y}_0$, respectively and wide enough in the \bar{z} - direction to have negligible side effects. The flow is driven by a combination of an externally imposed pressure gradient (maintained for example by a pump) and the motion of the upper plate at uniform speed \bar{U}_0 . The flow generated by the viscous fluid is generally referred to as Generalized Couette flow.

The equations for conservation of mass, linear momentum and energy are respectively

$$\text{div } \mathbf{v} = 0, \quad (1)$$

$$\rho \frac{d\mathbf{v}}{dt} = \text{div } \mathbf{T} + \rho \mathbf{b}, \quad (2)$$

$$\rho \frac{d\epsilon}{dt} = \mathbf{T} \cdot \mathbf{L} - \text{div } \mathbf{q} + QC_0 K_0(\bar{T}), \quad (3)$$

where \mathbf{v} is the fluid velocity, ρ is its density, \mathbf{T} is the stress tensor, \mathbf{b} is the body forces, ϵ is the specific internal energy, \mathbf{q} is the heat flux vector, Q is the heat of reaction, C_0 is the initial concentration of the reactant species, $K_0(\bar{T})$ is the reaction rate expression, \bar{T} is the temperature of the system, $\mathbf{L} = \text{grad } \mathbf{v}$ and d/ dt is the material derivative.

After Fosdick and Rajagopal [15], we assume the following constitutive relation

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3(\text{tr}\mathbf{A}_1^2)\mathbf{A}_1, \quad (4)$$

with the following restrictions

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \beta_3 \geq 0, \quad |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \quad (5)$$

where μ is the coefficient of viscosity; α_1, α_2 are the normal stress coefficients, β_3 is the material coefficient and all are functions of temperature in general. Here p denotes the pressure, $-\mathbf{I}$ is the identity tensor, tr the trace of a matrix, \mathbf{A}_1 and \mathbf{A}_2 are the first and second Rivlin - Ericksen tensors defined by

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \quad (6)$$

and

$$\mathbf{A}_2 = \frac{d}{dt}\mathbf{A}_1 + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1, \quad (7)$$

It is worth pointing out that important special cases of the model are second - grade fluids obtained by setting $\alpha_1 + \alpha_2 = \beta_3 = 0$ and fluids of grade 1 (Navier - Stokes fluids) obtained by letting $\alpha_1 = \alpha_2 = \beta_3 = 0$.

Now we look for solutions of (1) - (3) in the form

$$\mathbf{v} = (\bar{u}(\bar{y}), 0, 0), \quad \bar{T} = \bar{T}(\bar{y}), \quad (8)$$

where $\bar{u}(\bar{y})$ is the velocity in the direction of flow. For this flow the constraint of incompressibility (1) is identically satisfied. When we assume the Fourier's law of heat conduction with uniform thermal conductivity of the material, K , equations (2) and (3) reduces to

$$\frac{d}{d\bar{y}} \left(\mu \frac{d\bar{u}}{d\bar{y}} \right)^2 + 2 \frac{d}{d\bar{y}} \left(\beta_3 \left[\frac{d\bar{u}}{d\bar{y}} \right]^3 \right) = \frac{\partial p}{\partial \bar{x}}, \quad (9)$$

$$\frac{d}{d\bar{y}} \left((2\alpha_1 + \alpha_2) \left[\frac{d\bar{u}}{d\bar{y}} \right]^2 \right) = \frac{\partial p}{\partial \bar{y}}, \quad (10)$$

$$K \frac{d^2 \bar{T}}{d\bar{y}^2} + \mu \left(\frac{d\bar{u}}{d\bar{y}} \right)^2 + 2\beta_3 \left(\frac{d\bar{u}}{d\bar{y}} \right)^4 + QC_0 K(\bar{T}) = 0, \quad (11)$$

with the boundary conditions

$$\bar{u}(-\bar{y}_0) = 0, \quad \bar{u}(\bar{y}_0) = \bar{U}_0, \quad (12)$$

$$\bar{T}(-\bar{y}_0) = \bar{T}_0, \quad \bar{T}(\bar{y}_0) = \bar{T}_0 \quad (13)$$

in the absence of body forces and \bar{T}_0 is the the uniform wall temperature. We introduce the generalized pressure

$$P = p - (2\alpha_1 + \alpha_2) \left(\frac{d\bar{u}}{d\bar{y}} \right)^2. \quad (14)$$

Thus equations (9) and (10) takes the simpler form

$$\frac{d}{d\bar{y}} \left(\mu \frac{d\bar{u}}{d\bar{y}} \right)^2 + 2 \frac{d}{d\bar{y}} \left(\beta_3 \left[\frac{d\bar{u}}{d\bar{y}} \right]^3 \right) = \frac{\partial P}{\partial \bar{x}}, \quad (15)$$

with $P = P(\bar{x})$. The heat generated in the fluid is modeled through a general reaction - rate law

$$K_0(\bar{T}) = A_0 \left(\frac{k\bar{T}}{\nu h} \right)^m \exp \left(\frac{-E}{R\bar{T}} \right), \quad (16)$$

(see, Boddington et al. [23, 24] and okoya [25]) where A_0 is the rate constant, E is the activation energy, R is the universal gas constant, h is the Planck's number, k is the Boltzmann's constant, ν in the vibration frequency and m is an exponent from the pre - exponential factor. Physically, $m \in \{0.5, 0, -2\}$ correspond to Bimolecular temperature dependence, Arrhenius or zero - order reaction and sensitized temperature dependence, respectively.

The above mathematical formulations can now be written in dimensionless form as

$$\frac{d^2 u}{dy^2} \left(1 + 6\Lambda \left(\frac{du}{dy} \right)^2 \right) = C, \quad (17)$$

$$\frac{d^2\theta}{dy^2} + \Gamma \left(\frac{du}{dy} \right)^2 \left(1 + 2\Lambda \left(\frac{du}{dy} \right)^2 \right) + \delta(1 + \beta\theta)^m \exp\left(\frac{\theta}{1 + \beta\theta}\right) = 0, \quad (18)$$

subject to the boundary conditions

$$u(-1) = 0, \quad u(1) = 1, \quad (19)$$

and

$$\theta(-1) = 0, \quad \theta(1) = 0, \quad (20)$$

where

$$u = \frac{\bar{u}}{\bar{U}_0}, \quad \beta = \frac{R\bar{T}_0}{E}, \quad \Gamma = \frac{\mu\bar{U}_0^2}{K\bar{T}_0\beta}, \quad \theta = \frac{(\bar{T} - \bar{T}_0)E}{R\bar{T}_0^2},$$

$$y = \frac{\bar{y}}{\bar{y}_0}, \quad \Lambda = \frac{\beta_3\bar{U}_0^2}{\mu\bar{y}_0^2}, \quad \delta = \frac{QE A_0 \bar{y}_0^2 C_0 k^m \bar{T}_0^{m-2}}{\nu^m h^m R K} \exp\left(-\frac{E}{R\bar{T}_0}\right)$$

and we have assumed negligible reactant consumption. In essence our governing equations reduce to that which one would get for an appropriate generalized Newtonian fluid. It is worth noting that the mechanical and the thermal aspects of the flow are not coupled; that is equation (17) can be solved independently. Here Γ is the viscous heating parameter, δ is the Frank - Kamenetskii parameter, θ is the dimensionless temperature excess, u is the dimensionless velocity, β is the activation energy, Λ is the non - Newtonian material parameter of the fluid and y is the fractional distance from the central plane.

To summarize, we need to examine the system of equations (17) and (18) subject to the conditions (19) and (20) in order to investigate thermal stability. We note that for $\Lambda = m = 0$, we recover the viscous Newtonian model with chemical reaction which was investigated by Shonhiwa and Zaturka [8] while if we set $\Lambda = m = \Gamma = 0$ we obtain Newtonian model with chemical reaction (see Boddington et al. [23, 24]). In the event that only $\delta = 0$ (or $C = 0$) we are in Szeri and Rajagopal [16] (or Okoya [22]).

Velocity profile

The decoupling of velocity and temperature makes it possible to try to find and present velocity distribution in analytical form. Even where the validity of results is limited, analytical expressions are more illuminating and they often throw light on larger ranges. The simplest condition to solve equation (17) is the status in which $\Lambda = 0$, where the equation form changes from nonlinear to linear. When $\Lambda \neq 0$ it can be easily shown that the integral of the autonomous equation (17) is equivalent to

$$\dot{u} + 2\Lambda(\dot{u})^3 = Cy + A, \quad (21)$$

where the symbols over the letters in this paper signify derivatives with respect to the dimensionless space, y and A is a constant of integration. Equation (21) yields a unique positive real du/dy , as the solution of the Cardan cubic equation arising from (21) with the restriction that the discriminant $D(y, \Lambda, C, A) > 0$. The expressed analytical form of du/dy cannot be

integrated exactly as the integrand is a sum of two cubic roots. Since equation (17) do not have precise analytical solution, this nonlinear equation should be solved using other methods.

Perturbation method (P.) is one of the well - known methods (see [21] and the references cited there - in). Without giving details of the algebra involved in the perturbation technique we can write the solution of equations (17) and (19) to second order approximation as

$$u = 0.5(y^2 - 1)C + 0.5(y + 1) + \Lambda[0.5(1 - y^4)C^3 + y(1 - y^2)C^2 + 0.75(1 - y^2)C] + \Lambda^2[2(y^6 - 1)C^5 + (-4y + 6y^5 - 2y^3)C^4 + 0.5(-6y^2 + 15y^4 - 9)C^3 + 4.5y(y^2 - 1)C^2 + 9(y^2 - 1)C/8]. \quad (22)$$

The answer resulted by the perturbation method is correct for small Λ and if the small parameter is a bit greater than its range, the answer will change to a quite incorrect answer. So it is very necessary to seek another method which does not require small parameters at all.

A new method of homotopy - perturbation (H.P.) was recently developed and widely utilized in order to tackle purely nonlinear differential equations and to obtain rigorous asymptotic models which does not require small parameters at all. A suitable homotopy - perturbation structure for the equation (21) is shown as the following equation

$$H(\nu, p) = (1 - p)(L(\nu) - L(u_0)) + p(\dot{\nu} + 2\Lambda(\dot{\nu})^3 - Cy - A) = 0, \quad (23)$$

where

$$L(\nu) = \dot{\nu} + \nu, \quad L(u_0) = \dot{u}_0 + u_0, \quad \text{and } \nu(y, p) : (0, 1) \times [0, 1] \rightarrow \mathfrak{R}. \quad (24)$$

Here $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. Obviously, using equation (23) we have:

$$H(\nu, 0) = L(\nu) - L(u_0) = 0 \quad \text{and} \quad H(\nu, 1) = \dot{\nu} + 2\Lambda(\dot{\nu})^3 - Cy - A \quad (25)$$

The process of changes in p from zero to unity is that of $\nu(y, p)$ from u_0 to $u(y)$. We consider ν as following:

$$\nu = \nu_0 + p\nu_1 + p^2\nu_2 \quad (26)$$

And the best approximation for the solution is

$$u = \lim_{p \rightarrow 1} \nu = \nu_0 + \nu_1 + \nu_2. \quad (27)$$

The convergence for second - order approximation is discussed in earlier papers [26, 27, 28].

Substituting ν from equation (26) into equation (21) and expanding and collecting the coefficients of like powers of p , we have

$$p^0 : \dot{\nu}_0 + \nu_0 - \dot{u}_0 - u_0 = 0, \quad \nu(y = -1) = 0, \quad (28)$$

$$p^1 : \dot{\nu}_1 + \nu_1 - \nu_0 + 2\Lambda(\dot{\nu}_0)^3 - Cy - A = 0, \quad \nu(y = -1) = 0, \quad (29)$$

$$p^2 : \dot{\nu}_2 + \nu_2 - \nu_1 + 6\Lambda(\dot{\nu}_0)^2\nu_1, \quad \nu_2(y = -1) = 0. \quad (30)$$

To determine ν the above equations should be solved by the traditional perturbation techniques using the embedding variable as a "small parameter". We set the initial approximation of (21) as $u_0(y) = u_0 = 0$ and in view of equation (28), we obtain

$$\nu_0 = 1 - \exp(-[1 + y]). \quad (31)$$

Then we have

$$\nu_1 = 1 + A - C + Cy + (2C - \Lambda - A - 2 - y) \exp(-[1 + y]) + \Lambda \exp(-3[1 + y]), \quad (32)$$

$$\begin{aligned} \nu_2 = 1 + A - 2C + Cy + (5C - 5/2 - 2\Lambda + 3\Lambda^2/2 - 3\Lambda A - 2A + [2C - \Lambda - 2 - A]y \\ - y^2/2) \exp(-[1 + y]) + 6\Lambda C \exp(-2[1 + y]) + (4\Lambda - 6\Lambda C + 3\Lambda A + 3\Lambda^2 \\ + 3\Lambda y) \exp(-3[1 + y]) - (9/2)\Lambda^2 \exp(-5[1 + y]). \end{aligned} \quad (33)$$

So we have the following second - order approximation:

$$\begin{aligned} \nu = \nu_0(y) + \nu_1(y) + \nu_2(y) = 3 + 2A - 3C + 2Cy + (-11/2 - 3\Lambda + 7C - 3A + 3\Lambda^2/2 - 3\Lambda\Lambda \\ - \Lambda y + 2Cy - 3y - Ay - y^2/2) \exp(-1 - y) + 6\Lambda C \exp(-2 - 2y) + (-6\Lambda C + 3\Lambda^2 \\ + 3\Lambda\Lambda + 3\Lambda y + 5\Lambda) \exp(-3 - 3y) - (9/2)\Lambda^2 \exp(-5 - 5y). \end{aligned} \quad (34)$$

On using the boundary condition (19), A can be determined as

$$\begin{aligned} A = 1/(2 - [4 + 3\Lambda] \exp(-2) + 3\Lambda \exp(-6))(-2 + C + [4\Lambda - 9C + 9 - 3\Lambda^2/2] \exp(-2) \\ - 6C\Lambda \exp(-4) + [6\Lambda C - 8\Lambda - 3\Lambda^2] \exp(-6) + [9\Lambda^2/2] \exp(-10)). \end{aligned} \quad (35)$$

We now contrast the perturbation and homotopy - perturbation solutions with the numerical solution obtained by Maple Version 9.5.

Λ	C	Rel. error of u (Eq. 34)	Rel. error of u (Eq. 22)
0.1	-0.20	0.29 %	-0.07 %
0.3	-0.20	0.91 %	-1.49 %
0.41	-0.20	-0.84 %	-3.38 %
0.1	-0.25	-0.30 %	-0.10 %
0.3	-0.25	0.54 %	-2.08 %
0.41	-0.25	-0.86 %	-4.69 %
0.1	-0.30	-0.86 %	-0.15 %
0.3	-0.30	0.18 %	-0.84 %
0.41	-0.30	-0.91 %	-6.33 %

Table 1: Computations showing the relative errors of P. and H.P. solutions at the mid plane

Table 1 presents a comparison between the solutions of (22) and (34) for some selected cases. It is to be noticed that, for the solution (22) the mid plane velocity u has relative error less than 7%. As seen in Table 1, the result obtained by Homotopy - Perturbation exhibit a good agreement with relative error less than 1 %. Therefore Homotopy - Perturbation should

be preferred for $C = -0.25$ and $0 < \Lambda \leq 0.41$. However, perturbation solution (22) is useful for $\Lambda \ll 0$ and in some situations for small parameter values (e.g., [21], [29] and the references contained therein).

Having obtained the velocity distribution we now move on to discuss the heat transport in the aforementioned generalized Couette flow.

Heat - transfer analysis

In thermal explosion it is usually impossible analytically and quite a substantial task numerically, to locate all the properties at which transition from discontinuous to continuous behavior occurs. So the nonlinear equations (18) and (34) together with the boundary conditions (20) are solved numerically, for several sets of values of the parameters. The numerical procedure in Maple 9.5 used to solve the boundary value problem involves a shooting method with unknown parameter taken to be the Frank - Kamenetskii number δ . The numerical code written to solve the problem is tested by comparing our results with available literature. The numerical results are in agreement with Shonhiwa and Zaturka [8], Boddington et al. [23], [24] and Okoya [22] for $\Lambda = m = 0, \Gamma = 0$ with $m = 1/2$ and $C = 0$, respectively. Figures 1 - 3 describe the behavior of β_{tr} , δ_{tr} and $\theta_{max, tr}$ for several sets of values of the parameters Λ, Γ and m . Here θ_{max} is the value of the maximum temperature at $y = 0$.

As a final and less trivial example, we consider the case in which the non - Newtonian material parameter Λ is negligible. This special case corresponds to the generalized Couette device for Navier - Stokes with generalized chemical reaction rate term. The velocity is easily derived from equation (22) as the quadratic equation

$$u = (y^2 - 1)C/2 + (y + 1)/2. \quad (36)$$

Hence equation (18) reduces to

$$\frac{d^2\theta}{dy^2} + \Gamma(Cy + 1/2)^2 + \delta(1 + \beta\theta)^m \exp\left(\frac{\theta}{1 + \beta\theta}\right) = 0. \quad (37)$$

The same procedure, as in $\Lambda \neq 0$, may be followed to generate the solution set $\{\beta, \delta, \theta_{max}\}$ in $\Lambda = 0$. In Figures 4 and 5 the solution set $\{\beta_{tr}, \delta_{tr}, \theta_{max, tr}\}$ are plotted for various values of the viscous heating parameter Γ and the exponent m .

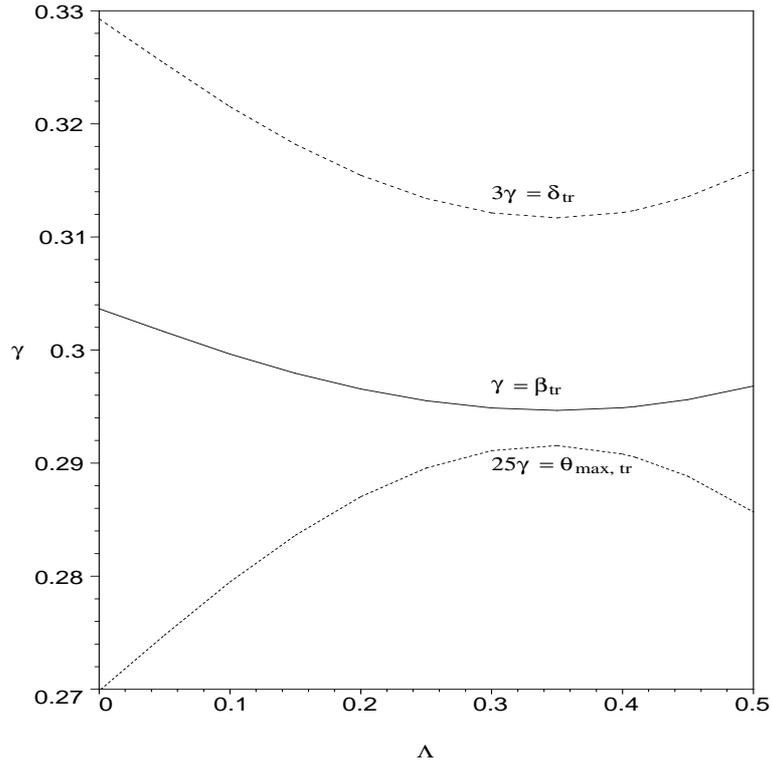


Figure 1: Plots of β_{tr} , δ_{tr} and $\theta_{max, tr}$ against Λ for $C = -0.25$ and $\Gamma = 10m = 5$.

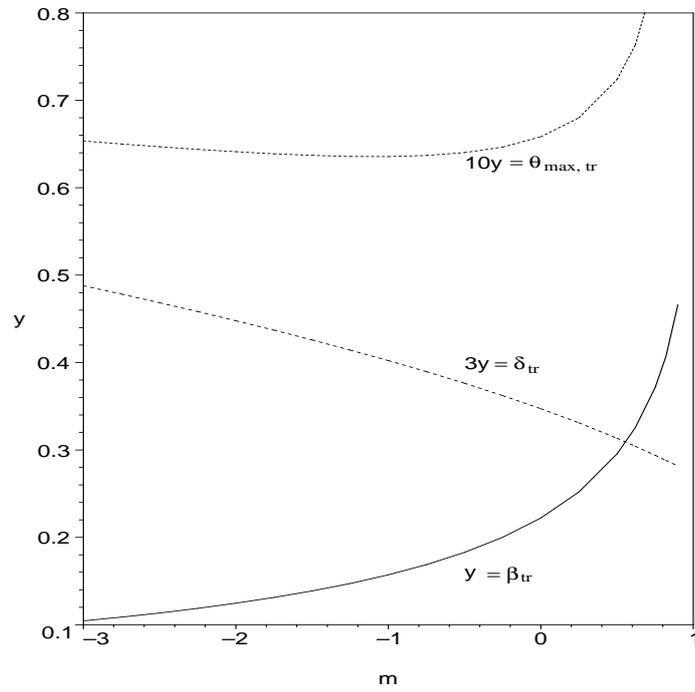


Figure 2: Variation of β_{tr} , δ_{tr} and $\theta_{max, tr}$ for different values of m with $C = -0.25$ and $\Gamma = 20\Lambda = 5$.

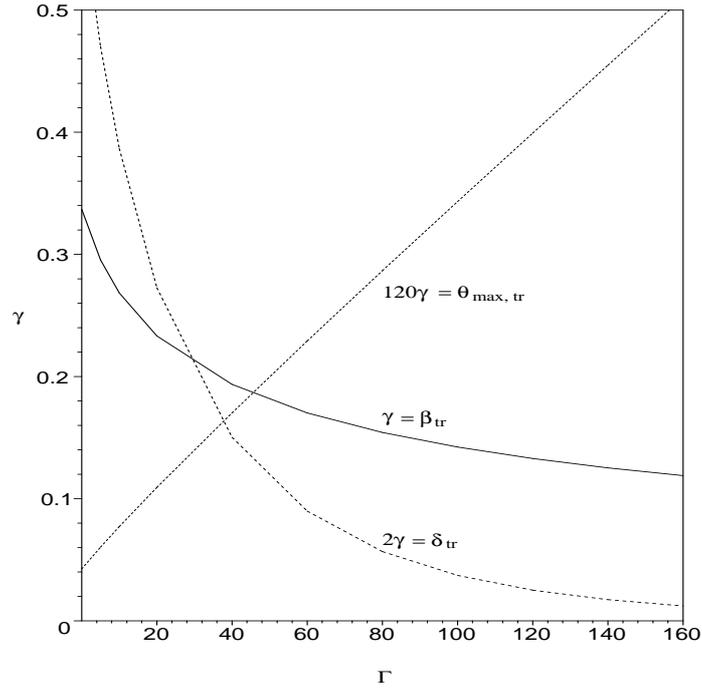


Figure 3: Profiles of β_{tr} , δ_{tr} and $\theta_{max, tr}$ as function of Γ when $C = -0.25$ and $m = 2\Lambda = 0.5$.

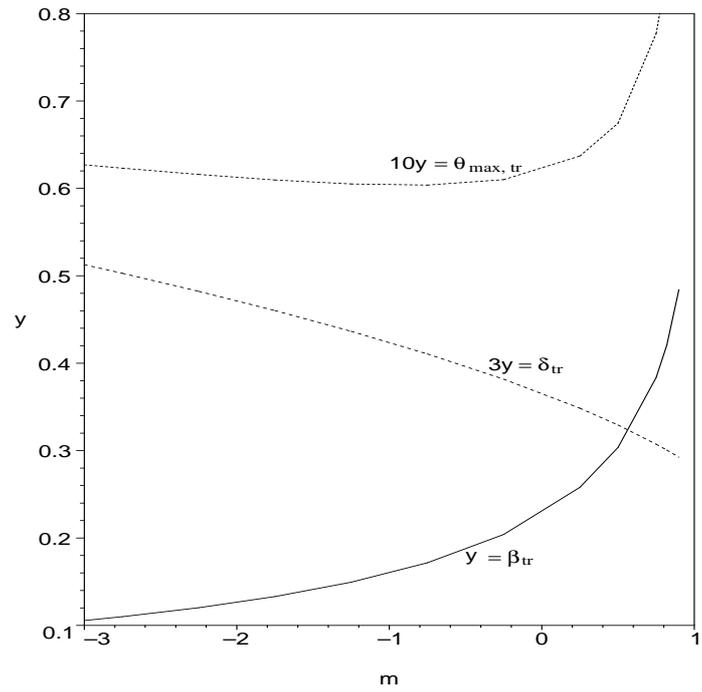


Figure 4: Graph of β_{tr} , δ_{tr} and $\theta_{max, tr}$ for different values of m with $C = -0.25$ and $\Gamma = 5$.

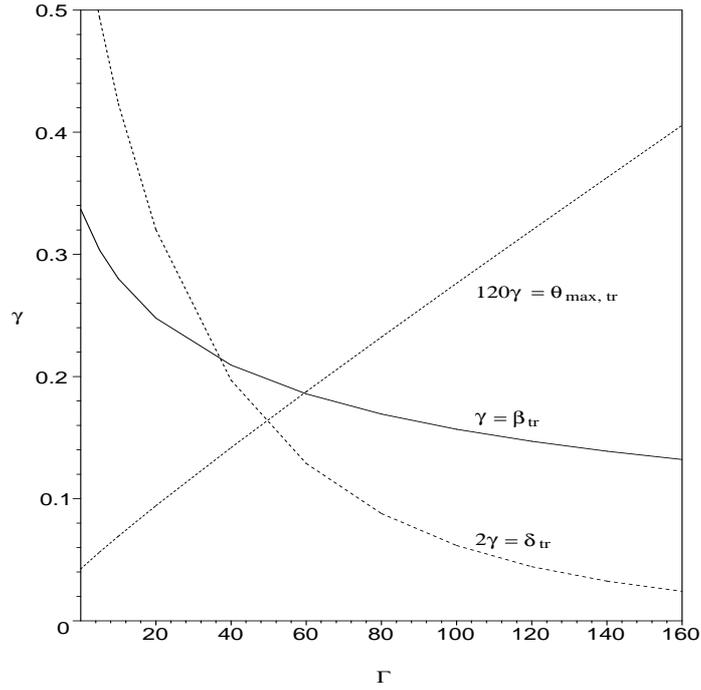


Figure 5: Curves of β_{tr} , δ_{tr} and $\theta_{max, tr}$ as function of Γ when $C = -0.25$ and $m = 0.5$.

Results and discussion

The investigation of the velocity, temperature, generalized chemical reaction rate, viscous heating and non-Newtonian effects of the flow of an incompressible fluid bounded between two long parallel flat plates, where the top plate is moving in its own plane with a constant speed and the bottom plate is fixed in the presence of externally imposed pressure gradient, has been carried out in the preceding paragraphs. The velocity is analyzed using homotopy - perturbation technique with the aid of the Maple 9.5 symbolic manipulation system and a new analytical solution is obtained. We established the range of validity for using a homotopy - perturbation technique in place of the perturbation method in evaluating the velocity. We pursue the heat problem numerically and presented transitional values over a physically reasonable composition range. The numerical computations are presented graphically and discussed in the following points:

The effects of the non-Newtonian material parameter Λ on the profiles of β_{tr} , δ_{tr} and $\theta_{max, tr}$ at $C = -0.25$ and $\Gamma = 10m = 5$ are shown in Figure 1 and it is noticed that as Λ increases, β_{tr} and δ_{tr} are convex functions of Λ while $\theta_{max, tr}$ is a concave function of Λ .

Next is to determine the effects of the exponent m on the response curves for the disappearance of criticality of parameters. When $C = -0.25$ and $\Gamma = 20\Lambda = 5$ for different values of m , the numerical results are shown in Figure 2. We observed that as m increases, β_{tr} (or δ_{tr}) increases (or decreases) while $\theta_{max, tr}$ is convex.

Variation of β_{tr} , δ_{tr} and $\theta_{max, tr}$ with an increase in viscous heating parameter Γ are shown in Figure 3 where $C = -0.25$ and $m = 2\Lambda = 0.5$. $\theta_{max, tr}$ increases with an increase in Γ

while β_{tr} and δ_{tr} decrease slowly as Γ increases.

The comparative examination of Figures 4 and 2 (or 5 and 3) underlines the effects of non-Newtonian material parameter on the Newtonian. A similar solution set $\{\beta_{tr}, \delta_{tr}, \theta_{max, tr}\}$ is observed in both cases. However, the non-Newtonian material parameter from Figures 4 and 2 lead to a significant reduction in the β_{tr} and δ_{tr} and tends to enhance the $\theta_{max, tr}$ as functions of m . It turns out that in Figures 5 and 3 the non - Newtonian material parameter enhances the β_{tr} and δ_{tr} while the $\theta_{max, tr}$ is reduced with respect to Γ . Non-Newtonian material parameter clearly appears to be an important phenomenon and cannot be neglected.

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