TA2 - Radiological Protection Systems and Regulation

Radiological Protection Optimization using Derivatives

**Clarice de Freitas Acosta Perez*
Gian Maria A. A. Sordi
Instituto de Pesquisas Energéticas e Nucleares, São Paulo, Brasil

Abstract

The aim of this paper is to provide a different approach related to the integral cost-benefit and extended cost-benefit analysis used in the decision-aiding techniques. In the ICRP publication 55 the annual protection cost is envisaged as a set of points, each of them representing an option, linked by a straight line. The detriment cost function is considered a linear function whose angular coefficient is determined by the alpha value. In this paper the uranium mine example considered in the ICRP publication 55 was used. But the potential curve was introduced both in the integral cost benefit analysis and in the extended cost-benefit analysis, which the individual dose distribution attribute is added. The result was obtained using derivatives. The detriment cost, Y, is not necessary because the alpha value is known. The Y derivative $\frac{dY}{dS}$ *is the alpha value itself and so, the attention is directed to the derivative* $-\frac{dX}{dS}$ *on the points that, along with the alpha value, present the optimum option. The results makes clear that the prevailing factor in the optimum option selection is the alpha value imputed, and those a single alpha value, as suggested now, probably as little efficiency on the optimization process. Obtaining a curve for the alpha value and using the derivative technique introduced in this paper, the analytical solution is more convincent and reliable compared to the one used now.*

1. Introduction

The simplest techniques in the quantitative decision-aiding recommended by the ICRP [1] for the radiation protection optimization are the cost-benefit analysis, which can be summarized by the following equations

$$\frac{dX}{dS} + \alpha \leq 0 \qquad (1)$$

e

$$\frac{dY}{dS} = \alpha \qquad (2)$$

designated as differential cost-benefit expressions and

$$(X+Y)_{\min} \quad or \quad (X+\alpha S)_{\min} \qquad (3)$$

designated as integral cost-benefit expression.

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The term $Y=\alpha S$ result, from the integration of differential cost benefit analysis equation (2). In the equation (1) the equal sign is valid when the function $X=f(S)$ is continuous and monotonous. In the opposite cases the minus sign is used.

Using the small mine example, Perez at al [2] showed that in a sensitivity study for the alpha value criterion we can fit the five radiation protection options to a potential function of $X=f(S)$ and by means of the function derivation, determine the alpha value for which each one of the five options remains itself optimum, in opposition to the value adopted by the ICRP, that is fitted by means of straight segments.

According to the ICRP recommendations [3] the workers individual doses were distributed in three regions:
Doses above the annual limits: unacceptable values;
Doses between the annual limits and 1/10 of these limits: tolerable values;
Doses below 1/10 of annual limits: acceptable values

The radiation protection optimization using the quantitative decision-aiding techniques is performed in the tolerable region and intends to decrease the doses until the acceptable region is reached. In this case the workers individual doses distribution becomes a relevant attribute to evaluate their doses decrease.

From this fact, we have thought, for the small mine example to introduce the potential function discussed in Perez et al [2] paper both in the integral cost-benefit analysis and in the extended cost-benefit analysis, which adds the individual doses distribution attribute, and shows the results using derivatives.

2. Cost-benefit analysis

2.1. Straight segment methodology used by ICRP

Table 1 and figure 1 present the results given by ICRP [1], showing that the optimum analytical solution, is option one.

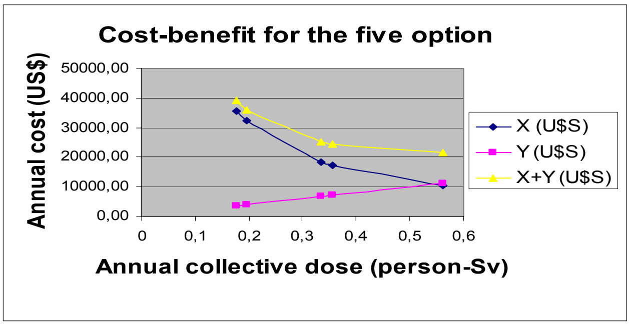
Table 1 - Options data considered in the small uranium mine example and its solution found in ICRP [1] publication

Option	S	X	Y	X +Y

	(person-Sv)	(US$)	(US$)	(US$)
1	0,561	10400,00	11220,00	21620,00
2	0,357	17200,00	7140,00	24340,00
3	0,335	18500,00	6700,00	25200,00
4	0,196	32200,00	3920,00	36120,00
5	0,178	35500,00	3560,00	39060,00

Source: ICRP publication 55, table 3

Figure 1 - Straight segment linking the options considered in the small uranium mine example



2.2. Methodology of potential to be used in this paper

In this paper we will present the same example, but considering the annual protection cost curve (X_A) fitted to a potential function, will be considered according to Perez et al [2], whose equation is

$$X_A = 5693 S^{-1,0648}$$

where,
X_A is the annual protection cost in US$
S is the annual collective dose in person-Sv

Table 2 and figure 2 present the results of the analysis considering the fitted curve.

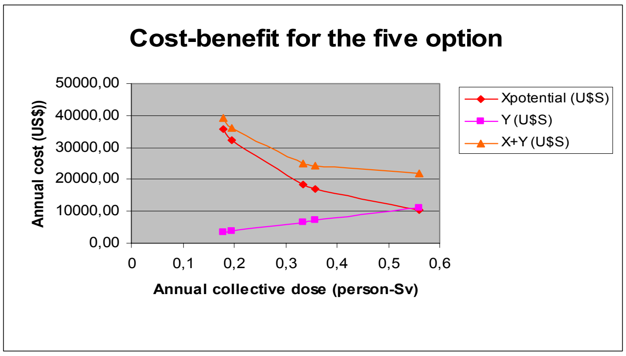
Table 2 - Option data considered in the small uranium mine example using a fitted potential curve (X_A)

Option	S (person-Sv)	X_A (US$)	Y (US$)	X_A + Y (US$)
1	0,561	10535,27	11220,00	21755,27
2	0,357	17047,48	7140,00	24187,48
3	0,335	18242,04	6700,00	24942,04
4	0,196	32281,00	3920,00	36201,00
5	0,178	35767,94	3560,00	39327,94

The optimum analytical solution is underlined.

As we supposed, as the function fitting is very good, the results are equal to those presented by the ICRP [1].

Figure 2 - Options curves considered in the small uranium mine example using a fitted potential curve (X_A)



3. Extended cost-benefit analysis

3.1. Straight segment methodology used by ICRP

The cost-benefit analysis technique is strictly limited to quantitative comparison between the protection cost and the collective dose. In order to include other relevant attributes in the analytical technique it is also possible to use the cost-benefit analysis for the extended one.

Considering that the radiation protection optimization intends to decrease the individual doses until they reach the acceptable region, the individual dose distribution becomes a relevant attribute and therefore it will be considered in this analysis.

To add this attribute the ICRP [1] has used the extended cost-benefit analysis modifying the value of unit collective dose, alpha. The basic value was substituted by other terms in the detriment cost evaluation. Different values were given to the collective dose unit depending on the individual doses values involved.

This new component of the detriment cost expressed as an additional term was introduced by the ICRP [4] and was then defined in a simplified form by the ICRP [5] itself:

$$Y = \alpha S + \sum_j \beta_j S_j$$

where:

- S_j is the collective dose originating from a per caput dose H_j delivered to the individual of the j th group, and
-$□_j$ is the additional value assigned by the decision maker to unit collective dose in the j th group.
In the small uranium mine example the ICRP adopted the following additional value for the individual doses distribution attribute:

$□ = US\$\ 20.000\ (person\text{-}Sv)^{-1}$

$□_1\ (\ < 5\ mSv) = US\$\ 0\ (person\text{-}Sv)^{-1}$

$□_2\ (5 - 15\ mSv) = US\$\ 40.000\ (person\text{-}Sv)^{-1}$

$□_3\ (15 - 50\ mSv) = US\$\ 80.000\ (person\text{-}Sv)^{-1}$

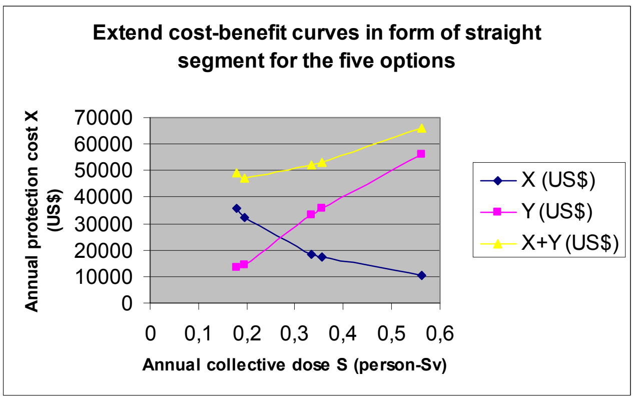
The cost of the detriment Y, is then the sum of the alpha term taking into account the collective dose and the beta terms, taking into account the individual dose distribution. The results presented by the ICRP are showed in table 3 and illustrated in figure 3.

Table 3 - Options data considered in the uranium small mine example and its analytical solution

Option	S (person-Sv)	X (US$)	Y Term □S (US$)	Y Term □□S (US$)	X +Y (US$)
1	0,561	10400,00	11200,00	44900,00	66000,00
2	0,357	17200,00	7100,00	28600,00	53000,00
3	0,335	18500,00	6700,00	26800,00	52000,00
4	0,196	32200,00	3900,00	10700,00	47000,00
5	0,178	35500,00	3600,00	9600,00	49000,00

The analytical solution is underlined, it is number four.

Figure 3 - Straight segment curves for the options considered in the small uranium mine example



3.2. Potential methodology used in this paper

We will use the same example but now considering the annual protection cost curve (X_A) fitted to a potential function, whose equation is

$$X_A = 5693 S^{-1,0648}$$

where,
X_A is the annual protection cost in US$
S is the annual collective dose in Person-Sv.

In table 4 the results of the analysis considering the fitted curve are presented.

Table 4 - Option data considering the small uranium mine example using the fitted curve (X_A)

Option	S (person-Sv)	X_A (US$)	Y Term □S (US$)	Y Term □□S (US$)	X +Y (US$)
1	0,561	10535,27	11200,00	44900,00	66635,27
2	0,357	17047,48	7100,00	28600,00	52747,48
3	0,335	18242,04	6700,00	26800,00	51742,04
4	0,196	32281,00	3900,00	10700,00	47610,00
5	0,178	35767,94	3600,00	9600,00	48967,94

As we supposed, because the function fitting is very good the results are the same as to those presented by the ICRP [1].

4. Analysis using the cost functions derivatives

Now we will present the results obtained from using the potential function in the cost-benefit and extended cost-benefit analysis for the small uranium mine example.

4.1. Cost-benefit analysis

The annual protection cost function, (X_A), is already defined

$$X_A = 5693 S^{-1,0648}$$

where,
X_A is the annual protection cost in US$
S is the annual collective dose in Person-Sv.

Those, the annual protection cost function derivative $-\frac{dX_A}{dS}$ is:

$$-\frac{dX_A}{dS} = 6061{,}9064 S^{-2{,}0648}$$

Otherwise, by equation (2), we have:

$$\frac{dY}{dS} = \alpha$$

or:

$$\frac{dY}{dS} = 20.000{,}00$$

Considering as relevant attributes the annual protection cost and the annual detriment cost, according to what was considered in the cost-benefit analysis, we see from table five that the analytical solution is still option one, underlined.

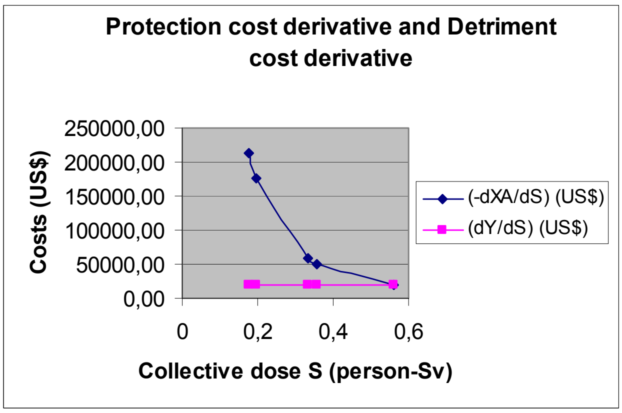
Table 5 - Option data considered the small uranium mine example using the fitted curve (X_A) and its derivative

Option	S (person-Sv)	X_A (US$)	dX_A US$/(Sv-pessoa)
1	0,561	10535,27	19996,35
2	0,357	17047,48	50846,36
3	0,335	18242,04	57982,47
4	0,196	32281,00	175371,45
5	0,178	35767,94	213964,64

The analytical solution is underlined.

Figure 4 plots the derivatives $-\frac{dX}{dS}$ *and* $\frac{dY}{dS}$

Figure 4 - Protection and Detriment costs functions derivatives



4.2. Extended cost-benefit analysis

Using the option data of the small uranium mine example showed in table 6 and by means of the costs derivatives techniques, we obtain the results presented in table 7.

Table 6 - Options data used in the small uranium mine example

Options	1	2	3	4	5
Collective Dose S (person-Sv)	0,561	0,357	0,335	0,196	0,178
	mSv	mSv	mSv	mSv	mSv
I (4 workers)	40,8	28,4	26,0	17,5	15,8
II (4 workers)	34,5	22,3	21,0	12,6	11,3
III (9 workers)	28,9	17,1	16,3	8,4	7,8

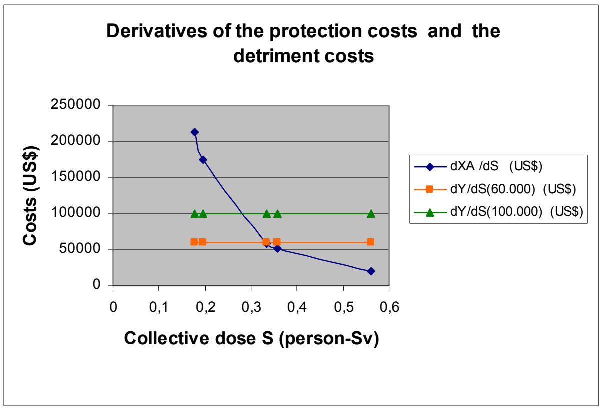
Table 7 - Options data of the uranium small mine example using the cost derivative analysis

Option	S (person-Sv)	X_A (US$)	$-\frac{dX_A}{dS}$ (US$)	$\frac{dY}{dS}$ (US$)	$\frac{dY}{dS}$ (US$)
1	0,561	10535,27	19996,35	60000,00	100000,00
2	0,357	17047,48	50846,36	60000,00	100000,00
3	0,335	18242,04	57982,47	60000,00	100000,00
4	0,196	32281,00	<u>175371,45</u>	60000,00	100000,00
5	0,178	35767,94	213964,64	60000,00	100000,00

The analytical solution is underlined.

$\frac{dY}{dS} = 20.000{,}00$ *was not used because there is not any individual dose smaller than 5 mSv. In figure 5 the derivatives* $\frac{dX}{dS}$, $\frac{dY}{dS}(60.000)$ and $\frac{dY}{dS}(100.000)$ *are shown.*

Figure 5 - Derivatives of the protection and detriment costs



Considering as a relevant attribute the annual protection cost, the annual detriment cost and the individual dose distribution according to what was considered in the extended cost-benefit analysis , we see from table 7 that the optimum analytical solution is still option 4, which is underlined.

5. Conclusions

We note that the function detriment cost, Y, is unnecessary if the alpha value is known because its derivative is the alpha value itself. Then what matters is the derivative $-\frac{dX}{dS}$ *in the options points, witch together with the alpha value provides the option optimum.*

The results show clearly that the prevailing element in the selection of the optimum option is the alpha value and that to select a single value, as suggested currently by ICRP, is not so efficient for optimization. If we use a curve for the alpha values as well as the derivative technique the analytical solution for optimization questions will certainly be more effective and reliable than using a single value as it is done today.

REFERENCES

1. *ICRP (1989) Optimization and Decision-Making in Radiological Protection, ICRP Publication 55. Ann, ICRP Vol. 20(1), Pergamonn Press. Oxford.*

2. *Perez, C.F.A. Sordi, G.M.A. Sensitivity Analysis: The Linearity Argument Between The Different Radiological Protection Options, IRPA, Paris, França, 2006*

3. *ICRP (1991) 1990 Recommendations of the International Commission of Radiological Protection, ICRP. Publication 60, Ann. ICRP Vol.21 no. 1-3, Pergamonn Press. Oxford..*

4. *ICRP (1983) Cost-Benefit Analysis in the Optimization of Radiation*

Protection, ICRP Publication 37, Pergamonn Press. Oxford.

5. *ICRP (1985) A compilation of the Major Concepts and Quantities in use by ICRP, ICRP Publication 42, Pergamonn Press. Oxford.*