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A STANDARD GRAPHITE BLOCK

by

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A graphite block was calibrated for the thermal neutron flux of the Ra-Be source using indium foils as detectors. Experimental values of the thermal neutron flux along the central vertical axis of the system were corrected for the self-shielding effect and depression of flux in the detector. The experimental values obtained were compared with the values calculated on the basis of solving the conservation neutron equation by the continuous slowing-down theory. In this theoretical calculation of the flux the Ra-Be source was divided into three resonance energy regions.

The measurement of the thermal neutron diffusion length in the standard graphite block is described. The measurements were performed in the thermal neutron region of the system. The experimental results were interpreted by the diffusion theory for point thermal neutron source in the finite system.

The thermal neutron diffusion length in graphite was calculated to be L = 50.9 ± 3.1 cm for the following graphite characteristics:

density – 1.7 g/cm^3;
boron content – 0.1 ppm;
absorption cross section – 3.7 mb.

1. Description of the block

The standard graphite block is made of nuclear pure graphite produced by the French firm "Pechiney", density 1.7 g/cm^3 and efficient cross section of thermal neutron absorption G_a = 3.7 mb. Dimensions of graphite bricks were 10 x 10 x 95 cm.

The standard graphite block was made in the form of a rectangular parallelopiped, dimensions 190 x 190 x 260 cm (Fig. 1). A movable graphite brick in aluminum shielding was placed 40 cm above the base of the block in the central horizontal axis and it was used to insert

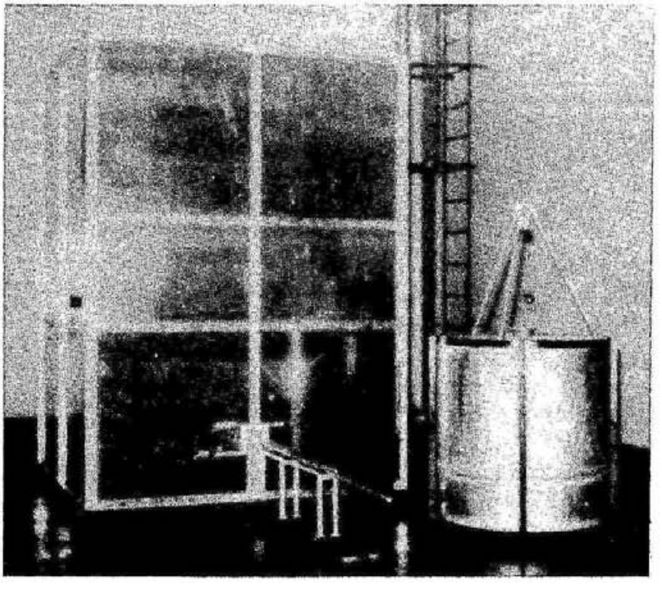


Fig. 1. - Standard graphite block

and to remove the source (Fig. 2). In the block there were 16 horizontal experimental channels, dimensions 0.7 x 3 x 190 cm (Fig. 3). The channels were mounted along the central horizontal axis of the layer rang-

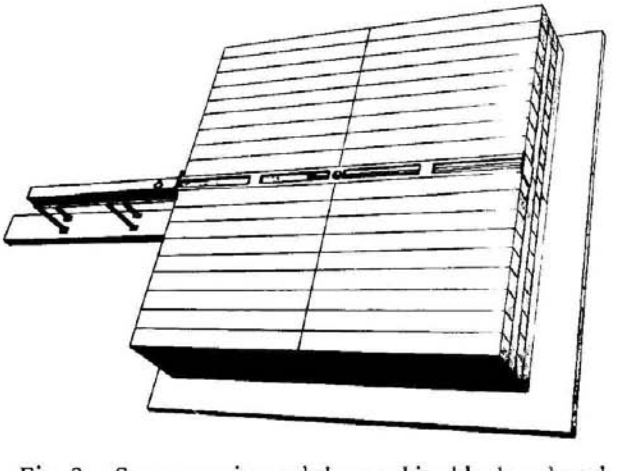


Fig. 2. - Source carrier and the graphite block pedestal

ing from 50 cm to 200 cm inclusive from the block base. The distance between the experimental channels was 10 cm. To place the foils - irradiation detectors - we used aluminum foil carriers (Fig. 4). The total

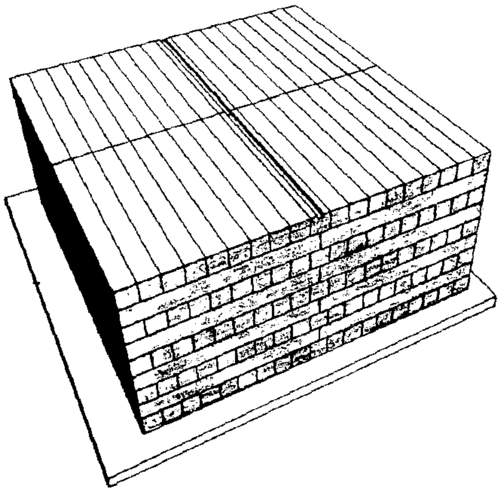


Fig. 3. - Experimental channel in the block

graphite block was covered with a cadmium tin layer 0.7 mm thick. For mechanical hardness the cadmium was reinforced with 1 mm aluminum tin and profiled aluminum armour.

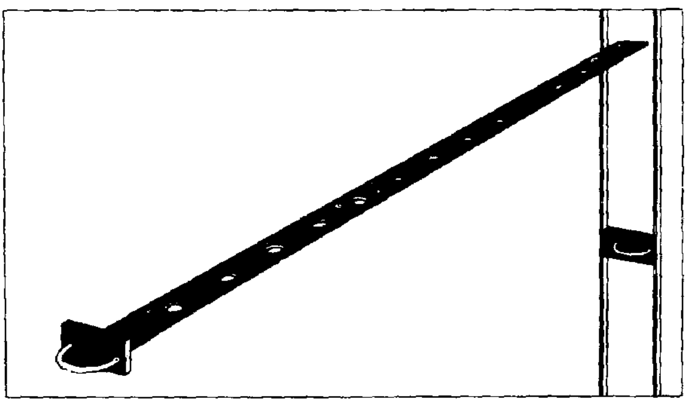


Fig. 4. - Aluminum foil holder

As a neutron source we used a polyenergy Ra-Be source 1C, intensity $1.157 \times 10^7 \pm 0.116$ n/sec.

2. Theoretical calculation of the flux distiribution of thermal neutrons

The standard graphite block was calibrated to the thermal neutron flux Ra-Be neutron source intensity 1C. The position of the source was $x' = y' = 96.95$ cm and $z' = 36.95$ cm measured according to the origin of coordinates at the corner of the column base with extrapolated dimensions. The thermal neutrons flux distribution was calculated using the solution of conservation neutron equation of the continous slowing-down diffusion model for the monoenergy point source and the finite medium. The solution for the given position of the source and the coordinate system (1) is as follows

$$\Phi\left(\frac{a_1}{2},\frac{a_2}{2},z\right) = v_0 \frac{8\tau Q}{a_1 a_2 a_3} \sum_{l,m,n=1}^{\infty} \frac{e^{-\left[\left(\frac{2l-1}{a_1}\right)^2 + \left(\frac{2m-1}{a_2}\right)^2 + \left(\frac{n}{a_3}\right)^2\right]\pi^2\theta_{th}}}{1+\pi^2 L^2\left[\left(\frac{2l-1}{a_1}\right)^2+\left(\frac{2m-1}{a_2}\right)^2+\left(\frac{n}{a_3}\right)^2\right]} \sin\frac{n\pi z'}{a_3} \sin\frac{n\pi z}{a_3} \qquad [2.1]$$

where

a_1, a_2, a_3 (cm)	–	extrapolated block dimensions
τ (sec)	–	average life of thermal neutrons
Q(n/sec)	–	Ra-Be source intensity
L (cm)	–	diffusion length of thermal neutrons
v_0 (cm/sec)	–	most probable thermal neutron velocity
$x' = y' = a_1/2$ (cm)	–	x, y source coordinates
z' (cm)	–	z - coordinate of the source position
θ_{th} (cm^2)	–	neutron age

Eq. [2.1] clearly shows the dependence of flux distribution on nuclear moderator properties, i.e. on the slowing length θ_{th} and diffusion length of thermal neutrons L. The diffusion length of the system is experimentally determined and given in the reference (2). The slowing down length θ_{th} is given for the polyenergy Ra-Be neutron source as the average slowing length over the whole output energy neutron spectrum. To avoid approximation introduced by observing the Ra-Be source as a monoenergy source of neutrons, eq. [2.1], the slowing-down length was calculated by dividing the energy spectrum of the Ra-Be neutron source in three energy regions.

2.1. *Calculation of slowing down length by introducing the Gauss source regions*

Fast neutrons of the Ra-Be source in graphite make a special distribution of slowing down density q. Taking into account the proportional activity of the cadmium covered resonant detectors and slowing down density, this spatial distribution can be measured using indium foils.

The spatial distribution of the slowing down density q given by the solution of the Fermi equation is

$$q(z) = \frac{4Q}{a_1^2\sqrt{\pi}} \frac{1}{r} \sum_{j,k=1}^{\infty} \exp\left[\left(\frac{-\pi^2 r^2}{4a_1^2}\right)(j^2+k^2)\right] e^{-(z-z')^2/r^2} \qquad [2.1.1]$$

where

Q /n/sec/ – neutron source intensity
r /cm/ – Gauss source region ($r = 2\sqrt{\theta}$)
z - z' (cm) – distance from the source along the vertical axis of the block
j,k = 1, 3, 5, ...

This equation is given for the monoenergy point source of neutrons, the origin of the coordinates given and the square block of an infinite height.

Since the Ra-Be source has a wide energy spectrum of emitted neutrons, the activity distribution of cadmium covered resonant detectors A_{Cd} cannot be represented well by a single Gaussian. The investigation of the least squares deviations has shown that the best agreement of the measured and theoretically calculated distribution is obtained using three Gaussians (3, 4)

$$A_{Cd}(z) = \sum_{i=1}^{3} B_i \cdot e^{-(z-z')^2/r_i^2} \qquad i = 1, 2 \text{ and } 3 \qquad [2.1.2]$$

where B_i and r_i are the empirical constants.

Dividing the source into three resonance energy regions, eq. [2.1.1] can be written in the form

$$q(z) = Q \sum_{i=1}^{3} f_i C_i e^{-(z-z')^2/r_i^2} \qquad [2.1.3]$$

where C_i given by

$$C_i = \frac{4}{a_i^2 \sqrt{\pi}} \frac{1}{r_i} \sum_{j,k=1}^{\infty} \exp / \left(\frac{-\pi^2 r_i^2}{.4a_i^2}\right) (j^2 + k^2) /$$

The magnitudes f_i are fractions of the total number of neutrons in resonant regions.

The Gauss regions r_i as well as the coefficients B_i are calculated by forming equations for the least square discrepancies of the calculated and measured activities of cadmium coated detectors.

Since the obtained values r_i regions give slowing-down length to indium resonance, the correction was made to thermal neutron energy (3).

Introducing the calculated ranges of r_i and fractions of the total neutron number f_i in eq. [2.1], the thermal neutron flux by the continuous slowing-down theory is given by

$$\phi\left(\frac{a_1}{2} \frac{a_2}{2}, z\right) = \sum_{i=1}^{3} f_i \phi_i(z) \qquad [2.1.4]$$

where $\phi_i(z)$ are the flux components in the resonance energy source regions. It is significant that eq. [2.1.4] makes it possible to calculate the thermal neutron flux using the components obtained from the division of the output polyenergy spectrum into three energy regions.

3. Experimental values of the thermal neutron flux in the standard graphite block

The measurement of the absolute thermal neutron flux was made via the activity of irradiated bare or cadmium covered indium foils. The foils were irradiated along the central vertical axis of the block at distances of $51.95 \leqslant z \leqslant 201.95$ cm in the horizontal experimental channels. The foil dimensions are $\phi = 17$ mm and 0.194 mm thick. They were calibrated by the analytical balance with accuracy, 0.2 mg. The absolute measurement of the foil activity was performed with the 4π-method with proportional counters (5).

In order to determine cadmium ratio R_{Cd} the activity of bare and cadmium coated indium foils were measured relatively using Geiger-Müller counters. To determine R_{Cd} the Ra-Be source, intensity 1,5 Ci was used.

3.1. *Results*

Applying the procedure described in section (2.1), with the constants given in Table I eq. [2.1.2] is as follows:

Table I

Values of coefficients r_i f_i B_i C_i

i	r	f	B	C
1	27	0.3473	2663.10	$0.8921 \cdot 10^{-5}$
2	40.5	0.5763	1335.89	$0.2697 \cdot 10^{-5}$
3	62	0.0764	49.50	$0.0753 \cdot 10^{-5}$

Table II gives the measured and calculated activity values of cadmium cavered detectors.

Table III and graph 1 give the values measured for the thermal neutron flux and the values calculated theoretically using eq. [2.1.4]. With regard to the fact that detectors perturb the flux, the values measured for the thermal neutron flux were corrected for the self-shielding effect and flux depression. Experimentally determined flux values

Table II

Experimental and calculated values of A_{Cd}

Distance form the source (cm)	A_{Cd} exp. imp/min)	A_{Cd} calculated (imp/min)	Deviation (%)
15	3191	3167.2	0.7
25	2094	2085.6	0.5
35	1169	1165.1	0.3
45	549	584.5	-6.3
55	252	276.8	-9.4
65	110	126.2	-14.8
75	52	56.9	-7.6
85	24	24.0	0
95	10	4.4	55.8

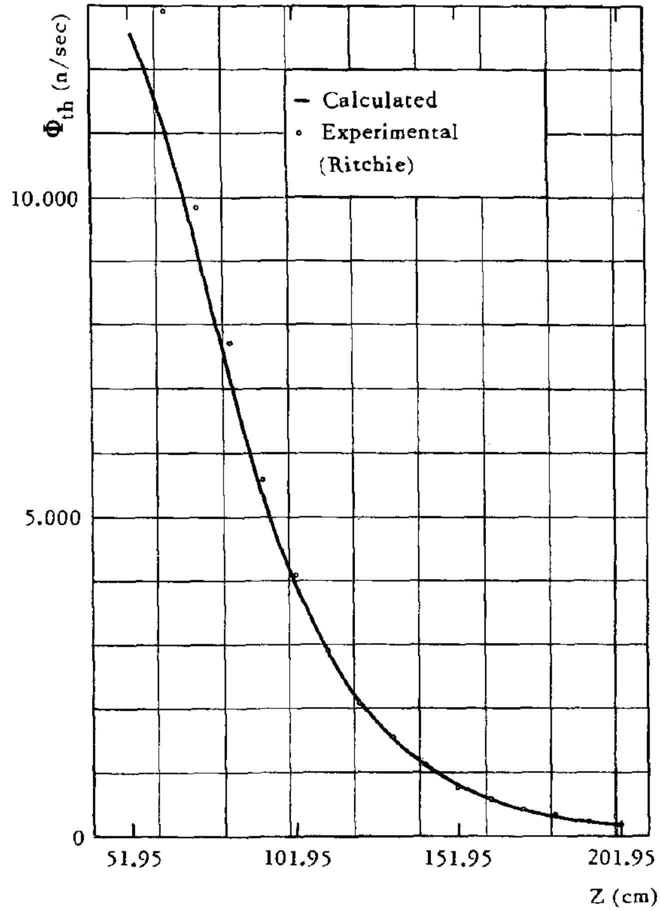
Table III

Experimental and calculated thermal flux values in the standard graphite block

z (cm)	Experimental*				Calculated
	Bothe	Bothe-Tittle	Skyrme	Ritchie-Eldridge	
51.95	13903	13912	14817	14607 ± 1445	12558
61.95	12272	12280	13078	12894 ± 1258	11083
71.95	9356	9362	9971	9830 ± 1506	9105
81.95	7349	7354	7832	7721 ± 807	7083
91.95	5316	5320	5666	5585 ± 556	5326
101.95	3899	3902	4155	4097 ± 446	3897
111.95	2764	2765	2945	2904 ± 371	2860
121.95	1970	1971	2100	2070 ± 244	2075
131.95	1451	1452	1547	1525 ± 177	1506
141.95	1049	1050	1118	1102 ± 168	1100
151.95	748	748	797	786 ± 95	798
161.95	543	543	579	571 ± 72	572
171.95	397	397	423	417 ± 75	417
181.95	314	314	334	329 ± 34	320
191.95	207	207	221	218 ± 23	223
201.95	153	153	163	160 ± 19	161

* For the perturbation factors calculated according to Bothe, Bothe-Tittle, Skyrme and Ritchie-Eldridge

given in Table III are given for the calculation of the perturbation factors using Bothe's, Title's, Skyrme's and Ritchie's theories (6, 7).



Graph 1

Comparing the values shown in Table III it appears that disagreement between the calculated and experimental flux is greater near the source. The disagreement was caused by the conditions of the perturbation factor application. In other words the theories for calculation of these factors require a monoenergy neutron flux, which is not the case near the Ra-Be source. In addition, it should be mentioned that the calculation of the thermal neutron flux was given using the continuous slowing-down theory which is also valid in the case of mono-energy fast neutrons. It is also characteristic that the calculated values of the thermal neutron flux considerably vary with the particular theory for calculation of self-shielding and flux depression applies. These variations in fact result from the different degrees of aproximation used in these theories for calculating this effect.

It should be pointed out that the errors of measured flux values result mainly from the indefinite self-absorption factor of γ-radiation in the detector and the indefiniteness of the effective cross section of indium.

4. Theoretical account of experimental determination of diffusion length

In order experimentally determine the diffusion length of thermal neutrons, the dimensions of the graphite block were calculated by determining the optimum values in relation to the decreasing effect of finite dimensions of the system and the method of measuring diffusion length. Namely, the column height mentioned is determined by the polyenergy Ra-Be neutron source and the determination method of diffusion length. The required measurements were made in the region of total thermalization of neutrons along the central vertical axis of the system.

The flux distribution of thermal neutrons in the diffusion model for point thermal neutron source and the system given in the form of parallelepiped is expressed by: (1)

$$\phi\left(\frac{a_1}{2},\frac{a_2}{2},z\right) = v_o \frac{4\tau Q}{a_1 a_2 L} \sum_{l,m=1}^{\infty} \frac{1}{\omega_{l,m}} \frac{\mathrm{sh}\,\omega_{l,m}\frac{z'}{L}}{\mathrm{sh}\,\omega_{l,m}\frac{a_3}{L}} \mathrm{sh}\,\omega_{l,m}\frac{a_3-z}{L} \qquad [4.1]$$

for $z > z'$ and $l,m = 1, 3, 5, \ldots$

where

a_1, a_2, a_3 (cm) – extrapolated system dimensions

τ (sec) – average thermal neutron life

Q (n/sec) – Ra-Be source intensity

L (cm)	– diffusion length of thermal neutrons
v_o (cm/sec)	– most probable thermal neutron velocity
z' (cm)	– z coordinate of the source

$\omega_{l,m}$ is given by

$$\omega_{l,m} = \sqrt{1 + \beta^2_{l,m} L^2} \text{ and } \beta^2_{l,m} = \pi^2 \left(\frac{l^2}{a_1^2} + \frac{m^2}{a_2^2}\right) \qquad [4.2]$$

The equation given for the thermal neutron flux distribution along the central vertical axis system, may be written:

$$C_e \cdot C_h \cdot \phi\left(\frac{a_1}{2}, \frac{a_2}{2}, z\right) = \frac{A_{11}}{2} e^{\omega_{11}\frac{a_3 - z}{L}} \qquad [4.3]$$

where C_e the end correction factor and C_h the correction for higher harmonics given by:

$$C_e = \frac{1}{1 - e^{-2\frac{\omega_{11}}{L}(a_3 - z)}}$$

$$C_h = \frac{1}{1 + \dfrac{\sum\limits_{l,m \neq 1,1}^{\infty} A_{l,m}\, sh\omega_{l,m} \dfrac{a_3 - z}{L}}{A_{11}\, sh\omega_{11} \dfrac{a_3 - z}{L}}}$$

By introducing logarithms in eq. [4.3] we obtain the equation of the straight line whose slope is:

$$\gamma = \frac{\omega_{11}}{L}$$

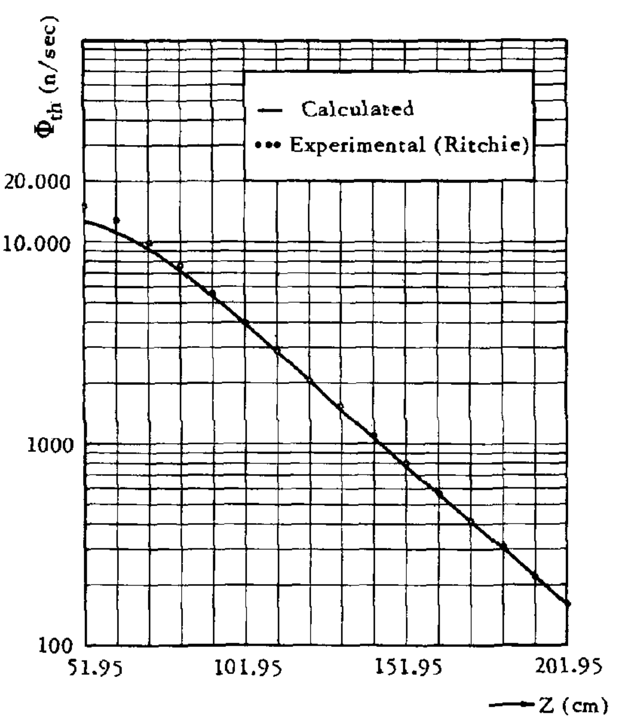
Using eq. [4.2] and the slope γ we obtain the equation for parameter L:

$$L^2 = \frac{1}{\gamma^2 - \left(\frac{\pi}{a_1}\right)^2 - \left(\frac{\pi}{a_2}\right)^2} \qquad [4.4]$$

5. Experimental determination of thermal neutron diffusion length

On the basis of the proportionality of the flux and activity, eq. [4.4] may be also used in determining the slope through the indium foils activity distribution. The indium foils were exposed in the thermal neutron region of the standard graphite block.

The diffusion length was determined from the slope of γ the distribution activities of foils Graph 2 by subtracting the higher harmonics contribution and end corrections in the following manner:



Graph 2

The preliminary value of diffusion length was obtained from the slope of the straight line using the least squares method and neglecting end corrections and the higher harmonics contribution. By dividing the experimental values of the indium foils activity by the corrections obtained for Ch and Ce and the repeated determination of slope γ from the diagram of the logarithms of In foils activity values as functions of z, a new value of the diffusion length was obtained. This procedure was repeated until the difference of calculated values became negligible as compared to the accuracy of the results.

The diffusion length obtained in this way is a diffusion length of the standard graphite block system. Owing to the presence of holes in the block resulting from the experimental channels and foil carriers the corresponding correction be made to obtain the diffusion length of the graphite itself. This correction, because of the presence of aluminum and nitrogen, was made by homogenizing graphite, aluminum and nitrogen and by calculating the diffusion of the systems L_{system} and graphite L_{gr} using (8):

$$L^2 = (3N\sigma_a N\sigma_{tr})^{-1} \tag{5.1}$$

The effective cross sections are given by:

$$\sigma_{tr} = \sigma_s(1 - \overline{\cos\theta})$$

$$N\sigma_a = \frac{\sum_{i=1}^{3} N_i \sigma_{ai} V_i}{V_{tot}} \qquad N\sigma_{tr} = \frac{\sum_{i=1}^{3} N_i \sigma_s (1 - \overline{\cos_i\theta}) V_i}{V_{tot}}$$

where the indexes i = 1, 2, and 3 refer to graphite, aluminum and nitrogen respectively, and V_i and N_i are the corresponding volumes and atomic numbers per cm^3. The following correction value was obtained for the holes

$$L_{system}/L_{graphite} = 0.986$$

6. Results

Several measurements were made of the activity distribution of irradiated In foils. The statistical error of measurement ranged up to 1%.

Table IV gives the foil activities and corrections of higher harmonics and end contributions.

Table IV

End and harmonics corrections

z (cm)	A_{In} (imp/min)	Ch	Ce
151.95	835	0.9277	1.00019
161.95	570	0.9443	1.00036
171.95	436	0.9564	1.00068
181.95	308	0.9658	1.00131
191.95	234	0.9731	1.00253
201.95	170	0.9788	1.00493

Taking into account the corrections due to the presence of holes in the system, the measured diffusion length as results from eq. [5.1], we obtain

$$L = 50.9 \pm 3.1 \text{ cm}.$$

It is essential to note that in determining diffusion length the standard method based on the exponential fall of the thermal neutron flux along higher dimensions of the finite system was used. Since the measurements are made in the region of total thermalization of neutrons along the central vertical axis of the system and far from the source, it was justified to use diffusion theory in interpreting experimental results.

The error of relaxation length was determined by the least squares method for calculating the error of the straight line slope. Student's rule was used for the correction of the error according to the number of experimental points and limits of the security interval chosen for the measured values. All the errors given in this work are within the 95% security range. The statistical error of measured values of the thermal neutron flux is 1%. This value of the statistical error was obtained by many repeated measurements in the region of low thermal flux.

The disadvantages of the method applied for measuring diffusion length consist in the effect of the final dimensions of the system on the accuracy of measurement, i.e. in finding out the differences of two close small values in the denominator of eq. [4.4]. Thus, the diffusion length error increases as a value calculated through the experimentally determined relaxation length: the relaxation length for the experiment described was determined with a 3% error whereas the diffusion length error was about 6%.

Résumé

BLOC-STANDARD EN GRAPHITE

Un bloc-standard en graphite a été calibré par les mesures du flux des neutrons thermiques effectuées aux sources Ra-Be, utilisant des feuilles d'indium comme détecteur. Les valeurs du flux des neutrons thermiques le long de l'axe central vertical, déterminées par des expériences, ont été corrigées pour l'effet de l'auto-protection et de la dépression du flux dans le détecteur. Les valeurs expérimentales ont été comparées à celles calculées à la base de la solution de l'équation de conservation des neutrons selon la théorie du ralentissement continu. C'est d'après ce calcul théorique du flux que les sources Ra-Be ont ete divisées en trois régions de résonance énergétique.

Les mesures de la longueur de diffusion des neutrons thermiques, effectuées dans le bloc-standard en graphite ont été décrites. C'est dans la région des neutrons thermiques de ce système que les mesures nécessaires ont été faites. Les résultats expérimentaux ont été interprétés d'après la théorie de diffusion appliqués à une source ponctuelle des neutrons thermiques dans un système fini.

La longueur de diffusion des neutrons thermiques dans le graphite est L = 50,9 ± 3,1 cm pour les caractéristiques suivantes: densité - 1,7 g/cm³, teneur en bore - 0,1 ppm, section d'absorption - 3,7 mb.

Резюме

СТАНДАРТНЫЙ ГРАФИТОВЫЙ БЛОК

Произведена калибровка графитового стандартного блока на тепловой нейтронный поток Ra-Be источника с помощью индиумовых фольг в качестве детектора. Приложены экспериментально определенные значения теплового потока нейтронов центральной вер-

тикальной оси системы, поправленной на эффект самозащиты и понижения потока в детекторе. Экспериментально определенные значения теплового нейтронного потока сопоставлены с вычисленными значениями полученными посредством решения уравнения сохранения нейтронов по непрерывной теории замедления. В этом теоретическом подсчете потока произведено разделение Ra-Be источника на три энергетические зоны.

Представлено измерение диффузионной длины тепловых нейтронов в графитовом стандартном блоке. Необходимые измерения проведены в тепловой нейтронной части системы. Выяснение экспериментальных данных основано на диффузионной теории для пунктированного источника тепловых нейтронов в окончательной системе.

Измеренная диффузионная длина тепловых нейтронов в графите составляет: $L = 50{,}9 \pm 3{,}1$ см для следующих графитовых характеристик: частота - 1,7 гр/см^3, содержание бора - 0,1 ppм, сечение поглощения - 3,7 mb.

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