INITIAL CONDITIONS CORRESPONDING TO OPTIMAL ION ACCELERATION IN THE VINCY CYCLOTRON

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Abstract – *The quality of a beam in a cyclotron depends a lot on the choice of initial conditions for acceleration. The criteria defining optimal acceleration as well as the choice of the corresponding initial conditions have been outlined. The results of beam dynamics simulations with optimal and non-optimal initial conditions are compared.*

1. INTRODUCTION

The VINCY Cyclotron [1] is a multipurpose machine whose function is to accelerate light ions as well as heavy ions with specific charges ranging from $\eta = 0.15$ to $\eta = 1$. The cyclotron magnet has four straight sectors per pole, a pole diameter of 2 m, a sector-to-sector gap of 36 mm, and a valley-to-valley gap of 190 mm. The maximum magnetic induction in the machine center is 1.97 T.

Ion beams are accelerated in the electric field produced by the two radio-frequency (RF) resonators with two accelerating gaps each. An ion gains maximal energy in each of the four accelerating gaps under two conditions. Firstly, RF frequency must be a multiple of an ion gyration frequency; a multiplier is called harmonic number, h. Secondly, the ion motion and RF voltage have to be synchronized, i.e., the RF phase must be properly chosen.

The test ion beams of the VINCY Cyclotron are 65 MeV H^-, 30 MeV per nucleon H_2^+, 7 MeV per nucleon $^4He^+$, and 3 MeV per nucleon $^{40}Ar^{6+}$ beams. These four ion beams have been chosen to check the four acceleration regimes, employing acceleration with harmonic numbers 1, 2, 3, and 4, respectively.

The quality of a beam extracted from a cyclotron depends greatly on the acceleration process a beam undergoes within the acceleration region. A quality of the accelerated beam prior to extraction is measured firstly by the energy spread within a beam bunch and the intensity of the extracted beam. Both of these are closely related to the azimuthal spread of the beam bunch. Secondly, transversal emittances are also beam quality indicators and thus should be investigated as well.

The acceleration region of the VINCY Cyclotron is taken to be the area between the radii of 12 cm and 84 cm. Therefore, theoretically, the radial position of 12 cm is a single initial acceleration parameter that is preset. Thus, the choice of other initial acceleration parameters determines the acceleration process and the resulting beam quality. The criteria for selecting the best out of a pool of different accelerated test ion trajectories with different initial parameters are outlined. According to these criteria a set of initial conditions corresponding to optimal acceleration of H^- ion beam is chosen. The results of the beam dynamics simulation with these initial conditions are compared to the results of the simulation corresponding to non-optimal initial conditions. We have discussed the results from the standpoint of validation of criteria that have been used.

2. CRITERIA FOR OPTIMAL ACCELERATION

Magnetic fields in an isochronous cyclotron, such as the VINCY Cyclotron, are designed in such a manner to account for radial increase of the relativistic mass of the particles and provide constant gyration frequencies [2], [3]. Given the desired extraction energy of a test ion, the one-to-one correspondence between the kinetic energy and related stationary equilibrium orbit (SEO) is fully determined. The dependence of the SEO mean, minimal and maximal radius on the kinetic energy is shown in Fig.1.

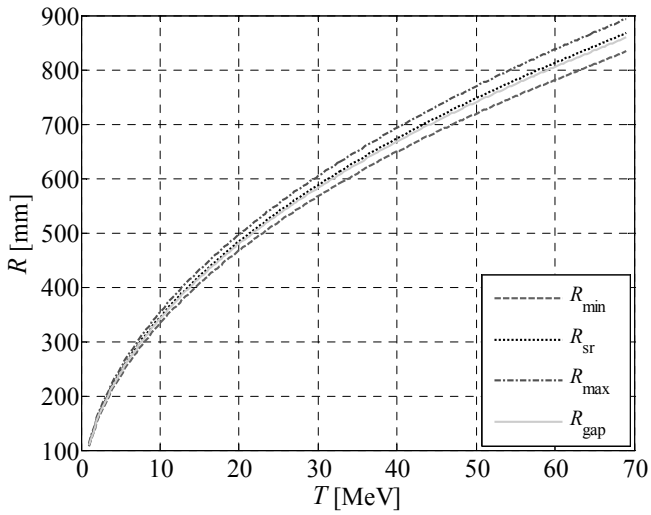

Fig.1. *The dependence of the SEO mean, R_{sr}, minimal, R_{min}, and maximal, R_{max}, radius on the ion kinetic energy. Also shown is the influence of the test ion kinetic energy on the ion radial position within the acceleration gap, R_{gap}. The test ion is H^- with the extraction energy of 65 MeV at the extraction radius R_{sr} = 84 cm.*

Depending on the radial and azimuthal position of the initial point of the accelerated equilibrium orbit (AEO) initial energy should take appropriate value (close to the one corresponding to R_{max} if starting from the middle of the sector, R_{min} if starting from the middle of the valley, etc.). If the mismatch between the initial energy and the one that is appropriate for the given initial point position is too large, a test ion will start moving along a trajectory with smaller or larger radius of curvature, leading to eccentric orbit and non-uniform change in orbit separations. It is a natural consequence of energy mismatch no matter how well the

initial position, momentum and RF phase are chosen. The initial momentum should be tangential to the AEO and the initial RF phase should provide that approximately the same amount of energy is collected over each of the four gaps during the acceleration process.

The prerequisite for an accelerated orbit to be an optimal accelerated orbit, i.e. accelerated equilibrium orbit, is that it is well centered relative to the machine center. Each time an accelerating gap is crossed ion picks up certain amount of energy and the local radius of curvature increases accordingly. Since ion turns on an orbit are denser at higher radii, acceleration is more efficient if the turns are well centered at higher radii. Therefore the initial turns are not well centered. The centering of an orbit is hard to visualize on an orbit itself, instead, the motion of the curvature center is investigated. Further, for each turn (four acceleration gaps) a barycenter of the curvature center trajectory can be determined. If a barycenter stays close to the machine center during the acceleration process the acceleration orbit is well centered. The trajectory of the curvature center of an orbit is obtained using the energy gain at the accelerating gaps. The motion of the barycenter is then obtained from this trajectory.

An ion that is well centered has the initial energy that is matched with the initial position and is well synchronized with the RF system is going to remain so over the course of acceleration if it [4]:

- exhibits the smallest degree of radial betatron oscillations;
- has the greatest mean energy increment per gap crossing;
- performs the least number of turns in the fixed energy range relevant to the acceleration region of the cyclotron, i.e. has the shortest acceleration time;
- attains the greatest final energy after a fixed number of turns.

The above are consequently the requirements for the best acceleration efficiency in a cyclotron.

3. RESULTS

To illustrate the importance of optimal acceleration two examples of beam dynamics simulation for H^- ion beam are presented. One set of initial conditions complies with the criteria discussed above. Another set of initial conditions is an output of a test central region beam dynamics simulation. Since the second set of initial conditions corresponds to the radius of approximately 30 cm, both test beam dynamics simulations in the accelerating region were performed from this radius on.

The initial conditions and the final results for the two examples of beam dynamics simulation for H^- ion beam are summarized in Table 1. The final results are given in the shaded part of Table 1. The gyration frequency of an ion in a realistic, isochronized magnetic field fluctuates slightly with radius as opposed to the ideal case of the constant gyration frequency. Therefore, the optimal RF frequency has been adopted as an average orbital frequency in the user-defined range of radii, taken here to be from 30 cm to 84 cm, multiplied by the harmonic number, as is explained in [5].

Table 1. *Summary of the two simulations.*

	Simulation No. 1 (optimal initial conditions)	Simulation No. 2 (non-optimal initial conditions)
f_{RF} [MHz]	20.0387067	20.0387067
φ_{RF} [°]	184.6824	274.9199
R_0 [mm]	299.9999	301.5869
θ_0 [°]	310.3176	229.9199
α_0 [°]	222.2059	137.6923
T_0 [MeV]	8.0490	7.8851
Δt [μs]	28.1333	28.6896
N_{turns}	564	575
R_{final} [mm]	810.4505	864.4415
θ_{final} [°]	44.9943	269.7537
T_{final} [MeV]	65.0143	65.0149

The initial phases of the cosine RF voltage have been adopted to provide the best synchronization of ions with the RF system for the most part of the acceleration process. The azimuthal angles of the initial as well as the final point have been denoted by θ, whereas α is the angle of the initial momentum. All angles are measured contraclockwise with respect to the positive x-axis. Radial coordinates are denoted by R, kinetic energies by T, Δt is the time the central particle spends traveling along the simulated orbit, and N_{turns} is the number of turns. Accuracy requirements for both simulations were set to x_{err} = 50 nm, p_{err} = 0.01 ppm.

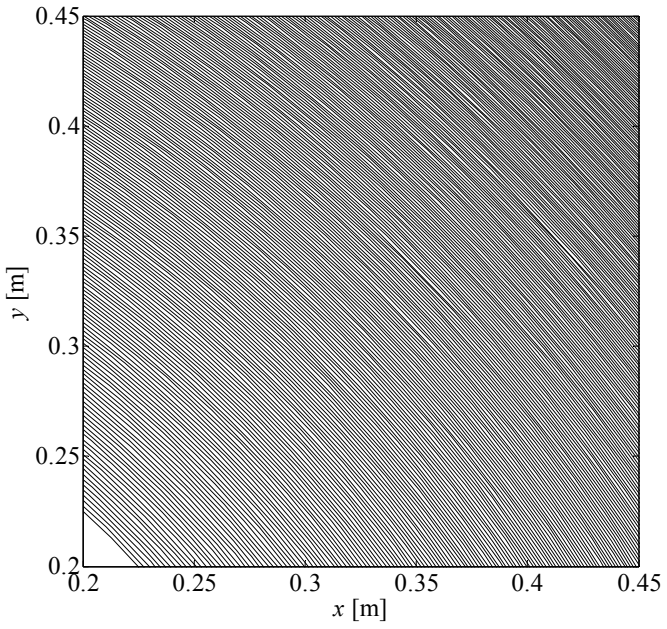

Fig.2. *The part of the central ion trajectory with the optimal initial conditions*

The parts of the central ion trajectories in the median plane for the two sets of initial conditions are shown in Fig.2. and Fig.3. In Fig.2, which corresponds to the optimal initial conditions, one can observe a uniform-like decrease in orbit separation, i.e. increase in orbit density, as one moves toward larger radii and higher energy. This is probably the most obvious consequence of the initial simulation parameters that are well matched with the optimal ones. Fig.3. depicts oscillations in orbit spacing, which coincide with the oscillations in energy gain per turn and appropriate increase in radius per one turn. This happens as a consequence of ions slipping out of phase during the acceleration process.

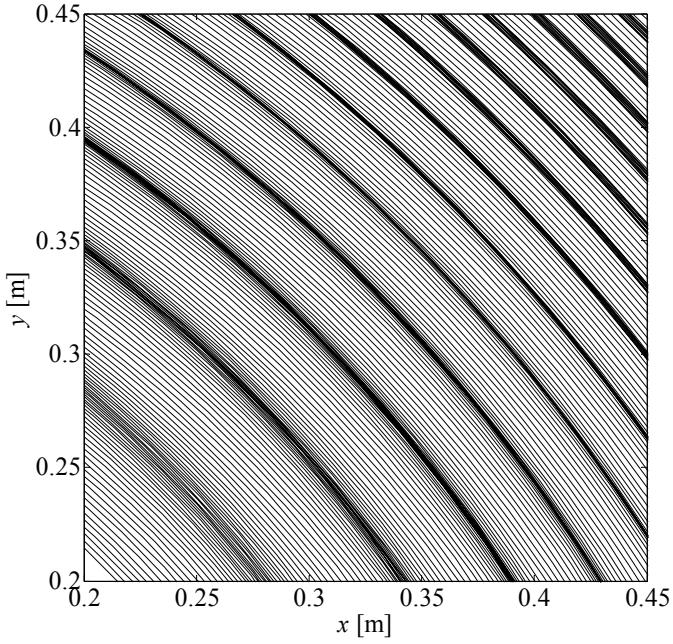

Fig.3. *The part of the central ion trajectory with the non-optimal initial conditions*

Ideally, an ion synchronized with the RF should gain the same amount of energy at each gap it crosses. This can not be achieved in reality due to the radial fluctuations of the gyration frequency of an ion, caused by the isochronized magnetic field. However, for optimal acceleration the differences between gap energy gains should be as small as possible. The gap energy gains for the two sets of initial conditions are shown in Fig.4. and Fig.5. The mean values of energy gains for each gap for the two cases, $\Delta T_{1,\,\mathrm{mean}}$ and $\Delta T_{2,\,\mathrm{mean}}$, are listed in Table 2.

Table 2. *Statistics of the gap energy gains.*

	Gap A	Gap B	Gap C	Gap D
Simulation No. 1 (optimal initial conditions)				
$\Delta T_{1,\,\mathrm{mean}}$ [keV]	27.855	22.812	27.904	22.819
$\Delta T_{1,\,\mathrm{mse}}$ [keV]	1.160	1.168	1.156	1.162
Simulation No. 2 (non-optimal initial conditions)				
$\Delta T_{2,\,\mathrm{mean}}$ [keV]	37.515	12.370	37.534	12.320
$\Delta T_{2,\,\mathrm{mse}}$ [keV]	1.187	1.269	1.158	1.353

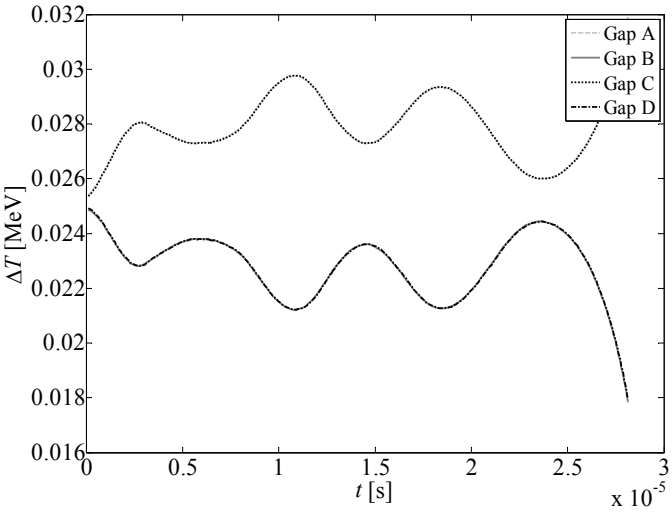

Fig.4. *The central ion energy gain within gaps for the optimal initial conditions*

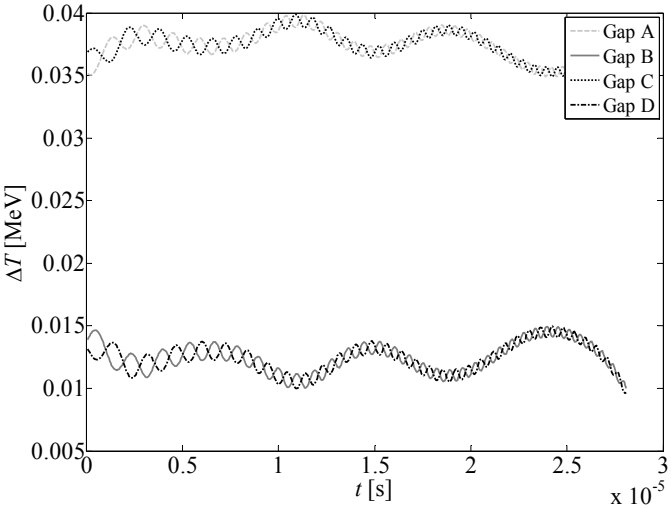

Fig.5. *The central ion energy gain within gaps for the non-optimal initial conditions*

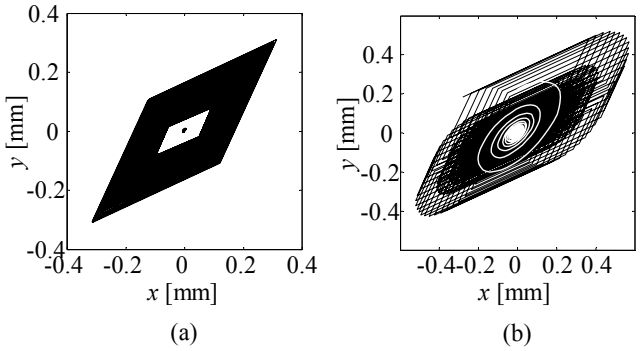

Fig.6. *The movement of the centers of the curvature for (a) the optimal initial conditions, and (b) the non-optimal initial conditions*

Mean square fluctuations in energy gain, denoted as $\Delta T_{1,\,\mathrm{mse}}$ and $\Delta T_{2,\,\mathrm{mse}}$, are also shown. The difference between energy gains in the first simulation is much smaller than in the

second one, although it seems that this could be even further improved.

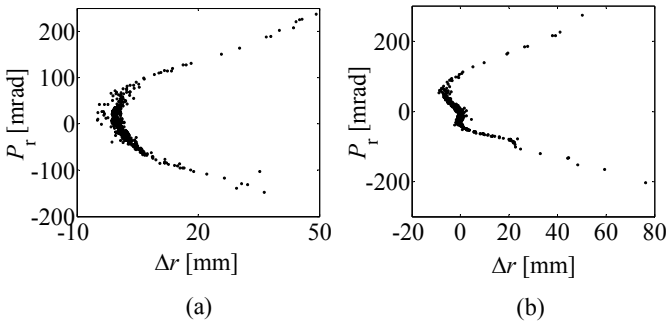

Fig.7. *The horizontal emittances for (a) the optimal initial conditions, and (b) the non-optimal initial conditions*

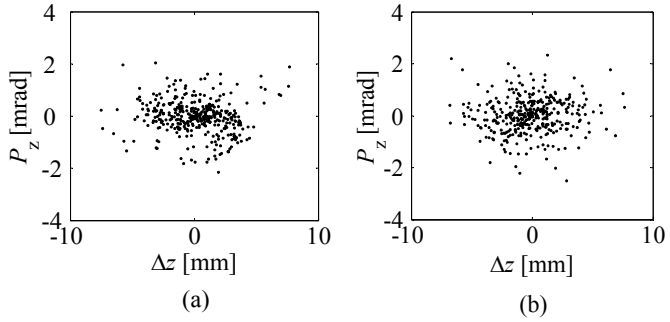

Fig.8. *The vertical emittances for (a) the optimal initial conditions, and (b) the non-otimal initial conditions*

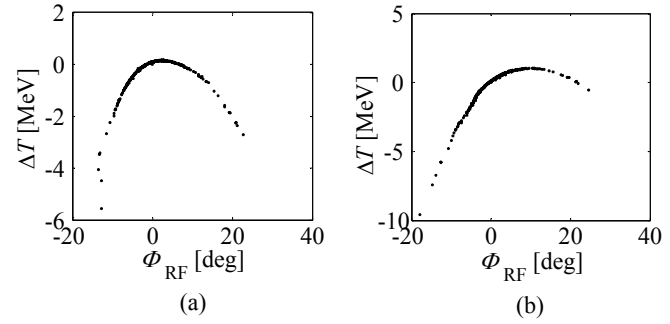

Fig.9. *The longitudinal emittances for (a) the optimal initial conditions, and (b) the non-optimal initial conditions*

Fig.6. illustrates the centering of the orbits through the motion of the centers of the curvature barycenter, for the two investigated sets of initial conditions. The movement of the centers of the curvature is plotted using the thin black line, whereas the movement of their barycenter is presented with a thick black line in the first case and with a thick white line in the second one. Both accelerated orbits become well centered at higher radii, close to the extraction radius. However for the optimal initial conditions the centering is good during the whole acceleration process and the barycenter is practically a point.

Finally, the obtained final emittances after the same number of turns are compared for the two simulations. The horizontal emittances, vertical emittances and longitudinal emittances are shown in Fig.7, Fig.8, and Fig.9, respectively. The energy spread of the bunch for the second set of initial conditions is almost twice the energy spread obtained with the optimal initial conditions. The radial emittance is also better for the first simulation, whereas the vertical emittances do not differ much since they are primarily shaped by the focusing capabilities of sectors.

4. CONCLUSION

The criteria to define the initial conditions for optimal acceleration of a test ion, i.e. for selecting the best out of a pool of different accelerated trajectories are outlined. The beam dynamics simulations of H^- ion beam, with the two different sets of initial conditions, have proven the validity of our methods. The results of these simulations have been compared and explained.

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