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STATISTICAL PROPERTIES OF QUANTUM ENTANGLEMENT AND INFORMATION ENTROPY

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1 General characteristic of the work

Actuality

The exciting new features of entanglement are burgeoning with revolutionary new advances in the areas of quantum communication, quantum information processing and quantum computing. Information cannot be separated from its physical representation: it is always stored in some physical system, manipulated by some physical process. This observation has a number of consequences for information theory. Perhaps, the most striking one is that, it makes a big difference whether the information is stored and processed in classical or quantum mechanical systems [1-4]. Until now this has always been done using systems governed by classical physics, e.g., one bit of information in a system that could take either of two states. A fundamental difference between quantum and classical physics is the possible existence of nonclassical correlations between distinct quantum. systems. The physical property responsible for the nonclassical correlations is called entanglement. From the beginning of the nineties, the field of quantum information theory opened up and expanded rapidly. A pair of quantum systems in an entangled state can be used as a quantum information channel to perform computational and cryptographic tasks that are impossible for classical systems. The Theory of Quantum Information also sheds new light on some of the traditional questions of the foundation of quantum mechanics. However, most workers in the field have adopted a very pragmatic attitude to these highly controversial questions [5-20].

The basic ideas behind quantum communication and computation are very simple. Quantum communication deals with sending quantum states from one place to another one in such a way that they arrive intact. The most important application so far in this field is the one in which a sender (traditionally called Alice) tries to convey a secret message to a receiver (traditionally called Bob). The message is encoded in the final state of a quantum system. Due to the fact that the quantum state of a system is distorted if somebody performs a measurement, Bob will receive a wrong state if a third (malevolent) party (Eve) tries to read the message. This way of secret communication is usually called quantum cryptography, and it is the only provably secure way in which two partners can share secret messages. In the context of quantum computation, the existence of entangled states of several particles offers the possibility of performing certain computational tasks in times much shorter than the ones taken by common (classical) computers. By acting on a system entangled to other systems, one modifies the state of the whole system at the same time, which leads to an important speed up in several computations.

Fortunately, owing to technological progress, experiments considered unrealistic up to recently, can now be carried out. As a result, various intriguing genuinely quantum effects traceable back to the superposition principle, entanglement, quantum interference, etc., are now within experimental reach. Experiments of increasing difficulty in cavity quantum electrodynamics over the last years, have, for example, made it possible to test fundamental radiation-matter interaction models involving single atoms. Moreover, cavity field states possessing remarkable nonclassical features have been generated and detected. In fact, since the foundations of quantum mechanics were laid, one of its most curious, and perhaps interning, features has been entanglement. In the past decade the focus has shifted to a more information-theoretic interpretation of entanglement in line with the global effort to understand and eventually build a quantum computer.

It is important to point out that the increased insight into the dynamics of the three-level systems may be helpful in developing quantum information theory. Recently, there is much interest in three-level quantum systems to represent information [1]. It was demonstrated that key distributions based on three-level quantum systems are more secure against eavesdropping than those based on two-level systems.

Entangled states are rather a result of a particular method of construction of states basis. Entering in a compound system, due to an interaction, subsystems properties are entangled and lose their individuality. While we manipulate with system as a wholeness (make calculations, perform observations), its entangled structure is preserved in a coherent way and never shows up. However, as soon as a compound system breaks down, we have absolutely different situation. Now, each subsystem should be considered as an independent one, described by its own state vector.

An existence of possible initial correlations between subsystems properties is a result of conservation rules, fulfilling in any case, rather than quantum ones. In pure-state quantum mechanics the state of the system is usually represented by a (normalized) wavefunction, which is a (unit) vector in a Hilbert space. If the system is in the pure state $|\psi(t)\rangle$ then $\rho(t)$ is simply the projector onto this state, i.e., $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$,in such a way that $\rho^2(t) = |\psi(t)\rangle\langle\psi(t)|\psi(t)\rangle\langle\psi(t)| = \rho(t)$, and $Tr\rho^2(t) = 1$. In the theory of open system or the reduction theory, one often considers two subsystems $\mathcal{H}_1$ and $\mathcal{H}_2$ represented by Hilbert space. Let $\mathfrak{S}(\mathcal{H}_i)$, $(i = 1, 2)$ be state spaces (the set of all density operators). Also $\mathfrak{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$ denotes the state space in the composite system $\mathcal{H}_1 \otimes \mathcal{H}_2$.

Within quantum information a unique measure of entanglement has not been forthcoming, however there is considerable agreement on the properties that an entanglement measure must satisfy. A coherent set of properties for an entanglement measure, E, for an arbitrary state ,ρ, should satisfy [2-6].

(1) $E(\rho) = 0$, if ρ is separable. This property refers to the most fundamental premise,

that if we can write the system's state vector as a product of the subsystem state vectors then the system has zero entanglement.

(2) $E(\rho)$ is invariant under local unitary operations. This property means that if there is no entanglement, no quantum correlations, to start with then it cannot be created by any unitary operations carried out locally on the separate subsystems.

(3) The expected entanglement cannot increase under any combination of local general measurements, classical communication and post selection.

Property 3 provides a caveat for entanglement transformation protocols. This last property allows the two subsystems to have ancilla locally introduced and to classically communicate results, regarding measurements, operations etc to one another so that they can select a subensemble of the system that has an increase in entanglement. Property 2 is not violated by this as the total entanglement of the system is still the same, all that has taken place is that part of the ensemble has increased its entanglement whilst the rest of the ensemble has had its decreased on average. This is an underlying principle of many entanglement transformation schemes.

In the meantime squeezing states of light offer possibilities of improving the performance of optical devices since they can reduce fluctuations in one of quadrature below the level associated with the vacuum states. This situation is relevant for the optical communication networks as well as for many optical devices. Such light has recently been used in a power-recycled interferometer and in a phase- modulated signal-recycled interferometer, aiming to improve significantly the sensitivity of these devices. It has been shown that this light can be used to tune the resonant frequency of the cavity without actually moving the signal recycling mirror or changing the bandwidth of the interferometer without substantially decreasing the sensitivity at the resonant frequency.

The controlled manipulation of entangled states of N-particle systems is fundamental to the study of basic aspects of quantum theory, and provides the basis of applications such as quantum computing and quantum communications. Engineering entanglement in real physical systems requires precise control of the Hamiltonian operations and a high degree of coherence. Achieving these conditions is extremely demanding, and only a few systems, including trapped ions, cavity QED and mesoscopic electronics, have been identified as possible candidates to implement quantum logic in the laboratory. On the other hand, experiments with cold and ultra-cold atoms have led to many recent achievements in different fields. Because the atoms move so slowly, not only are they sensitive to long-range interaction potentials but, in the presence of light, they can also undergo changes of internal states during a collision (the interaction time can be larger than the typical spontaneous emission time) [21-36].

Actuality of the considered problems are proved by regularity of international conferences and workshops. Among the latest international conferences are: The First International Conference on Quantum, Nano, and Micro Technologies ICQNM 2007, January 2-6, 2007, Guadeloupe, French Caribbean; 4th International Conference on Quantum Dots May 1-5, 2006, Chamonix-Mont Blanc, France; International Workshop on Solid State Based Quantum Information Processing May 24 - 26, 2006, Herrsching, Bavaria, Germany; and International Conference on Quantum Information, 2001 OSA Technical Digest Series (Optical Society of America, 2001

Aim of the work

Our aim was to introduce some new features or models of the interaction between particles (atom or trapped ion) with field (cavity field or laser field). We focused particularly on the relation between this interaction and its impact on the quantum information and quantum computation (entanglement, information entropy, geometric phase and ultracold atoms). Intuitive pictures of the interaction between a single trapped ion (or atom) and a laser field (cavity field) commonly involve the expectation that the atomic level populations must change as both systems exchange excitations over the course of time. This work completes the analytical treatment of two- and three-level trapped ion (or atoms) interacting with a single or bimodal laser field (cavity field). In particular, under certain conditions the systems become solvable and analytical solutions are obtained and have been used to discuss the mixedness and entanglement measures, entropy squeezing, geometric phase and mazer interaction. We studied how the entropy and entanglement are influenced by different regions of the parameters. In the phenomenon of the collapses and revivals, the coherent state causes two basic effects: the revival amplitude is strongly suppressed and the revival time is halved.

We focused on using initially factored and entangled states to measure the entanglement degree. We would especially like to draw attention to the results on the entropy squeezing and entanglement degree, which are presented here for the sake of comparison. The applications discussed here show the power and the versatility of information entropy and quantum entropy methods in attacking problems of quantum information theory. As a result we have predicted the existence of super-revivals for a particular choice of the involved parameters. Several conclusion can be drawn from our results: One of the insights provided by quantum information theory is that the von Neumann entropy has an interpretation as a measure of the resources necessary to perform an information task.

One of our aim was to provide an acceptable entanglement measure for a coupled system when the system starts form its mixed state. We have used for this purpose the quantum mutual entropy and the new measure is called DEM (degree of entanglement due to quantum mutual entropy). Also, the entanglement degree due to quasi-mutual entropy of a three-level atom interacting with a single cavity field is investigated. We consider the situation for which the three-level system is initially in a mixed state, whereas the field may start from either a coherent or a squeezed state. We present a derivation of the unitary evolution operator on the basis of the dressed state formalism taking into account an arbitrary form of nonlinearity of the intensity-dependent coupling, by means of which we identify and numerically demonstrate the region of parameters where significantly large entanglement can be obtained. Most interestingly, it is shown that features of the degree of entanglement are influenced significantly by different forms of the nonlinearity. The atom and radiation subsystems exhibit alternating sets of collapses and revivals due to the initially mixed states of the atom and radiation employed here.

The issue we have in mind involves the behavior of the emission probability of the micromaser with an ultra-cold two or three three-level atom one- or two-mode system, taking into account spatial variation along the cavity axis. This is most conveniently accomplished in a quantum theory of the mazer formalism in terms of the dressed-state approach. A related treatment that discusses the quantum theory of the micromaser with an ultracold, three-level, two-mode system without spatial dependence was presented very recently. Our aim was to study and present the generalization of the complete wave-function equation, usually obtained in the micromaser with an ultra-cold, to the three-level case taking into account the spatial dependence. Fortunately, due to technological progress, experiments that were until recently considered unrealistic can now be carried out. Over the last few years, experiments of increasing difficulty in cavity quantum electrodynamics (QED) have made it possible to test fundamental radiation–matter interaction models involving single atoms. Previous work in cavity QED has been plagued by fluctuations in the number of atoms interacting with a cavity field at any instant. For a three-level atom coupled to one or two quantized cavity field modes, both the atomic dynamics and statistical properties of the re-emitted field have been studied extensively over the last several years in many contexts, both in a cavity and in free space. These show interesting phenomena, the most outstanding being perhaps the existence of collapses and revivals.

Thus we may say that, we have covered some points of the main important points which treat the effect of the particles-field interaction.

2 Achieved results and their novelty

All the results of the dissertation are new.

In this work, various types of entanglement have been discussed for systems start from a pure state or mixed states and many new features have been discovered in this work. Particularly, we focused on the most recent theoretical studies and applications of the quantum entanglement of different systems such as, atoms or trapped ions interacting with a cavity field or a laser field. The main phenomena and observations of two-, three- and four-level systems are presented. We explored the influence of the various parameters of these systems on the entanglement and quantum information. A general expression of the mixed state entanglement is obtained with the physical significance (with or without the diagonal approximation treatments). The particular advantages of using Shannon entropy are highlighted and the information entropy for different cases has been discussed.

The atomic inversion of cold atoms in a microcavity when its center-of-mass (CM) motion is described quantum mechanically is presented, but is distinguished from other treatments by the inclusion of the spatial variation along the cavity axis. It is shown that the spatial dependence of the atomic inversion is very sensitive to changes of the atomic CM motion, as well as the cavity length. For a three-level ultra-cold atoms, different behaviors in the atomic emission are demonstrated for the change in cavity length. Here is a list of the new findings

- We discussed the most general case of the pure state entanglement. Although various special aspects of the quantum entropy have been investigated previously, the general features of the dynamics, when a multi-level system and a common environment are considered, have not been treated before and our work therefore filed a gap in the literature.

- We introduced a new entanglement measure due to quantum mutual entropy (mixed-state entanglement) we called it DEM. We have applied the measure on different systems and compared with the well known measures when the system starts form its pure state. In all those cases the new measure gave a good agreement.

- We introduced a new treatment of the atomic information entropy in higher level systems. We have completely solved the problem in the case of three-level system, and found an extension in the case where ion–field coupling is time dependent. Treating the time-dependent ion–field interaction to manipulate the quantum information entropy has many new and important dynamical quantities.

- We introduced a new solution of the interaction between the ultra-cold atoms and cavity field.
- We introduced some new models of the atom-filed interaction.

3 Methods of investigation

The ability of different theoretical physics and computing physics(namely statistical physics and quantum optics) methods to predict general features of the atom-field interactions are used. In our work we have used analytical or numerical treatments depending on the considered problem. Analytical solutions are obtained by solving Schödinger or Heisenberg equations of motions while the $C++$ and Mathematica have been used to obtain the numerical results.

4 Theoretical and practical value

The general results of this dissertation have a theoretical character. Theoretical work on quantum computation is very important since experiments have been carried out in which quantum computational operations were executed on a very small number of qubits. Research in both theoretical and practical areas continues at a frantic pace, and many research agencies support quantum computing research to develop quantum computers and cryptanalysis. It is widely believed that if large-scale quantum computers can be built, they will be able to solve certain problems asymptotically faster than any classical computer.

5 Approbation

Some results from this dissertation have been reported in International and National Conferences and Workshops: Egyptian Mathematical Society Conference 2004, the Second International Conference on Basic Sciences and Advanced Technology, Nov. 5-7 Assuit , Egypt (2002), the XXIV International School of Theoretical Physics, Ustron 2001, Poland, Al-Azhar Engineering Sixth Inter. Conference, Sep. 1-4, 2004 Cairo , Egypt, the International Conference on Mathematics and 21 Century, Jan. 15-20 Cairo , Egypt (2001), several seminars in International Islamic University Malaysia, 2002, 2006 and some seminars in Faculty of Science, Behard Malaysia 2006, seminars in Max-Plank-Insinuate for Quantum Optics, Germany, Bahrain University, Bahrain, Sohag University, Egypt; Institute of Nucluar Physics, Academy of Sciences, Republic of Uzbekistan, Theoretical Physics (Academician M M Mosakhanov) and Mathematical Physics (Academician A A Alimov) in National University, Republic of Uzbekistan.

6 Structure and volumes of the dissertation

The dissertation consists of an introduction, six chapters, conclusion and references. Each chapter contains some paragraphs and the paragraphes have some items. The dissertation contains 200 pages and 11 pages for 196 references.

7 The content of the work

In the introduction chapter we formulated the problems and gives a short overview of these problems. The actuality of the problem is given followed by our results and their novelty. In chapter 1, we consider the pure-state entanglement and extend earlier investigations on the entanglement degree of a two-level or three-level atoms to include any forms of nonlinearities of both the field and the intensity-dependent atom-field coupling. The influences of the nonlinearity, Stark shifts and detuning on the degree of entanglement are examined. As far as we are aware, we have presented the pure state entanglement for a three-level atom interacting with a cavity field in [1-4]. The time evolution of the field (atomic) entropy reflects the time evolution of the degree of entanglement between the atom and the field. An expression for the field entropy for the entangled state of a single two-level atom interacting with a single electromagnetic field mode in an ideal cavity with the atom undergoing either a one- or a two-photon transition has been studied previously. We extend the model in another direction to consider a model consists of a single three-level atom in a single-mode field surrounded by a nonlinear Kerr-like medium contained inside a very good quality cavity. The cavity mode is coupled to the Kerr medium as well as to the three-level atom. The Kerr medium can be modelled as an anharmonic oscillator with frequency. Physically this model may be realized as if the cavity contains two different species of atoms, one of which behaves like a three-level atom and the other behaves like an anharmonic oscillator in the single-mode field. Such a model is interesting by itself as another exactly solvable quantum model that gives non-trivial results, but we can also think of its possible applications. This Hamiltonian is natural for local modes in molecular physics or for a nonlinear Jahn-Teller effect, although the long-time behavior in either case might be obscured by omnipresent damping.

Such systems are potentially interesting for their ability to process information in a novel way and might find application in models of quantum logic gates. For small values of the Kerr-like medium, there is an increase of the sustainment time of the maximum field entropy, and strong entanglement of the field with the atom, while for large values, it results in a decrease of the field entropy, and the field is disentangled from the atom during the time evolution. On the other hand, the maximum field entropy and the atom–field entanglement are reduced as the Kerr-like medium increases.

The most important and interesting work to understand relation between entropy and information was done by Shannon [20], who introduced the entropy (the Shannon entropy) into communications theory. The Shannon information entropy [20], corresponding to the photon-number operator is given by

$$S_H = \sum_{n=0}^{\infty} P\left(n,t\right) \ln P\left(n,t\right), \tag{1}$$

where $P(n,t) = \langle n \left| \hat{\rho}_F\left(t\right) \right| n \rangle$,is the photon number distribution, $\hat{\rho}_F\left(t\right) = Tr_A\left[\hat{\rho}\left(t\right)\right]$. Investigating the dynamical behavior of a physical system from a theoretical point of view, very often involves more or less tacit assumptions about the possibility of preparing the system in a given state at $t = 0$. It is noted that the Shannon information entropy for photon number operator uses only diagonal elements of density operator and contains no phase information, but it provides some useful information about the behavior of photon-number distribution (i.e. entropy is the amount of random information in a system).

In general the quantum information entropy S introduced by, and given by the expectation value of the logarithmic operator $\hat{\rho}(t)$ in the following form

$$S(t) = -Tr\left[\hat{\rho}(t) \ln\left(\hat{\rho}(t)\right)\right], \tag{2}$$

where $\hat{\rho}(t)$ is the density operator for a given quantum system and we have set Boltzmann's constant $k = 1$. If $\hat{\rho}$ describes a pure state, then $S = 0$, and if $\hat{\rho}$ describes a mixed state, then $S \neq 0$. Consider F and A that interact with each other. How are the entropies of these systems related to the entropy of the composite system that comprises them both?.

The entropies of the atom and the field, when treated as a separate system, are defined through the corresponding reduced density operators by [21]

$$S_{A(F)} = -Tr_{A(F)}\{\hat{\rho}_{A(F)} \ln \hat{\rho}_{A(F)}\}. \tag{3}$$

The field entropy is used as a measurement of the degree of entanglement between the field and the atom of the system under consideration. In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues of the reduced field density operator.

As remarked above, it is our purpose to present a formal constructive mechanism to purify any given ensemble, showing at the same time how, by resorting to this procedure, one can use the obtained pure state to generate all ensembles of systems equivalent to the one has purified. The procedure is based on a formalism which parallels strictly the one proposed by von Neumann for implementing ideal measurement processes. Using the matrix elements which represent the state of the field, we are able to determine under which conditions we may attain a reasonable purification of the field, namely, departing from a mixed state. The coherence loss of the field by means of linear entropy, is a convenient way to study the coherence properties of the density operator. By analogy with definition of the quantum entropy, the linear entropy of entanglement can be defined as [6]

$$S_a(t) = 1 - Tr_a \left\{ \hat{\rho}_a^2(t) \right\}. \tag{4}$$

Equation (4) has been used in the literature as an entropy-like quantity, and sometimes referred to as the linear entropy.

A quantity also used to compare distributions as well as quantum states is the mutual information or mutual entropy (total correlation) [23]. The quantum mutual information I relative to two subsystems (A and B) may be written as

$$I = S_A + S_B - S_{AB}, \tag{5}$$

where $S_A(S_B)$ is the entropy relative to the subsystem $A(B)$, and S_{AB} is the entropy of the overall state, described by a density operator $\widehat{\rho}_{AB}$. The reduced density operators relative to the subsystems, $\widehat{\rho}_A$ and $\widehat{\rho}_B$ are obtained from $\widehat{\rho}_{AB}$ through the usual partial tracing operation, as the form of equation (5).The mutual information is then calculated using the quantum entropies of the different systems. We expect that the quantum mutual entropy techniques will be central to further work in quantum information theory. In particular, we think that they show promise in resolving some of the many perplexing additivity problems that face the theory at present. As the mutual information indicates the total correlations, it should be larger than the lower bound on entanglement we compute.

With the reliance in the processing of quantum information on a cold trapped ion, we analyzed the entanglement entropy in the ion-field interaction with pair cat states [30]. We have investigated a long-living entanglement allowing the instantaneous position of the center-of-mass motion of the ion to be explicitly time-dependent. An analytic solution for the system operators is obtained. We show that different nonclassical effects arise in the dynamics of the population inversion, depending on the initial states of the vibrational motion. We study in detail the entanglement degree and demonstrate how the input pair cat state is required for initiating the long-living entanglement. This long-living entanglement is damp out with an increase in the number difference q. Owing to the properties of entanglement measures, the results are checked using another entanglement measure (high order linear entropy). The main motivation was twofold: first, we demonstrate how a pair cat state affects the entanglement for the ion-field interaction, and second to see how the time-dependent amplitude of the irradiating laser field affect this entanglement. In particular, we consider a single trapped ion which can be laser cooled to the ground state of the trapping potential and discuss the roles played by the initial state setting and time-dependent amplitude of the laser field on the entanglement. The main topics of the chapter have been published in Refs. [1-14].

In chapter 2, we consider the mixed-state entanglement due to quasi-mutual entropy of two- or three-level atoms interacting with a single or a bimodal cavity fields. We consider the situation for which the atomic system is initially in a mixed state, whereas the field may start from either a coherent or a squeezed state. The scope of this chapter is essentially to examine the entanglement for an initial mixed state of the atom. We adopted the method using quantum mutual entropy to measure the degree of entanglement in the time development of the Jaynes–Cummings model. In this paper, we formulate the entanglement in the time development of the Jaynes–Cummings model with squeezed states, and then show that the entanglement can be controlled by means of squeezing. A systematic study of the entanglement properties of the atom–field interaction that have emerged from the quantum relative entropy has not been performed in multi-level atoms. It is the objective of the present contribution, which is a progress report in character, to contribute to this systematic study. We shall here focus on what is perhaps the simplest situation in this context, namely a three-level atom interacting with a single cavity field, including an arbitrary form of the intensity-dependent coupling. Thus the present work sheds some light on the entanglement behavior of multi-level systems when initially an entangled mixed state of the coupled system is considered and how this is affected by different parameters of the multi-level system. The scheme we are going to discuss exploits the passage of a single atom only through the cavity. We wish to underline from the beginning the relevance of this aspect from an experimental point of view.

Preparing and controlling a single atom is certainly much easier to achieve with respect to the case when the manipulation of many atoms is required. In addition, taking into consideration the low efficiency of the atomic state detectors today used in laboratory, conditional measurement procedures involving one atom only instead of many ones, have to be preferred. The dynamics of several Hamiltonian models describing such systems is exactly treatable and, in most cases, testable in the laboratory. Also, a more intriguing reason is that investigating these systems is likely to shed light on basic questions of quantum mechanics. The point to be appreciated is indeed that, studying such systems, one has the opportunity to induce entanglement and to control its evolution in a multipartite physical system. The physical scenario relative to the problems we shall be faced with in a quantum electrodynamics context, involves three-level atoms interacting, one at a time, with a single quantized electromagnetic mode sustained by a high-Q resonator. In this case, the dynamical properties of the system are investigated using a Hamiltonian model characterized by the presence of bosonic variables describing the quantized electromagnetic mode and of pseudo-spin atomic operators.

Our main results: in principle, it is possible to address the characterization of the entanglement degree due to quasi-mutual entropy. We have presented an analytical solution to a three-level system interacting with a single-mode taking into account an arbitrary form of the nonlinear intensity-dependent coupling on the basis of the dressed-state formalism. It is appropriate to emphasize that the work here extends previous studies in this context. In particular, we have explored the influence of the various parameters of the system on the entanglement degree. It is found that entanglement is affected strongly when nonlinear intensity-dependent coupling is taken into account. As expected, the maximum value of the entanglement degree decreases with decreasing occupation probability of the upper atomic level. We have found that in general the shape of revival envelopes is a direct reflection of the form of a continuous interpolation of the probability distribution. In the particular case of a field initially in a coherent state, this explains the appearance of a doublet structure in the revivals in the limit of greatest populations. It is of interest to remark that at a special choice of the nonlinear intensity-dependent coupling we have obtained longer period of the entanglement. We expect that the results of this paper can be of help for some problems, especially for quantum computation or quantum information processing, because research into the dynamical properties of multi-level atoms or trapped ions locates completely in the field of quantum computation. We also expect that this paper can lead to some other interesting discussions for systems of multi-level atoms with arbitrary form of the nonlinear intensity-dependent coupling, such as generating nonclassical states of one or more modes of a single cavity field. The next obvious step in the progression of this work would be to damp the quantum field interacting with the atom.

In chapter 3, we have considered the new concept of entropy squeezing to discuses the quantum information dynamics. To do so we have to calculate the quadrature variances for the atomic operators, which can be achieved by employing the Heisenberg uncertainty principle. However, the Heisenberg uncertainty relation cannot give us sufficient information on the atomic squeezing for some cases, but it can be used as a general criterion for the squeezing in terms of information entropy of a two-level system.

Recently the new field of quantum information and computation has emerged, not only offering the potential of immense practical computing power, but also suggesting deep links between the well-established disciplines of quantum theory, information theory and computer science. Theoretically quantum computers can perform some types of calculations much faster than classical computers, but the technological difficulties of manipulating quantum information have so far prevented researchers from constructing a quantum computer which is able to perform useful tasks. In this chapter, we have shown that in the two-level system the effect of a Kerr-like medium on the entropy is negative and the effect of detuning on the atomic variable squeezing is positive. This emphasizes the fact that the atomic coherence has a remarkable effect on the squeezing of the entropy, and the system of the Jaynes-Cumming model with a Kerr-like medium can have a potential application in the field of quantum information.

Trapped and laser-cooled ions are increasingly used for a variety of modern high-precision experiments, frequency standard applications and quantum information processing. Therefore, in this communication we present a comprehensive analysis of the pattern of information entropy arising in the time evolution of an ion interacting with a laser field. A general analytic approach is proposed for a three-level trapped-ion system in the presence of the time-dependent couplings. By working out an exact analytic solution, we conclusively analyze the general properties of quantum information entropy. It is shown that the information entropy is affected strongly by the time-dependent coupling and exhibits long time periodic oscillations. This feature attributed to the fact that in the time-dependent region Rabi oscillation is time dependent. Using parameters corresponding to a specific three-level ionic system, a single beryllium ion in a RF-(Paul) trap, we obtain illustrative examples of some novel aspects of this system in the dynamical evolution. Our results establish an explicit relation between the exact information entropy and the entanglement between the multilevel ion and the laser field. We show that different nonclassical effects arise in the dynamics of the ionic population inversion, depending on the initial states of the vibrational motion/field and on the values of Lamb-Dicke parameter η.

From a very fundamental point of view, we have discovered a new feature of the quantum information entropy, which shows how far the quantum field entropy lies from information entropy in the pure state case. A particularly interesting aspect of our work is the introduction of the ansatz which results from the observation that one can use the eigenvalues and eigenvectors of the atomic operators to define the quantum information entropy in the multi-level systems. The general forms of the information entropy of the three-level trapped ions taking into account a time-dependent ion–field interaction are clearly exhibited and they are new as far as we are aware.

In chapter 4, we discuss the geometric phase in different quantum systems. Geometric phases in quantum theory have attracted great interest since Berry showed that the state of a quantum system acquires a purely geometric feature in addition to the usual dynamical phase when it is varied slowly and eventually brought back to its initial form. For the two atoms case, and under the exact resonance condition and in the presence of both the Stark shift and phase shift an exact solution to the Schrödinger equation is obtained.

Geometric phases in quantum theory have attracted great interest since Berry showed that the state of a quantum system acquires a purely geometric feature in addition to the usual dynamical phase when it is varied slowly and eventually brought back to its initial form. The Berry phase has been extensively studied, generalized in various directions, observed in several systems and has very interesting applications, such as the implementation of quantum computation by geometrical means. Pancharatnam was the first to introduce the concept of a geometric phase in his study of interference of light in distinct states of polarization. Its quantal counterpart was discovered by Berry, who proved the existence of geometric phases in cyclic adiabatic evolutions. This was generalized to the case of nonadiabatic and noncyclic evolutions.

In this chapter, we have presented some aspects of a system of two-level atom interacting with a cavity field via a multi-photon process in the presence of both Stark and Kerr-like medium. This appears to be even more important today with a lot of new effects to be described and experimentally observed. We have used the Heisenberg uncertainty relation as a general criterion for the squeezing in terms of information entropy of a two-level atom multiphoton process, taking into account arbitrary forms of nonlinearities of both the field and the intensity-dependent atom-field coupling. In particular, we have explored the influence of the various parameters of the system on the entropy squeezing. Such systems are potentially interesting for their ability to process information in a novel way and might find application in models of quantum logic gates. An idealized situation when the cavity losses are negligible is considered here, however, the case of real experiment the losses must be introduced. It can be expected that for a non-ideal but high-quality cavity our results are of relevance in the case the Hamiltonian is appropriate for the experimental setup. Thus, the model presented here has not only demonstrated the effect on entropy squeezing of the nonlinearity of a single-mode field theoretically, but is of experimental importance in measuring the entropy squeezing of an atom interacting with single-field mode in a cavity containing any kind of the nonlinearities in the future, providing some guidelines to experimentalist in the identification of the kind of unknown nonlinear medium and the utilization of the nonlinearities of atom-field interaction.

A nonlinear Kerr-type medium and detuning effects are taken into account and the dependence of the entropy squeezing and atomic inversion is considered. It is found that entropy squeezing is affected strongly when a nonlinear medium is taken into account. We have shown in our system the effect of Kerr like medium on the entropy is negative and the effect of detuning on the atomic variable squeezing is positive. This emphasis on the fact that the intensity coupling has a remarkable effect on the squeezing of the entropy and the system of a single two-level atom with Kerr-like medium can have a potential application in the field of quantum information.

Also, we consider a Hamiltonian modelling a pair of two-level atoms within a perfect cavity for the case of a two-photon transition. Under the exact resonance condition and in the presence of both the Stark shift and phase shift an exact solution to the Schrödinger equation is obtained. Employing this solution gives an analytical expression for the temporal evolution of the Pancharatnam phase when a second atom is allowed to interact with the cavity mode and becomes entangled with the first atom. Numerical investigations are carried out for the phenomena of collapses and revivals for different Stark shift values, the distance between the two atoms and the phase shift. It has been shown that the Pancharatnam phase is affected by the Stark shift and the distance between the two atoms when a nonlinear two-photon process is involved.

Of paramount importance is acquiring the Pancharatnam phase due to an ensemble of two two-level atoms (or trapped ions) as it is becoming ubiquitous in different fields of application such as quantum computation and nanotechnology; fields that might well merge. We have obtained an exact solution that can be easily physically interpreted, and thus provides insight into the behavior of more complicated multi-atom systems. We have focused on the behavior of the Pancharatnam phase under this very general model, where an analytical expression for the Pancharatnam phase has been given. We have shown that the Pancharatnam phase can be sensitive to the Stark shift and the distance between the two atoms, while the phase shift has no effect on the Pancharatnam phase. However, the timing of an applied phase step has a significant influence on the Pancharatnam phase. The phases discussed above are physically observable interferometrically through a structured approach, and it is known that the Pancharatnam phase is very important in the propagation of a light beam where its polarization state is changing periodically.

In chapter 5, we consider the ultra-cold atoms. The previous studies of cold atoms system discussed this problem when its CM motion is described quantum mechanically.

In those treatments the wave-function only described the reflected and transmitted parts. However, in this problem three regions are distinguished: in front of the cavity where incident and reflected waves are present, inside the cavity where the transient regime occurs; and after leaving the cavity where transmitted waves are present.

The micromaser system is an experimental realization of the idealized system of a two-level atom interacting with a second quantized single-mode of the electromagnetic field. In the micromaser a beam of two-level atoms is sent through a microcavity where each atom intersects with the photon field inside the cavity during a transit time. After exit from the cavity the atoms are detected in either of its two states. It is assumed that subsequent atoms arrive at time intervals which are much longer then the atom-field interaction such that at most one atom at a time is inside the cavity, which is the operating condition for the one-atom maser.

Previous studies of the cold atom in the Jaynes-Cummings when its center-of-mass (CM) motion are described quantum mechanically. However in those treatment the wave function only describes the reflected and the transmitted parts. However in this problem three regions are distinguished: in front of the cavity described by $\theta(-z)$ where incident and reflected waves are present, inside the cavity represented by $\theta(z) - \theta(z - L)$ where transient regime occurs; and after leaving the cavity described by $\theta(z - L)$ where transmitted waves are present. There is an investigation of the internal operation and field damping during the atom-field interaction for a single atom mazer.

The target in the present chapter was to extend the previous work to include the three regions mentioned above. Also another extension is made namely the off-resonance case is considered. Thus we concentrate on the influence of the quantization of the center-of-mass motion on the atomic inversion when the atom initially prepared in the excited state and the quantized cavity mode in the coherent state taking into account the spatial dependence. We apply the dressed state approach to an ultra-cold atom interacting with an arbitrary field mode of a high-Q cavity whether in resonance or off-resonance cases. It is noted that different dressed-state components of the atom-field system experience different potentials. The one-dimensional scattering problem and the results are applied for computing the atomic inversion.

The full solution is given and the case of the intercavity is considered in particular. This effect shows the spatial dependence of the atomic internal variables. When a beam of ultracold atoms is considered some remarkable new phenomena are observed in the spatial behavior. These features depend on the length of the cavity (or, rather, the parameter γL). With the rapid progress in the cooling and manipulation of single atoms, an experimental demonstration of the quantized-z-motion-induced emission, e.g. by detecting reflected atoms that have deposited a photon in the cavity, seems to be feasible. In such an experiment, the cavity potential could be enhanced by an injected field. The full solution for the problem is given for both cases of resonance and off-resonance and the spatial variation and detuning on the internal variables of the atomic beam are clearly exhibited and they are new as far as we are aware.

The main results of the dissertation have been published in international refereed journals. Here is a list of the main publications of our results

1 M. Abdel-Aty, *(Influence of a Kerr-like medium on the evolution of field entropy and entanglement in a three-level atom)* Journal Physics B: Atomic, Molecular & Optical Physics, Vol. 33, pp. 2665-2676 (2000).

2 M. Abdel-Aty (*Quantum field entropy and entanglement of a three-level atom two-mode system with an arbitrary nonlinear medium*) Journal of Modern Optics, Vol. 50, No. 2 pp. 161-177 (2003).

3 M. Abdel-Aty, (*Generation of long-living entanglement using cold trapped ions with pair cat states*) Applied Physics B: Laser and Optics, Volume 84, No. 3, pp. 471-478 (2006)

4 M. Abdel-Aty, (*Qualitative aspects of the entanglement in the three-level model with photonic crystal*) Applied Physics B: Laser and Optics, 81, Numbers 2-3 (2005) 193 - 203

5 M. Sebawe Abdalla, A.-S. F. Obada, and M. Abdel-Aty, (*Von Neumann entropy and phase distribution of two mode parametric amplifier interacting with a single atom*) Annals of Physics 318 (2005) 266–285

6 M Sebawe Abdalla, A-S F Obada and M. Abdel-Aty, (*Isotropic-coupled oscillators interacting with a single atom via two-photon processes: quantum information aspects*) Journal of Physics B: Atomic, Molecular and Optical Physics,. 37 No 4 (28 February 2004) 775-790.

7 M. Abdel-Aty, (*An investigation of entanglement and quasiprobability distribution in a generalized Jaynes-Cummings model*) Journal of Mathematical Physics, 44, pp. 1457-1471 (2003) and it has been selected for Virtual Journal of Quantum Information – April 2003, Volume 3, Issue 4.

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9 M. S. Abdalla, M. Abdel-Aty, A.-S. F. Obada (*Entropy and entanglement of time dependent two-mode Jaynes-Cummings model*) Physica A: Statistical Mechanics and its Applications: Volume 326, Issues 1-2, pp. 203-219 (2003)

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Summary

of thesis of M. Abdel-Aty on the academic degree of the Doctor of Science (D. Sc.) of Theoretical Physics and Applied Mathematics, speciality 01.04.02 - Theoretical Physics.

Subject of the inquiry: STATISTICAL PROPERTIES OF QUANTUM ENTANGLEMENT AND INFORMATION ENTROPY

Key worlds: entropy, entanglement, atom-field interaction, trapped ions, cold atoms, information entropy

Subject of inquiry: Pure state entanglement, mixed state entanglement, entropy squeezing, mazer

Aim of inquiry: Study of the new entanglement features and new measures for both pure-state and mixed state of particle-field interaction. Also, the impact of the information entropy on the quantum information theory.

Methods of inquiry: Methods of theoretical physics and applied mathematics (statistical physics, quantum information, quantum optics) are used.

Achieved results and their novelty: All the results of the dissertation are new and many new features have been discovered. Particularly: the most general case of the pure state entanglement has been introduced. Although various special aspects of the quantum entropy have been investigated previously, the general features of the dynamics, when a multi-level system and a common environment are considered, have not been treated before and our work therefore filed a gap in the literature. Specifically: 1) A new entanglement measure due to quantum mutual entropy (mixed-state entanglement) we called it DEM, has been introduced, 2) A new treatment of the atomic information entropy in higher level systems has been presented. The problem has been completely solved in the case of three-level system, 3) A new solution of the interaction between the ultra-cold atoms and cavity field has been discovered, 4) Some new models of the atom-filed interaction have been adopted.

The practical value and sphere of usage. The subject carries out theoretic character and its results can be used in quantum computer developments. Also, the presented results can be used for further developments of the quantum information and quantum communications.