

ON THE DIRAC OSCILLATOR

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Abstract

In the present work we obtain a new representation for the Dirac oscillator based on the Clifford algebra Cl_7 . The symmetry breaking and the energy eigenvalues for our model of the Dirac oscillator are studied in the non-relativistic limit.

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I. INTRODUCTION

The relativistic tridimensional isotropic harmonic oscillator has been introduced many years ago by Itô, Mori and Carriere [1], with the Dirac Hamiltonian linear in the position \vec{r} and momentum \vec{p} , with the replacement of \vec{p} by $\vec{p} - im\omega\beta\vec{r}$, where $i = \sqrt{-1}$, m the mass and ω the oscillator frequency. This system is an exactly soluble model which has unusual accidental degeneracies in its spectrum [2].

The system analyzed in [1], was denominated Dirac oscillator by Moshinsky and Szczepaniak [3]. The Dirac oscillator has been investigated in several contexts [4–20]. The Dirac oscillator with a generalized interaction was treated by Castaños *et al.* [9]. Dixit *et al.* [10] have obtained a parity invariant Dirac oscillator with scalar coupling by doubling the number of components and using a representation of the Clifford algebra Cl_7 . These works motivate the construction of a new linear Hamiltonian in terms of the momentum, position and mass coordinates, through a set of seven mutually anticommuting 8×8 -matrices yielding a representation of the Clifford algebra Cl_7 .

In the present work we study a new formulation of the Dirac oscillator using the Clifford algebra Cl_7 which, in the non-relativistic limit leads to the 3D isotropic oscillator with a correction term for both signs of energy. The correction term is different from those in the other formulations and will be interpreted in the following.

II. GENERALIZED DIRAC OSCILLATOR

The Clifford algebra Cl_7 is defined by a set of 7 objects satisfying the anticommutation relations

$$[\Gamma_a, \Gamma_b]_+ = 2\delta_{ab}\mathbf{1}, \quad a, b = 1, 2, \dots, 7. \quad (1)$$

The irreducible representations of Γ_a are provided by the 8×8 matrices given by

$$\vec{\Gamma} = \mathbf{1}_{2 \times 2} \otimes \vec{\alpha}, \quad \Gamma_4 = \mathbf{1}_{2 \times 2} \otimes \beta, \quad \Gamma_{4+i} = \tilde{\Gamma}_i = \rho_i \otimes \gamma_5, \quad (2)$$

where $\rho_i, i = 1, 2, 3$, are a set of the Pauli matrices and

$$\vec{\alpha} = \tau_1 \otimes \vec{\sigma}, \quad (i = 1, 2, 3), \quad \beta = \tau_3 \otimes \mathbf{1}_{2 \times 2}, \quad \gamma_5 = \alpha_1 \alpha_2 \alpha_3 \beta = \tau_2 \otimes \mathbf{1}_{2 \times 2}. \quad (3)$$

Here, ρ_i, τ_i and σ_i are three sets of the Pauli matrices which act in different space. Now, we build the Dirac oscillator Hamiltonian linear in the position \vec{r} , momentum \vec{p} and mass M as:

$$H = c\vec{\Gamma} \cdot \vec{p} + \Gamma_4 M c^2 + cM\omega\vec{\Gamma} \cdot \vec{r}. \quad (4)$$

The above Hamiltonian gives

$$H^2 = c^2\vec{p}^2 + M^2c^4 + c^2M^2\omega^2r^2 - i\hbar c^2M\omega\vec{\Gamma} \cdot \vec{\Gamma}. \quad (5)$$

To interpret the last term in H^2 we analyze the structure of the total angular momentum associated with the Hamiltonian H .

It is easy to verify the commutation relations:

$$[H, L_i]_- = -i\hbar c(\vec{\Gamma} \wedge \vec{p})_i - i\hbar cM\omega(\vec{\Gamma} \wedge \vec{r})_i \quad (6)$$

$$[H, S_i]_- = i\hbar c(\vec{\Gamma} \wedge \vec{p})_i \quad (7)$$

where $\vec{L} = \mathbf{1}_{8 \times 8} \otimes \vec{r} \wedge \vec{p}$ and $\vec{S} = -\frac{i\hbar}{4}\mathbf{1}_{2 \times 2} \otimes (\vec{\alpha} \wedge \vec{\alpha})$.

Thus

$$[H, L_i + S_i]_- = -i\hbar cM\omega(\vec{\Gamma} \wedge \vec{r})_i \neq 0, \quad i = 1, 2, 3. \quad (8)$$

Now we compute the commutator of H with another spin like operator \vec{I} , which we define as being

$$\vec{I} = -i\frac{\hbar}{4}\vec{\Gamma} \wedge \vec{\Gamma} \quad (9)$$

with

$$[I_i, I_j]_- = i\hbar\epsilon_{ijk}I_k, \quad (i, j, k = 1, 2, 3). \quad (10)$$

We obtain

$$[H, I_i]_- = i\hbar c M \omega (\vec{\Gamma} \wedge \vec{r})_i, \quad i = 1, 2, 3. \quad (11)$$

Thus we see that the operator

$$\vec{J} = \vec{L} + \vec{S} + \vec{I} \quad (12)$$

with

$$[J_i, J_j]_- = i\hbar \epsilon_{ijk} J_k \quad i, j, k = 1, 2, 3, \quad (13)$$

satisfies the equation

$$[H, \vec{J}]_- = [H, \vec{L} + \vec{S} + \vec{I}]_- = 0. \quad (14)$$

Thus we may identify \vec{J} as the total conserved angular momentum. When $\omega = 0$, $\vec{J}_D = \vec{L} + \vec{S}$ commutes also with the Hamiltonian in equation (6), i.e.

$$[H(\omega = 0), \vec{L} + \vec{S}]_- = 0. \quad (15)$$

Note that the operator $H(\omega = 0)$ is a direct sum of two Hamiltonians of the free Dirac particle, viz.,

$$H(\omega = 0) = c\vec{\Gamma} \cdot \vec{p} + M c^2 \Gamma_4 = \begin{pmatrix} c\vec{\alpha} \cdot \vec{p} + M c^2 \beta & 0 \\ 0 & c\vec{\alpha} \cdot \vec{p} + M c^2 \beta \end{pmatrix}. \quad (16)$$

In this case with $\omega = 0$, the operator \vec{I} commutes also with $H(\omega = 0)$:

$$[H(\omega = 0), I_i]_- = 0, \quad (i = 1, 2, 3). \quad (17)$$

Therefore, the operators I_i , commuting with $H(\omega = 0)$, generate a global symmetry of $SU(2)$ between the Dirac particles described by the Hamiltonians in the lower and upper sectors. The doublet of fermionic particles described by this Hamiltonian can be labeled by value $1/2$ of the I-spin, and the eigenvalues of $I_3 = \frac{\hbar}{2}$ and $I_3 = -\frac{\hbar}{2}$. From equation (9), we obtain:

$$I^2 = \frac{3}{4} \hbar^2 \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & \mathbf{1}_{4 \times 4} \end{pmatrix}, \quad I_3 = \frac{\hbar}{2} \begin{pmatrix} \mathbf{1}_{4 \times 4} & 0 \\ 0 & -\mathbf{1}_{4 \times 4} \end{pmatrix}. \quad (18)$$

The interaction term in equation (4) is dependent on the operators of the ordinary spin S_i and of the I-spin I_i . Indeed, using the definitions given by equations (3), (6) and (9) we can write the forms of H and H^2 , respectively, as

$$\begin{aligned} H &= \mathbf{1}_{2 \times 2} \otimes (c\vec{\alpha} \cdot \vec{p} + \beta M c^2) + \frac{2}{\hbar} c M \omega \vec{I} \cdot \vec{r} \\ H^2 &= c^2 \vec{p}^2 + M^2 c^4 + c^2 M^2 \omega^2 r^2 + \frac{4}{\hbar} c^2 M \omega \vec{S} \cdot \vec{I}. \end{aligned} \quad (19)$$

At this stage, we can justify that the matrices of I-spin (1/2 in our case), represent an inner symmetry of the doublet of free Dirac particles, given by above expression for the total angular momentum. Indeed, we notice that $\vec{S} + \vec{I}$ is the true total spin of the Dirac oscillator described by the Hamiltonian (19).

Next, we consider the solution of our model of the Dirac oscillator in the non-relativistic limit. If we decompose the eigenfunction Φ of H with eigenvalue E_R in the form

$$\Phi = \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix}, \quad (20)$$

where v_1, w_1, v_2 and w_2 are two-component spinors, the eigenvalue equation

$$H\Phi = \{ \mathbf{1}_{2 \times 2} \otimes (c\vec{\alpha} \cdot \vec{p} + \beta M c^2) + \frac{2}{\hbar} c M \omega (\vec{I} \cdot \vec{r}) \} \Phi = E_R \Phi \quad (21)$$

gives:

$$\begin{aligned} & \begin{pmatrix} c\tau_1 \otimes \vec{\sigma} \cdot \vec{p} + \tau_3 \otimes \mathbf{1}_{2 \times 2} M c^2 & 0 \\ 0 & c\tau_1 \otimes \vec{\sigma} \cdot \vec{p} + \tau_3 \otimes \mathbf{1}_{2 \times 2} M c^2 \end{pmatrix} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix} \\ & + c M \omega (\vec{\rho} \cdot \vec{r}) \otimes \begin{pmatrix} 0 & -i\mathbf{1}_{2 \times 2} \\ i\mathbf{1}_{2 \times 2} & 0 \end{pmatrix} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix} = E_R \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix}, \end{aligned} \quad (22)$$

where

$$\vec{\rho} \cdot \vec{r} = \begin{pmatrix} x_3 & r_- \\ r_+ & -x_3 \end{pmatrix}, \quad r_{\mp} \equiv x_1 \mp ix_2. \quad (23)$$

Thus we get for the spinors v_1, w_1, v_2 and w_2 , the following relations:

$$\begin{aligned} E_R v_1 &= M c^2 v_1 + c \vec{\sigma} \cdot \vec{p} w_1 + c M \omega (-ix_3 w_1 - ir_- w_2), \\ E_R w_1 &= -M c^2 w_1 + c \vec{\sigma} \cdot \vec{p} v_1 + c M \omega (ix_3 v_1 + ir_- v_2), \\ E_R v_2 &= M c^2 v_2 + c \vec{\sigma} \cdot \vec{p} w_2 + c M \omega (-ir_+ w_1 + ix_3 w_2), \\ E_R w_2 &= -M c^2 w_2 + c \vec{\sigma} \cdot \vec{p} v_2 + c M \omega (ir_+ v_1 - ix_3 v_2), \end{aligned} \quad (24)$$

which show that in the non-relativistic limit, for $E_R \rightarrow E'_+ + M c^2$ (positive energy), the components v_1 and v_2 are predominant and $w_1 \rightarrow (v_1/c) \rightarrow 0$ and $w_2 \rightarrow (v_2/c) \rightarrow 0$. On the other hand, for $E_R \rightarrow -E'_- - M c^2$ (negative energy), the components w_1 and w_2 are predominant and $v_1 \rightarrow (w_1/c) \rightarrow 0$ and $v_2 \rightarrow (w_2/c) \rightarrow 0$.

Now, the eigenvalue equation for H^2 can be simplified to give

$$H^2 \Phi = E_R^2 \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix} = \left\{ c^2 p^2 + M^2 c^4 + c^2 M^2 \omega^2 r^2 + M c^2 \hbar \omega \vec{\rho} \otimes \tau_3 \otimes \vec{\sigma} \right\} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix}, \quad (25)$$

or

$$\frac{E_R^2 - M^2 c^4}{2 M c^2} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix} = \left\{ \frac{p^2}{2 M} + \frac{1}{2} M \omega r^2 + \hbar \omega \begin{pmatrix} \frac{1}{2} \sigma_3 & 0 & \sigma_- & 0 \\ 0 & -\frac{1}{2} \sigma_3 & 0 & -\sigma_- \\ \sigma_+ & 0 & -\frac{1}{2} \sigma_3 & 0 \\ 0 & -\sigma_+ & 0 & \frac{1}{2} \sigma_3 \end{pmatrix} \right\} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix}, \quad (26)$$

where

$$\frac{1}{2} \vec{\rho} \otimes \tau_3 \otimes \vec{\sigma} = \begin{pmatrix} \frac{1}{2} \sigma_3 & 0 & \sigma_- & 0 \\ 0 & -\frac{1}{2} \sigma_3 & 0 & -\sigma_- \\ \sigma_+ & 0 & -\frac{1}{2} \sigma_3 & 0 \\ 0 & -\sigma_+ & 0 & \frac{1}{2} \sigma_3 \end{pmatrix}, \quad \sigma_{\pm} \equiv \frac{1}{2} (\sigma_1 \pm i \sigma_2). \quad (27)$$

Observing that equation (26) gives us coupled relations only between (v_1, v_2) or (w_1, w_2) , we have:

$$\begin{aligned} \frac{E_R^2 - M^2 c^4}{2Mc^2} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} &= \left\{ \frac{p^2}{2M} + \frac{1}{2}M\omega r^2 + \hbar\omega \begin{pmatrix} \frac{1}{2}\sigma_3 & \sigma_- \\ \sigma_+ & -\frac{1}{2}\sigma_3 \end{pmatrix} \right\} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \\ \frac{E_R^2 - M^2 c^4}{2Mc^2} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} &= \left\{ \frac{p^2}{2M} + \frac{1}{2}M\omega r^2 - \hbar\omega \begin{pmatrix} \frac{1}{2}\sigma_3 & \sigma_- \\ \sigma_+ & -\frac{1}{2}\sigma_3 \end{pmatrix} \right\} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \end{aligned} \quad (28)$$

where

$$\begin{pmatrix} \frac{1}{2}\sigma_3 & \sigma_- \\ \sigma_+ & -\frac{1}{2}\sigma_3 \end{pmatrix} = \frac{1}{2} \{ \vec{\rho} \otimes \mathbf{1}_{2 \times 2} \} \cdot \{ \mathbf{1}_{2 \times 2} \otimes \sigma \} = \frac{1}{2} \vec{\rho} \otimes \cdot \vec{\sigma}. \quad (29)$$

Next, using the fact that v_1 and v_2 are large compared to w_1 and w_2 in the case $E_R \rightarrow E'_+ + Mc^2 \Rightarrow E_R^2 - M^2 c^4 \rightarrow 2Mc^2 E'_+$, and $E_R \rightarrow -E'_- - Mc^2 \Rightarrow E_R^2 - M^2 c^4 \rightarrow -2Mc^2 E'_-$, equations (28) give us

$$\begin{aligned} E'_+ \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 + \frac{1}{2}\hbar\omega \vec{\rho} \otimes \cdot \vec{\sigma} \right) \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \\ E'_- \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 - \frac{1}{2}\hbar\omega \vec{\rho} \otimes \cdot \vec{\sigma} \right) \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \end{aligned} \quad (30)$$

where E'_\pm are assumed to be small in comparison with Mc^2 . The operator $\vec{\rho} \otimes \cdot \vec{\sigma}$ commutes with all the other terms of these two Hamiltonians, so that we can substitute it by the eigenvalues 1 and -3, when acting on the triplet states, V_T and W_T , and on the singlet states, V_S and W_S , respectively. Hence we get:

$$\begin{aligned} E'_+ V_T &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 + \frac{1}{2}\hbar\omega \right) V_T, & E'_- W_T &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 - \frac{1}{2}\hbar\omega \right) W_T, \\ E'_+ V_S &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 - \frac{3}{2}\hbar\omega \right) V_S, & E'_- W_S &= \left(\frac{p^2}{2M} + \frac{1}{2}M\omega r^2 + \frac{3}{2}\hbar\omega \right) W_S. \end{aligned} \quad (31)$$

The energy spectra of these Hamiltonians are then given by:

$$\begin{aligned} [E'_+]_T^{(n)} &= (n+2)\hbar\omega, & [E'_-]_T^{(n)} &= (n+1)\hbar\omega, & [E'_+]_S^{(n)} &= n\hbar\omega \\ [E'_-]_S^{(n)} &= (n+3)\hbar\omega, & (n=0, 1, 2 \dots), & n &= \ell + 2m, \end{aligned} \quad (32)$$

which reveal an asymmetry between the positive and negative energy spectra. For example, when $[E'_+]_S^{(0)} = 0$, we have $E_R = [E'_+]^{(0)} + Mc^2 = Mc^2$; on the other hand, when $[E'_-]_S^{(0)} = 3\hbar\omega$, $E_R = -Mc^2$ is absent, since $E_R = -[E'_-]_S^{(0)} - Mc^2 = -Mc^2 - 3\hbar\omega$. The interesting question about the existence or not of an interaction that inverts this asymmetry of the spectra derived above, in the non-relativistic limit, can be responded in the affirmative. This interaction corresponds to changing the sign of ω in equations (4) and (5). The nonequivalence of the spectra in these two cases follows from the nonexistence of a unitary transformation satisfying the following conditions: $\Gamma_i \rightarrow \Gamma_i$, $\Gamma_4 \rightarrow \Gamma_4$ and $\tilde{\Gamma}_i \rightarrow -\tilde{\Gamma}_i$, i.e., $\vec{\alpha} \rightarrow \vec{\alpha}$, $\beta \rightarrow \beta$, $\gamma_5 \rightarrow \gamma_5$ and $\vec{\rho} \rightarrow -\vec{\rho}$, since the representations $\vec{\rho}$ and $-\vec{\rho}$ are inequivalent.

III. CONCLUSION

We have found a new representation for a Dirac oscillator via the Clifford algebra Cl_7 . With the introduction of the interaction dependent on the I -spin in (9), for $\omega \neq 0$, the global symmetry $SU(2)$ that exists in the case $\omega = 0$ is broken and the I -spin degrees of freedom convert to the degrees of freedom of spin and orbital angular momentum, according to equation (12).

In the context of a gauge field theory with the local symmetry of $SU(2)$ spontaneously broken, such a phenomenon of convention of the degrees of freedom I -spin to spin [21] occurs. However, the breaking of the global symmetry $SU(2)$, as in our Dirac oscillator model, has not been investigated in the literature. Our Dirac oscillator model is not manifestly covariant, however, it is quantum-mechanically well-defined. Interestingly, the Hamiltonian does not commute with the ordinary angular momentum operator $\vec{J} = \vec{L} + \vec{S}$, but a new I -spin must be added.

The symmetry breaking brought out here for the Dirac oscillator was studied in the non-relativistic limit when the additional constraint provided by the Dirac equation is fully implemented.

The formulation of the Dirac oscillator Hamiltonian [3] in terms of the Wigner ladder operators [22] permits a purely algebraic treatment for the relativistic problem [23], the details of which will be published separately.

Let us conclude with a discussion on the relationship between the new Dirac oscillator and other proposals including the 4×4 oscillator with vector coupling and the 8×8 oscillator scalar coupling. The usual Dirac oscillator Hamiltonian in the non-relativistic limit leads to that of a 3-dimensional isotropic oscillator shifted by a constant term plus a $\vec{L} \cdot \vec{S}$ coupling term for both signs of energy. In another work, Dixit *et al.* [10] have considered the Dirac oscillator with scalar coupling which in the non-relativistic limit leads to a harmonic oscillator Hamiltonian plus a $\vec{\sigma} \cdot \hat{r}$ coupling term, where $\hat{r} = \frac{\vec{r}}{r}$. In the new Dirac oscillator presented in this paper the correction term is different. Indeed, our Dirac oscillator Hamiltonian in the non-relativistic limit leads to that of a 3-dimensional isotropic oscillator plus a $\vec{\rho} \otimes \cdot \vec{\sigma}$ coupling term.

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References

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- [1] D. Itô, K. Mori and E. Carriere, *Nuovo Cimento* **A51**, 1119 (1967).
 - [2] P. A. Cook, *Lett. Nuovo Cimento* **1**, 419 (1971).
 - [3] M. Moshinsky and A. Szczepaniak, *J. Phys. A: Math. Gen.* **22**, L817 (1989).
 - [4] M. Moreno and A. Zentella, *J. Phys. A: Math. Gen.* **22**, L821 (1989).
 - [5] J. Beckers and N. Debergh, *Phys. Rev.* **D42**, 1255 (1990).
 - [6] C. Quesne and M. Moshinsky, *J. Phys. A: Math. Gen.* **23**, 2263 (1990).
 - [7] J. Benítez, R. P. Martínez y Romero, H. N. Núñez-Yépez and A. L. Salas-Brito, *Phys. Rev. Lett.* **65**, 2085 (1990); R. P. Martínez y Romero, Matías Moreno and A. Zentella, *Phys. Rev.* **D43**, 2036 (1991).
 - [8] O. L. de Lange, *J. Phys. A: Math. Gen.* **24**, 667 (1991); O. L. de Lange and R. E. Raab, *J. Math. Phys.* **32**, 1296 (1991).

- [9] O. Castaños, A. Frank, R. López and L. F. Urrutia, *Phys. Rev.* **D43**, 544 (1991).
- [10] V. V. Dixit, T. S. Santhanam and W. D. Thacker, *J. Math. Phys.* **33**, 1114 (1992).
- [11] V. Villalba, *Phys. Rev.* **A49**, 586 (1994).
- [12] R. P. Martinez-y-Romero, H. N. Núñez-Yépez and A. L. Salas-Brito, *Eur. J. Phys.* **16**, 135 (1995).
- [13] R. Szymtkowski and M. Gruchowski, *J. Phys. A: Math. Gen.* **34**, 4991 (2001).
- [14] M. H. Pacheco, R. R. Landim and C. A. S. de Almeida, *Phys. Lett.* **A311**, 93 (2003).
- [15] A. D. Alhaidari, *Int. J. Theor. Phys.* **43**, 939 (2004).
- [16] J. N. Ginocchio, *Phys. Rev.* **C69**, 034318 (2004).
- [17] R. Lisboa, M. Malheiro, A. S. de Castro, P. Alberto and M. Fiolhais, *Phys. Rev.* **C69**, 024319 (2004).
- [18] C. Quesne and V. M. Tkachuk, *J. Phys. A: Math. Gen.* **38**, 1747 (2005).
- [19] O. Mustafa, *Energy-levels crossing and radial Dirac equation: supersymmetry and quasi-parity spectral signatures*, quant-ph/0703078.
- [20] R. de Lima Rodrigues, J. Jayaraman and A. N. Vaidya, *Nonrelativistic limit of the generalized Dirac oscillator*, paper presented in the *XXIV Encontro de Físicos do Norte e Nordeste*, João Pessoa (2006), <http://www.sbf1.sbfisica.org.br/eventos/efnne/xxiv/sys/resumos/R0771-3.pdf>.
- [21] R. Jackiw and C. Rebbi, *Phys. Rev. Lett.* **36**, 1116 (1976); P. Hasenfratz and G. t'Hooft, *Phys. Rev. Lett.* **36**, 1119 (1976).
- [22] J. Jayaraman and R. de Lima Rodrigues, *J. Phys. A: Math. Gen.* **23**, 3123 (1990).
- [23] R. de Lima Rodrigues and A. N. Vaidya, *Dirac Oscillator via R-deformed Heisenberg algebra* preprint CBPF-NF-030/02 December 2002, hep-th/0301093